A Progress Report on a Least Squares Regression Mineral Magnetic Modelling Programme

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INTRODUCTION

A computer printout from the least squares regression development programme PEMM8 is shown in exhibits 1 to 10.

The notes below discuss some points of interest in the printout. They are designed to be referred to while the printout itself is being read.

Three examples are investigated in the course of the computer listing

(i) a gneiss bedrock sample
(ii) a stream bedload sample and
(iii) the effect of changing proportions of multi-domain to single-domain magnetite on the SIRM / X ratio.

Turn now to the printout and consult notes 1 to 28 as and when indicated by the circled numbers in the listing.

NOTES

1. Draper and Smith (1966) write the following about predictive models in their chapter on 'Multiple Regression and Mathematical Models' in their very readable textbook 'Applied Regression Analysis'.

> 'When the function model is very complex and when the ability to obtain independent estimates of the effects of the control variables is limited, one can often obtain a linear predictive model which, though it may be in some senses unrealistic, at least reproduces the main features of the behaviour of the response under study. These predictive models are very useful and under certain conditions can lead to real insight into the process or problem. It is in the construction of this type of predictive model that multiple regression techniques have their greatest contribution to make. These problems are usually referred to as "problems with messy data"--that is, data in which much intercorrelation exists. ..... If nothing else ‹a predictive model› provides guidelines for further experimentation, it pinpoints important variables, and it is a very useful variable screening device. It is necessary , however, to be very careful in using multiple regression, for it is easily misused and misunderstood.'

Bearing these pertinent comments on model building in mind, a multivariate regression programme is being developed. It attempts to explain mineral magnetic measurements in terms of mixtures of natural magnetic minerals. The present version of the programme, called PEMM8, accepts up to eleven magnetic measurements and eight mineral components. It has been writen in such a way as to be extendable to include additional observations or additional components relatively easily.

2. The programme developed during the late Summer and Autumn of 1987 runs on an IBM compatible PC in an interactive manner. Any uncertainty about a question posed by the programme can be dealt with by entering the number 0.

3. The programme begins by asking for the sample name followed by the weight of the sample in gms.

4. Eleven observations can now be entered. The particular measurements used here were selected simply in order to cover the range of instrumentation available in the Lund laboratory and for their likely value in discriminating between certain mineral magnetic components.

5. Susceptibility after oxidation, a very useful parameter for detecting greigite (Snowball and Thompson, 1988), is not appropriate for the gneiss sample under consideration. So after a zero has been entered the programme ignores this parameter.

((Note. Any other missing measurements can be similarly removed from the modelling process by entering a zero.))

6. Following the entry of ten observations, in this case, two initial susceptibility and eight forward isothermal remanence measurements, the programme immediately carries out a diamagnetic correction based on the weight of the sample. Then it announces the total number of measurements to be used in all subsequent calculations.

7. The main branching point in the programme is now reached. Five options are presented. We choose the main route of option number 3, that is of setting up a model. The other

four options are all dealt with below.

8. We are asked which of eight possible mineral magnetic components we would like to include in the model. As this gneiss bedrock sample demonstrates no frequency dependence and as it is extremely unlikely to contain greigite we reject viscous magnetite and greigite. However we accept the single-domain, multi-domain and pseudosingle-domain magnetite, single-domain and multi-domain haematite and (super)paramagnetic components.

((Note. The measurements included in PEMM8 do not enable the programme to distinguish a paramagnetic contribution from a superparamagnetic contribution. Low temperature measurements would need to be included for that distinction. Furthermore results expressed, as here, in terms of an SPM magnetite component have to be multiplied by several hundred to a thousand for a paramagnetic component such as olivine or chamosite.))

9. The programme announces that it will model the ten observations in terms of six variables. The heart of the programme now comes into operation.

Following the terminology of Draper and Smith (1966) the model is set down in the form of the matrix equation

$$Y = X\beta + \varepsilon$$

(The equivalent equation $Ax=b$ was used in Thompson,1986)

where, in this first Troll Hill gneiss example, Y is a 10 by 1 column vector consisting of our 10 magnetic observations, and X is a 10 by 6 array of 'known' independent variables provided automatically by the programme. (These 'known' parameters (of for example the low frequency susceptibility of single-domain magnetite or the IRM of multi-domain magnetite grown in a 60 mT field) are taken mainly from Table 1 of Thompson,1986 and Tables 3.4 and 4.2 of Thompson and Oldfield,1986.) β is a 6 by 1 vector of parameters to be estimated (our six mineral components) and ε is a 10 by 1 vector of errors. In deriving a least squares solution it is assumed that the errors have zero mean, are uncorrelated and are normally distributed. In practice, of course, this is by no means the case, so some manipulations of the matrix equation have to be carried out in an attempt to alleviate the situation. Nevertheless, continuing to follow Draper and Smith's matrix approach to linear regression, we find it convenient to express the problem in terms of the normal equations as

$$X'Xb = X'Y$$

where X' is the transpose of X and b provides the least squares estimates of β. All that remains is to solve the normal equations in matrix form. This is achieved by premultiplying both sides of the above equation by $(X'X)^{-1}$ where $^{-1}$ signifies the inverse matrix and hence we obtain the important

$$b = (X'X)^{-1}X'Y$$

since $(X'X)^{-1}X'X = I$. Where I is the unit matrix. The significant point about this equation is that a solution can always be obtained, provided that $X'X$ is nonsingular. Bjorn Holmquist most kindly explained how to use the wonderful subroutine HDIAG, which calculates eigenvalues of a symmetric matrix, to carry out the above calculations at a time when the first version of this programme was failing miserably.

Having found b, we can then calculate the fitted values $\hat{Y}$ very simply by evaluating $Y = Xb$, and we are almost through with the matrix algebra.

10. The ten magnetic observations (after diamagnetic correction of the susceptibilities) and their fitted values are listed out along with their errors, $Y_i - \hat{Y}$, in the middle three columns. Their associated percentage errors $100 * (Y_i - \hat{Y}) / Y_i$ are listed out in the final column. In fact because of the matrix manipulations alluded to above the whole programme is based on the percentage errors.

11. Two measures of the overall 'fit' of the model are listed. First we have the correlation coefficient R^2, which in this case is 0.999 . Basically this is the sum of squares due to the regression model divided by the total sum of squares of the observations. A coefficient of 1.000 represents a perfect fit. Secondly we have the root mean square (percentage) error which divides the mean square error by the degrees of freedom. Here it is 3.65 . We will later use this second measure, the root mean square error, when 'searching' for the 'best' regression equation.

((Note. We take the degrees of freedom to equal the number of magnetic observations (M) minus the number of components (N) i.e. M-N rather than M-N-1 which is the number normally encountered in regression work. This is because in our magnetic models we are not dealing with regression about a mean.))

12. The least squares estimates are now written out in parts per thousand. Having performed the analysis in matrix terms we can obtain errors for the estimates, with little extra work, from the diagonal terms of the variance-covariance matrix of the vector b, namely $(X'X)^{-1}$.

To calculate 95% confidence limits we need the percentage points of a t-distribution with the appropriate degrees of freedom. These percentage points, however, are easily included in the programme and the resulting confidence limits are also listed out. All the calculations of this first modelling attempt are now over.

13. This brings us back to the main branching point of the programme. Inspecting the six modelled variables (of note 12), their error estimates and the ten fitted values (of note 10) we see how a good fit has been found but that some of the mineral magnetic estimates are negative.

Mathematically these negative results are quite acceptable and this is how the generally excellent fit is produced, but physically it is, of course, totally wrong. Consequently we still have a little room for improvement. We can try to produce a more acceptable model either through the automatic search algorithm, option number 4, or through the simplification procedure, option number 2. We select option 2.

14. The programme now asks us which of the six components we wish to retain. We choose to remove the pseudosingle-domain magnetite component which was negative in our first model with a value of -3.494 +/- 2.361 . We retain the other five components including the multi-domain haematite, which could have been positive, within its extremely wide confidence limits. A second model based on the same original ten observations is now produced and listed out.

15. As a result of removing the negative pseudosingle-domain magnetite the five component model now has a negative single-domain magnetite contribution. We again simplify by removing this negative component from our model through the use of option number 2.

16. The mathematical fit is naturally getting worse as we continue to remove these negative components, while the model is becoming physically more realistic. The haematite contribution, however, still looks equivocal with its large errors and opposite signs. Perhaps it is time to resort to the search algorithm option number 4.

17. We hit key 4 and initiate the search procedure. The programme begins an exhaustive search of the remaining four control variables. It looks for the 'best possible' regression model based on minimizing the root mean square error subject to the constraint that all magnetic components must be non-negative. As part of the search the programme has to evaluate and test 2 to the power N regression equations where N is the number of model components. In this case, because of our preselections, N is just 4, The programme sorts through the appropriate 16 models and selects the best one. This all takes around one second on an IBM-PC-AT with 640K RAM memory and a 80287 coprocessor chip.

((Note. A non ANSI FORTRAN bit testing procedure had to be included in this search algorithm - I could not see a way of coding it efficiently in standard FORTRAN.))

18. The search algorithm has reduced the root mean square error from 10.64 to 9.39, and found a solution with positive values and just two components. It combines 6 parts per thousand (0.6%) MD magnetite by weight with a paramagnetic contribution in its best fit. Furthermore the two selected components have values very similar to those they carried in the previous four component model. Examining the fitted values, we find a tolerable fit in all measurements, the largest discrepancy occurring in the forward IRM measurement at a 20 milli Tesla field.

19. We might like to investigate this model further, for example by considering how good a one component model might be. This can be achieved by again branching to option number 2 in the main menu and removing the (super)paramagnetic component.

20. We see how this single component model fails to fit the susceptibility observations, so justifying the search algorithm's choice of a two component model. Satisfied, for the moment, with the above two component model, reached at note 18, we move on to a second sample and our second example by selecting option number one.

21. The programme reminds us again that it is experimental and that we are just involved in some preliminary exploratory data analysis. It then asks for the new sample name, weight and magnetic observations. We enter the name, weight and the same ten magnetic measurements but this time for our stream bedload sample.

22. We rather lazily include all eight possible components.

23. Following the production of the eight component model we immediately hit the search key and wait some 50 seconds for an answer to appear.

24. This time we are presented with a four component model and a very good fit. Haematite is revealed as being by far the dominant magnetic mineral.

((Note. This 'quick fix' route, of immediately including all eight components in option 3 followed by option 4, will, as a matter of interest, yield the identical two component model for the gneiss sample in example one that we arrived at in note 18.))

25. A third and final example illustrates how the programme can also be used for data transformation. We investigate the effect of changing SD to MD magnetite contributions on SIRM / X ratios. Here we enter just two magnetic observations and request a model with two components. This leaves us with no degrees of freedom and naturally a perfect fit. In this final short section of printout we see how an SIRM / X ratio of 5 kAm^{-1} can be accounted for by an SD/MD magnetite ratio of 0.057 : 1.848, whereas a SIRM / X ratio of 80 kAm^{-1} is accounted for by a SD/MD ratio of 2.622 : 0.008

26. The final option number 5, quit, stops the programme and returns us to MS-DOS command level.

27. Again many thanks to Bjorn Holmquist for patiently explaining details of the matrix approach to solving simultaneous linear equations and multiple regression problems.

28. Any comments ?

REFERENCES

Draper, N.R. and Smith, H., 1966. Applied Regression Analysis. J. Wiley : New York.

Snowball, I. and Thompson, R., 1988. The occurrence of greigite in sediments of Loch Lomond. (Under Review).

Thompson, R., 1986. Modelling magnetization data using SIMPLEX. Phys. Earth Planet. Ints. , 42, 113-127.

Thompson, R. and Oldfield, F., 1986. Environmental Magnetism. Allen and Unwin : London.

Exhibit 1

```
D:\ROY -> MODEL

D:\ROY -> echo off


    VERY PRELIMINARY 8 COMPONENT ENVIRONMENTAL                 ①
          MAGNETIC MODEL   ( PEMM8 )
          VERSION 2.7



    ****************************************
    *                                      *
    *  IF IN DOUBT ABOUT ANY  MEASUREMENTS  *                  ②
    *  EXCEPT  FOR Xlf THEN JUST  ENTER  0  *
    *                                      *
    ****************************************



  ENTER NEW SAMPLE NAME : TROLL HILL GNEISS                    ③

  ENTER WEIGHT (IN gms) : 7.98

  ENTER TOTAL LOW FREQUENCY SUSCEPTIBLIITY : 70                ④

  ENTER TOTAL HIGH FREQUENCY SUSCEPTIBILITY : 70

  ENTER SUSCEPTIBILITY AFTER OXIDATION : 0                     ⑤

**** NO MEASUREMENT ASSUMED ****

  ENTER FORWARD IRM AT  20mT FIELD (Am2) : 48.7

  ENTER FORWARD IRM AT  40mT FIELD (Am2) : 68.7

  ENTER FORWARD IRM AT  60mT FIELD (Am2) : 76.3

  ENTER FORWARD IRM AT  80mT FIELD (Am2) : 81.4

  ENTER FORWARD IRM AT 100mT FIELD (Am2) : 84.6

  ENTER FORWARD IRM AT 200mT FIELD (Am2) : 93.6

  ENTER FORWARD IRM AT 300mT FIELD (Am2) : 94.1

  ENTER FORWARD IRM AT    1T FIELD (Am2) : 94.5
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             DIAMAGNETIC CORRECTION =     0.0114 %          ⑥

        10 MEASUREMENTS TO BE INCLUDED IN CALCULATION

Menu ¦ NEW SAMPLE=1  SIMPLIFY MODEL=2  MODEL=3  SEARCH=4  QUIT=5   : 3   ⑦
    INCLUDE SD MAGNETITE ?    YES = 0  ;  NO = 1  :0                     ⑧
                  ++++ SD MAGNETITE INCLUDED IN MODEL ++++

    INCLUDE MD MAGNETITE ?    YES = 0  ;  NO = 1  :0

                  ++++ MD MAGNETITE INCLUDED IN MODEL ++++

    INCLUDE PSD MAGN     ?    YES = 0  ;  NO = 1  :0

                  ++++ PSD MAGN     INCLUDED IN MODEL ++++

    INCLUDE VISCOUS MAG  ?    YES = 0  ;  NO = 1  :1

    INCLUDE GREIGITE     ?    YES = 0  ;  NO = 1  :1

    INCLUDE SD HAEMATITE ?    YES = 0  ;  NO = 1  :0

                  ++++ SD HAEMATITE INCLUDED IN MODEL ++++

    INCLUDE MD HAMATITE  ?    YES = 0  ;  NO = 1  :0

                  ++++ MD HAMATITE  INCLUDED IN MODEL ++++

    INCLUDE (SUPER)PARAM ?    YES = 0  ;  NO = 1  :0

                  ++++ (SUPER)PARAM INCLUDED IN MODEL ++++

           SAMPLE : TROLL HILL GNEISS    WEIGHT (gms)     7.980

      6 COMPONENTS INCLUDED IN PEEM8 MODEL BASED ON 10 MEASUREMENTS     ⑨
```

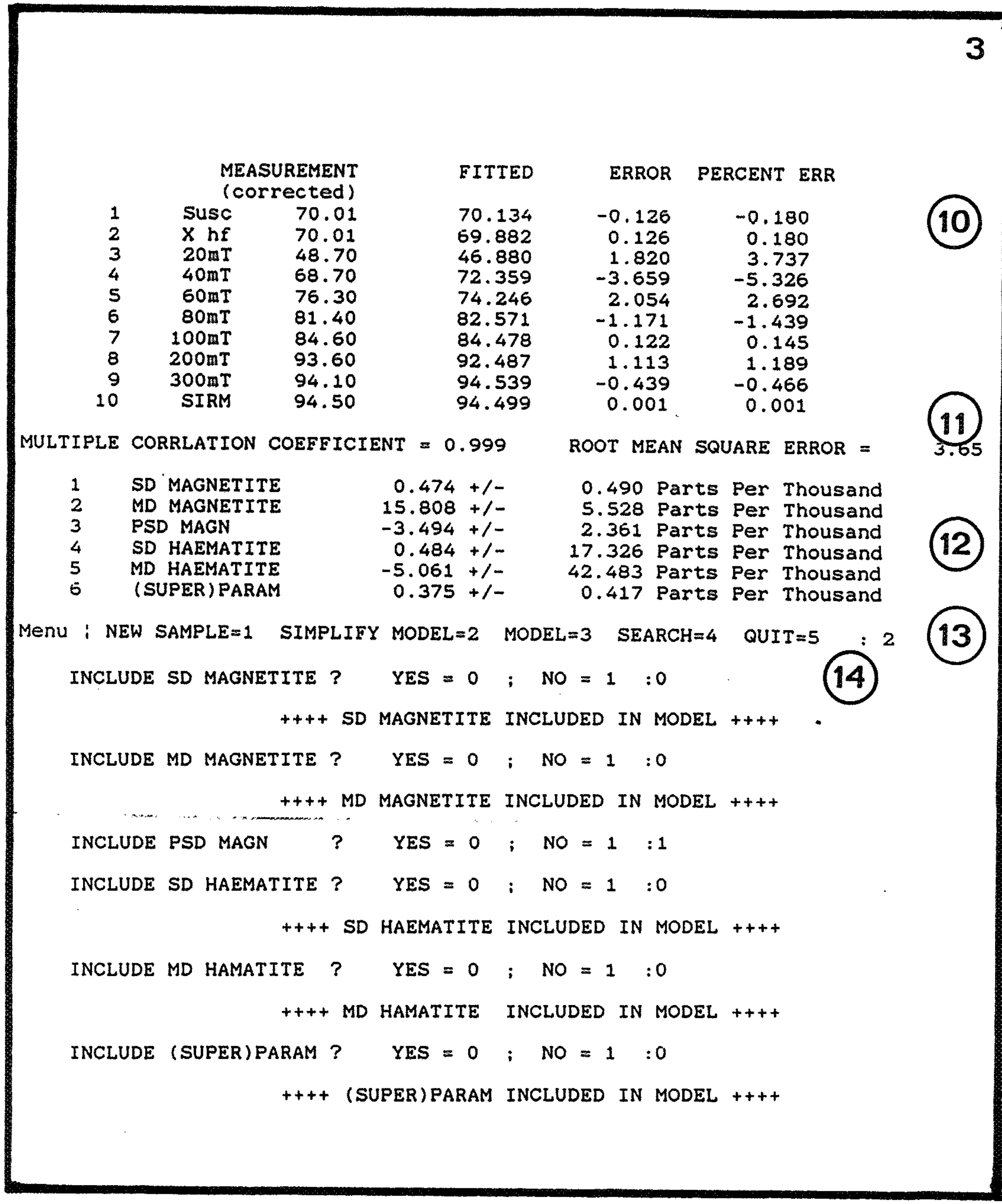

```
          MEASUREMENT          FITTED      ERROR   PERCENT ERR
          (corrected)
  1    Susc      70.01          70.134     -0.126     -0.180
  2    X hf      70.01          69.882      0.126      0.180
  3    20mT      48.70          46.880      1.820      3.737
  4    40mT      68.70          72.359     -3.659     -5.326
  5    60mT      76.30          74.246      2.054      2.692
  6    80mT      81.40          82.571     -1.171     -1.439
  7   100mT      84.60          84.478      0.122      0.145
  8   200mT      93.60          92.487      1.113      1.189
  9   300mT      94.10          94.539     -0.439     -0.466
 10    SIRM      94.50          94.499      0.001      0.001
MULTIPLE CORRLATION COEFFICIENT = 0.999    ROOT MEAN SQUARE ERROR =     3.65

   1     SD MAGNETITE          0.474 +/-     0.490 Parts Per Thousand
   2     MD MAGNETITE         15.808 +/-     5.528 Parts Per Thousand
   3     PSD MAGN             -3.494 +/-     2.361 Parts Per Thousand
   4     SD HAEMATITE          0.484 +/-    17.326 Parts Per Thousand
   5     MD HAEMATITE         -5.061 +/-    42.483 Parts Per Thousand
   6     (SUPER)PARAM          0.375 +/-     0.417 Parts Per Thousand
Menu ¦ NEW SAMPLE=1  SIMPLIFY MODEL=2  MODEL=3  SEARCH=4  QUIT=5   : 2
    INCLUDE SD MAGNETITE ?     YES = 0  ;  NO = 1  :0

                 ++++ SD MAGNETITE INCLUDED IN MODEL ++++

    INCLUDE MD MAGNETITE ?     YES = 0  ;  NO = 1  :0

                 ++++ MD MAGNETITE INCLUDED IN MODEL ++++

    INCLUDE PSD MAGN     ?     YES = 0  ;  NO = 1  :1

    INCLUDE SD HAEMATITE ?     YES = 0  ;  NO = 1  :0

                 ++++ SD HAEMATITE INCLUDED IN MODEL ++++

    INCLUDE MD HAMATITE  ?     YES = 0  ;  NO = 1  :0

                 ++++ MD HAMATITE  INCLUDED IN MODEL ++++

    INCLUDE (SUPER)PARAM ?     YES = 0  ;  NO = 1  :0

                 ++++ (SUPER)PARAM INCLUDED IN MODEL ++++
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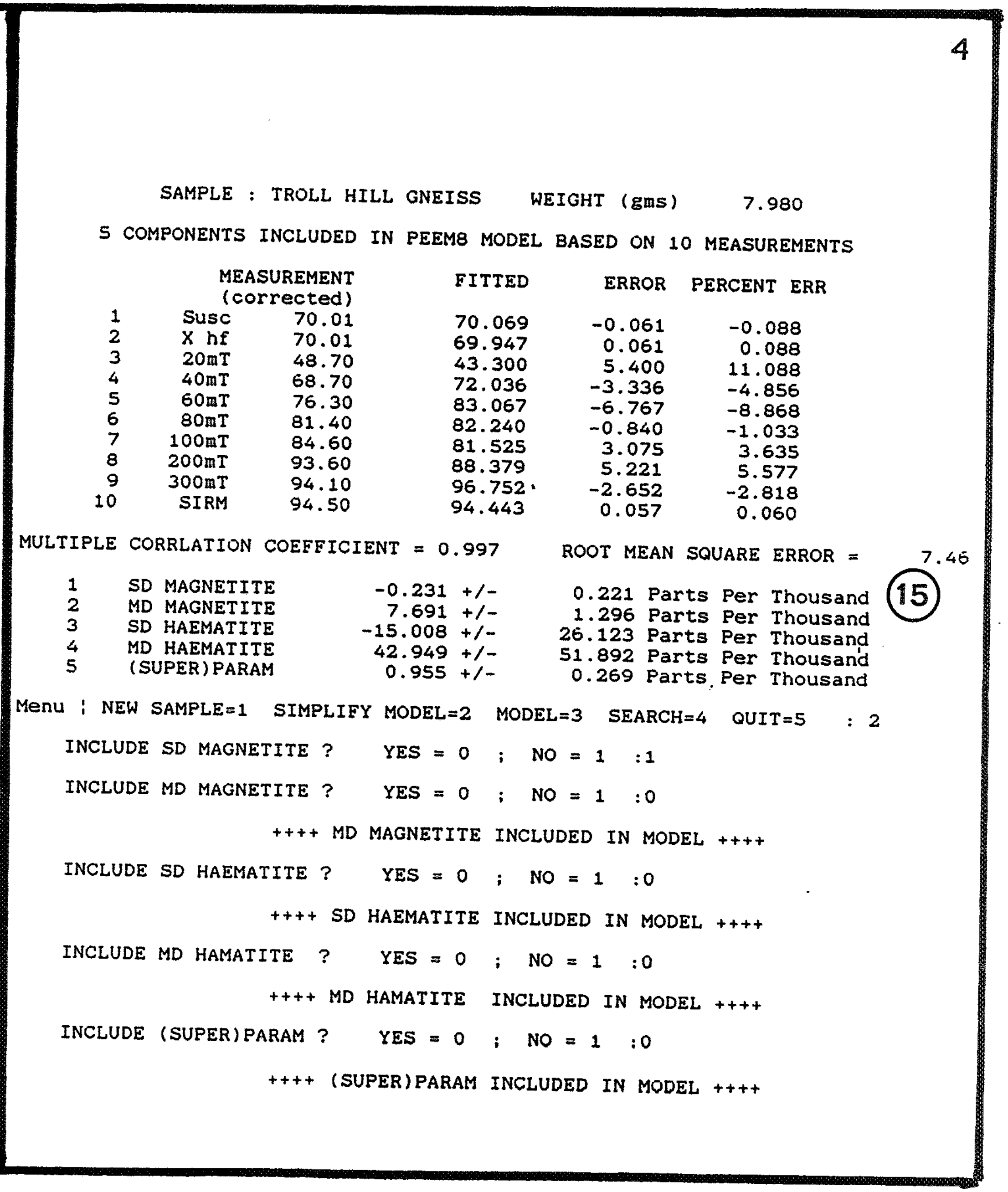

```
      SAMPLE : TROLL HILL GNEISS     WEIGHT (gms)     7.980

   5 COMPONENTS INCLUDED IN PEEM8 MODEL BASED ON 10 MEASUREMENTS

           MEASUREMENT          FITTED     ERROR   PERCENT ERR
           (corrected)
  1    Susc      70.01          70.069    -0.061     -0.088
  2    X hf      70.01          69.947     0.061      0.088
  3    20mT      48.70          43.300     5.400     11.088
  4    40mT      68.70          72.036    -3.336     -4.856
  5    60mT      76.30          83.067    -6.767     -8.868
  6    80mT      81.40          82.240    -0.840     -1.033
  7   100mT      84.60          81.525     3.075      3.635
  8   200mT      93.60          88.379     5.221      5.577
  9   300mT      94.10          96.752    -2.652     -2.818
 10    SIRM      94.50          94.443     0.057      0.060
MULTIPLE CORRLATION COEFFICIENT = 0.997    ROOT MEAN SQUARE ERROR =     7.46

   1     SD MAGNETITE         -0.231 +/-     0.221 Parts Per Thousand
   2     MD MAGNETITE          7.691 +/-     1.296 Parts Per Thousand
   3     SD HAEMATITE        -15.008 +/-    26.123 Parts Per Thousand
   4     MD HAEMATITE         42.949 +/-    51.892 Parts Per Thousand
   5     (SUPER)PARAM          0.955 +/-     0.269 Parts Per Thousand
Menu ¦ NEW SAMPLE=1  SIMPLIFY MODEL=2  MODEL=3  SEARCH=4  QUIT=5   : 2
    INCLUDE SD MAGNETITE ?     YES = 0  ;  NO = 1  :1

    INCLUDE MD MAGNETITE ?     YES = 0  ;  NO = 1  :0

                 ++++ MD MAGNETITE INCLUDED IN MODEL ++++

    INCLUDE SD HAEMATITE ?     YES = 0  ;  NO = 1  :0

                 ++++ SD HAEMATITE INCLUDED IN MODEL ++++

    INCLUDE MD HAMATITE  ?     YES = 0  ;  NO = 1  :0

                 ++++ MD HAMATITE  INCLUDED IN MODEL ++++

    INCLUDE (SUPER)PARAM ?     YES = 0  ;  NO = 1  :0

                 ++++ (SUPER)PARAM INCLUDED IN MODEL ++++
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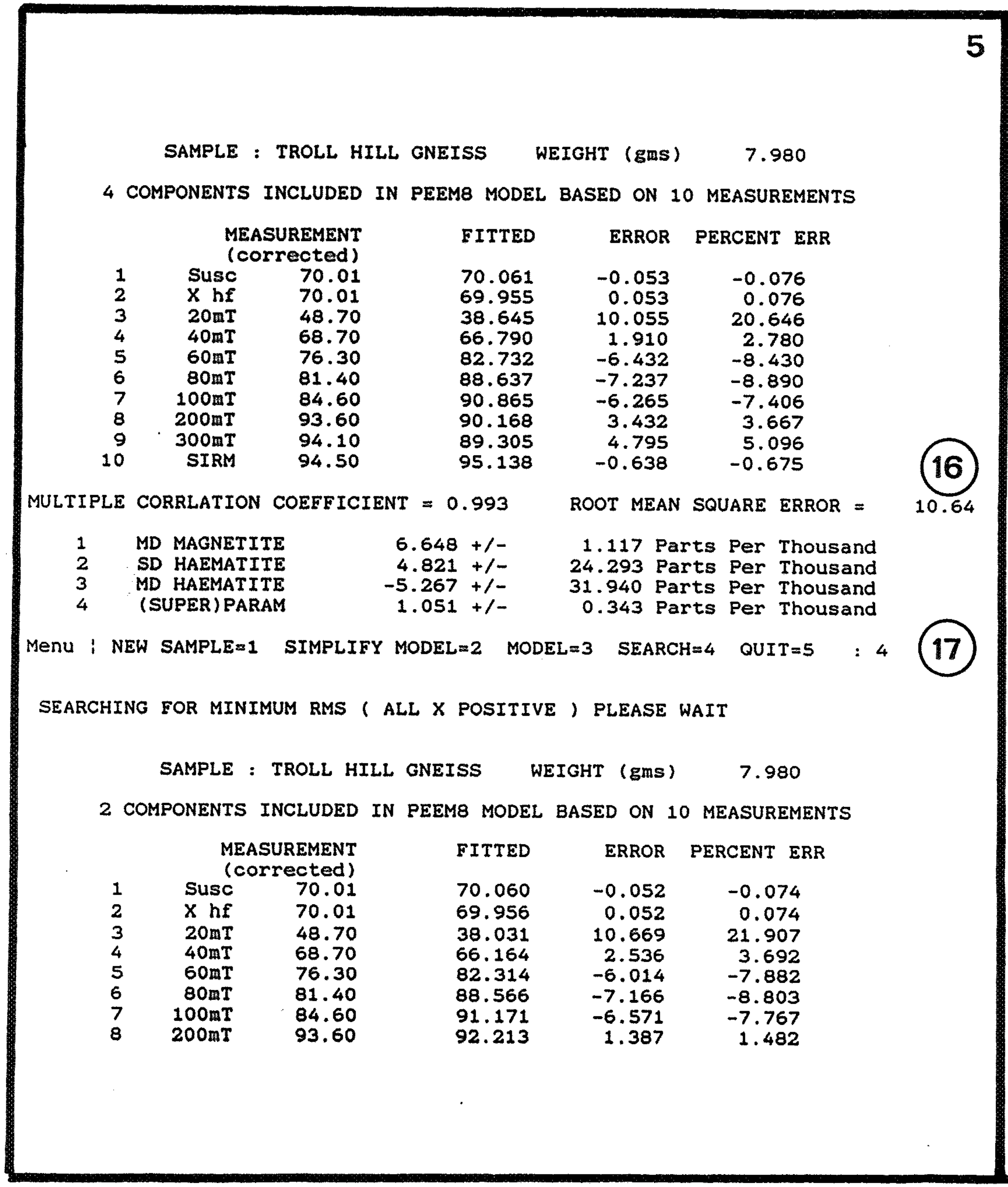

```
      SAMPLE  :  TROLL HILL GNEISS     WEIGHT (gms)      7.980

   4 COMPONENTS INCLUDED IN PEEM8 MODEL BASED ON 10 MEASUREMENTS

                 MEASUREMENT            FITTED       ERROR   PERCENT ERR
                  (corrected)
      1      Susc       70.01           70.061      -0.053      -0.076
      2      X hf       70.01           69.955       0.053       0.076
      3      20mT       48.70           38.645      10.055      20.646
      4      40mT       68.70           66.790       1.910       2.780
      5      60mT       76.30           82.732      -6.432      -8.430
      6      80mT       81.40           88.637      -7.237      -8.890
      7     100mT       84.60           90.865      -6.265      -7.406
      8     200mT       93.60           90.168       3.432       3.667
      9     300mT       94.10           89.305       4.795       5.096
     10      SIRM       94.50           95.138      -0.638      -0.675        ⑯
MULTIPLE CORRLATION COEFFICIENT = 0.993     ROOT MEAN SQUARE ERROR =        10.64

    1     MD MAGNETITE            6.648 +/-      1.117 Parts Per Thousand
    2     SD HAEMATITE            4.821 +/-     24.293 Parts Per Thousand
    3     MD HAEMATITE           -5.267 +/-     31.940 Parts Per Thousand
    4     (SUPER)PARAM            1.051 +/-      0.343 Parts Per Thousand
Menu ¦ NEW SAMPLE=1   SIMPLIFY MODEL=2   MODEL=3   SEARCH=4   QUIT=5    : 4   ⑰

 SEARCHING FOR MINIMUM RMS ( ALL X POSITIVE ) PLEASE WAIT


      SAMPLE  :  TROLL HILL GNEISS     WEIGHT (gms)      7.980

   2 COMPONENTS INCLUDED IN PEEM8 MODEL BASED ON 10 MEASUREMENTS

                 MEASUREMENT            FITTED       ERROR   PERCENT ERR
                  (corrected)
      1      Susc       70.01           70.060      -0.052      -0.074
      2      X hf       70.01           69.956       0.052       0.074
      3      20mT       48.70           38.031      10.669      21.907
      4      40mT       68.70           66.164       2.536       3.692
      5      60mT       76.30           82.314      -6.014      -7.882
      6      80mT       81.40           88.566      -7.166      -8.803
      7     100mT       84.60           91.171      -6.571      -7.767
      8     200mT       93.60           92.213       1.387       1.482
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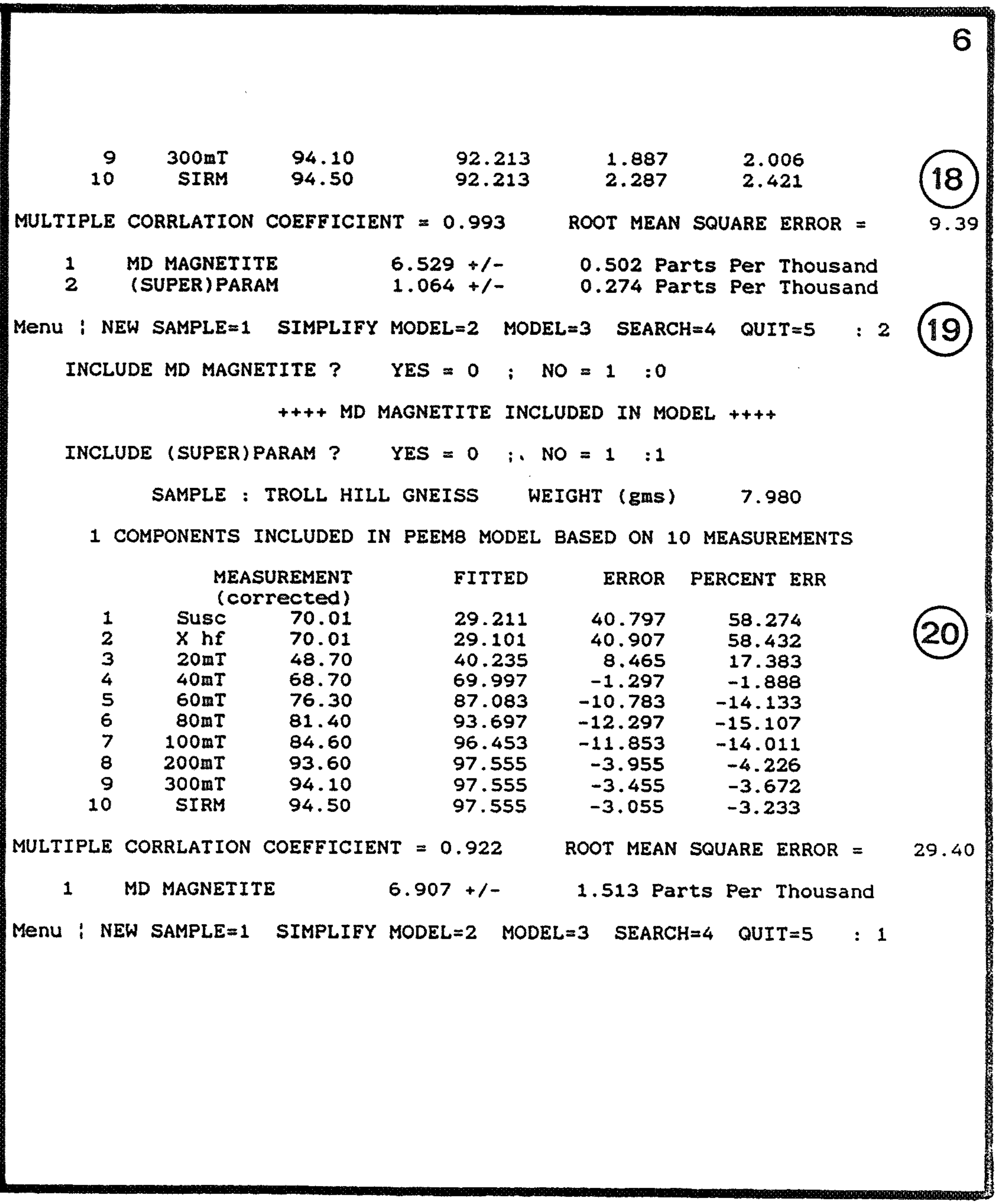

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      9     300mT       94.10           92.213       1.887       2.006
     10      SIRM       94.50           92.213       2.287       2.421        ⑱
MULTIPLE CORRLATION COEFFICIENT = 0.993     ROOT MEAN SQUARE ERROR =         9.39

    1     MD MAGNETITE            6.529 +/-      0.502 Parts Per Thousand
    2     (SUPER)PARAM            1.064 +/-      0.274 Parts Per Thousand

Menu ¦ NEW SAMPLE=1   SIMPLIFY MODEL=2   MODEL=3   SEARCH=4   QUIT=5    : 2   ⑲

    INCLUDE MD MAGNETITE ?      YES = 0   ;   NO = 1   :0

                  ++++ MD MAGNETITE INCLUDED IN MODEL ++++

    INCLUDE (SUPER)PARAM ?      YES = 0   ;.  NO = 1   :1

      SAMPLE  :  TROLL HILL GNEISS     WEIGHT (gms)      7.980

   1 COMPONENTS INCLUDED IN PEEM8 MODEL BASED ON 10 MEASUREMENTS

                 MEASUREMENT            FITTED       ERROR   PERCENT ERR
                  (corrected)
      1      Susc       70.01           29.211      40.797      58.274
      2      X hf       70.01           29.101      40.907      58.432        ⑳
      3      20mT       48.70           40.235       8.465      17.383
      4      40mT       68.70           69.997      -1.297      -1.888
      5      60mT       76.30           87.083     -10.783     -14.133
      6      80mT       81.40           93.697     -12.297     -15.107
      7     100mT       84.60           96.453     -11.853     -14.011
      8     200mT       93.60           97.555      -3.955      -4.226
      9     300mT       94.10           97.555      -3.455      -3.672
     10      SIRM       94.50           97.555      -3.055      -3.233

MULTIPLE CORRLATION COEFFICIENT = 0.922     ROOT MEAN SQUARE ERROR =        29.40

    1     MD MAGNETITE            6.907 +/-      1.513 Parts Per Thousand

Menu ¦ NEW SAMPLE=1   SIMPLIFY MODEL=2   MODEL=3   SEARCH=4   QUIT=5    : 1
```

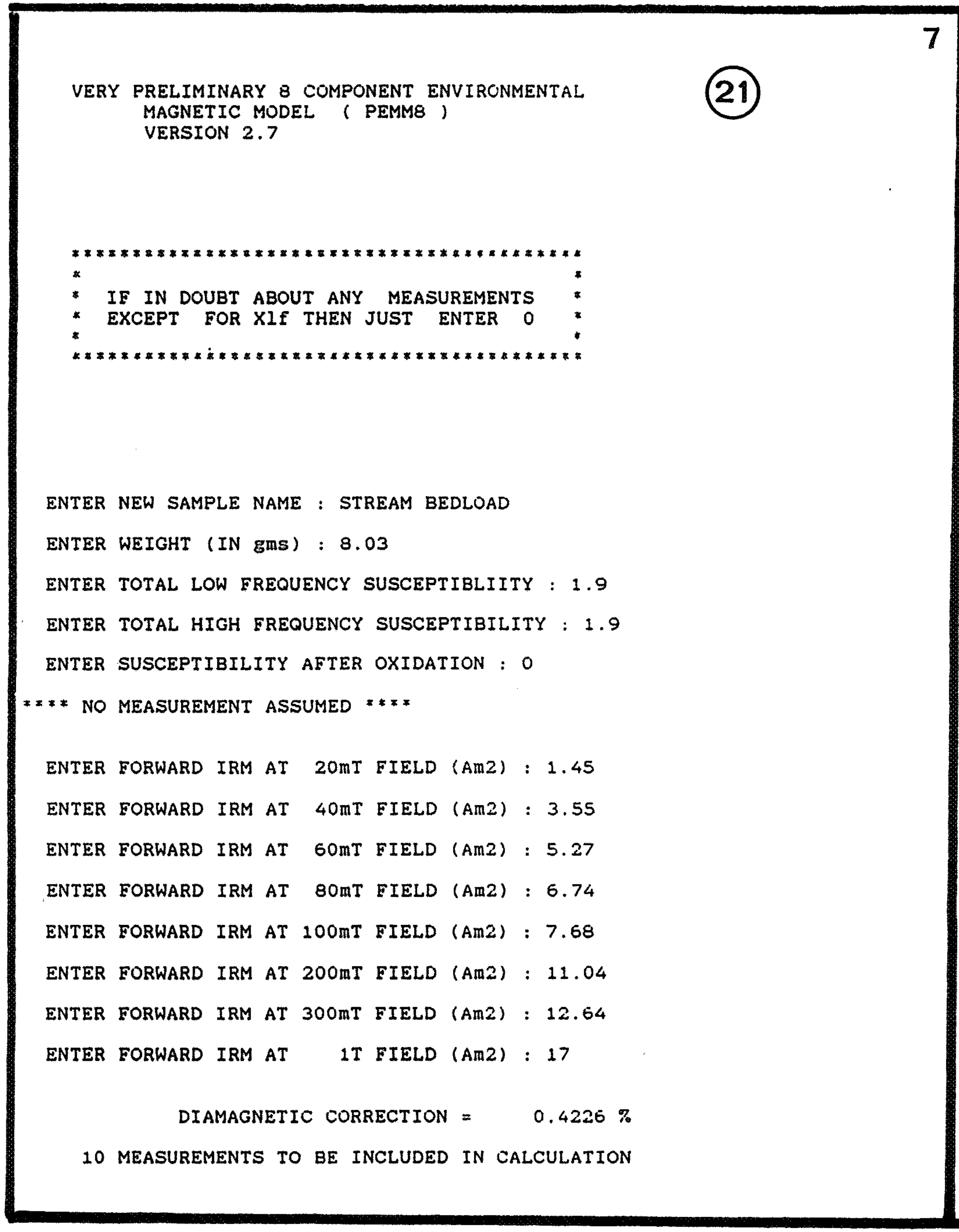

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VERY PRELIMINARY 8 COMPONENT ENVIRONMENTAL
      MAGNETIC MODEL   ( PEMM8 )
      VERSION 2.7



******************************************
*                                        *
*  IF IN DOUBT ABOUT ANY  MEASUREMENTS   *
*  EXCEPT  FOR Xlf THEN JUST  ENTER  0   *
*                                        *
******************************************




 ENTER NEW SAMPLE NAME : STREAM BEDLOAD

 ENTER WEIGHT (IN gms) : 8.03

 ENTER TOTAL LOW FREQUENCY SUSCEPTIBLIITY : 1.9

 ENTER TOTAL HIGH FREQUENCY SUSCEPTIBILITY : 1.9

 ENTER SUSCEPTIBILITY AFTER OXIDATION : 0

**** NO MEASUREMENT ASSUMED ****

 ENTER FORWARD IRM AT  20mT FIELD (Am2) : 1.45

 ENTER FORWARD IRM AT  40mT FIELD (Am2) : 3.55

 ENTER FORWARD IRM AT  60mT FIELD (Am2) : 5.27

 ENTER FORWARD IRM AT  80mT FIELD (Am2) : 6.74

 ENTER FORWARD IRM AT 100mT FIELD (Am2) : 7.68

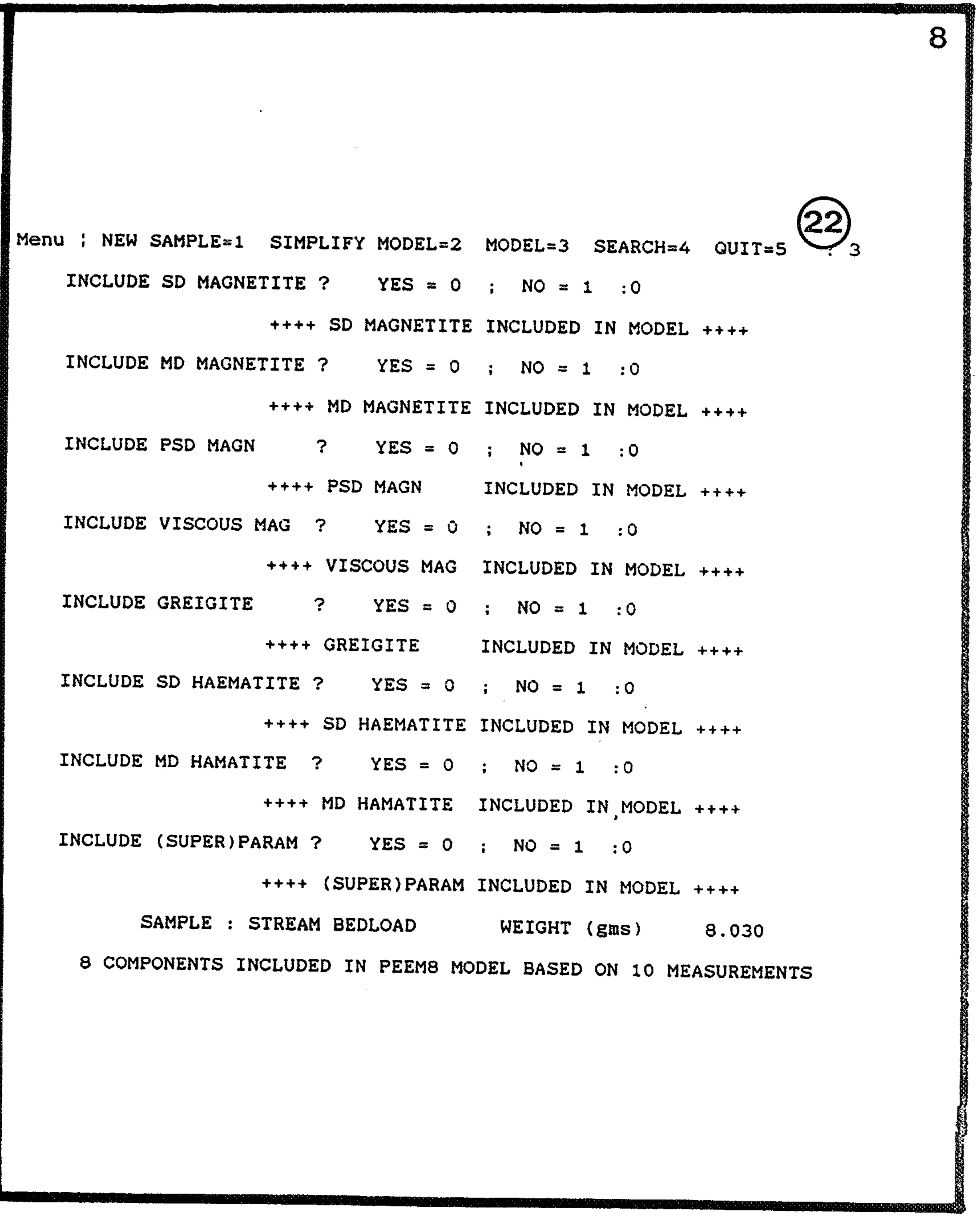
 ENTER FORWARD IRM AT 200mT FIELD (Am2) : 11.04

 ENTER FORWARD IRM AT 300mT FIELD (Am2) : 12.64

 ENTER FORWARD IRM AT    1T FIELD (Am2) : 17

            DIAMAGNETIC CORRECTION =     0.4226 %

   10 MEASUREMENTS TO BE INCLUDED IN CALCULATION
```



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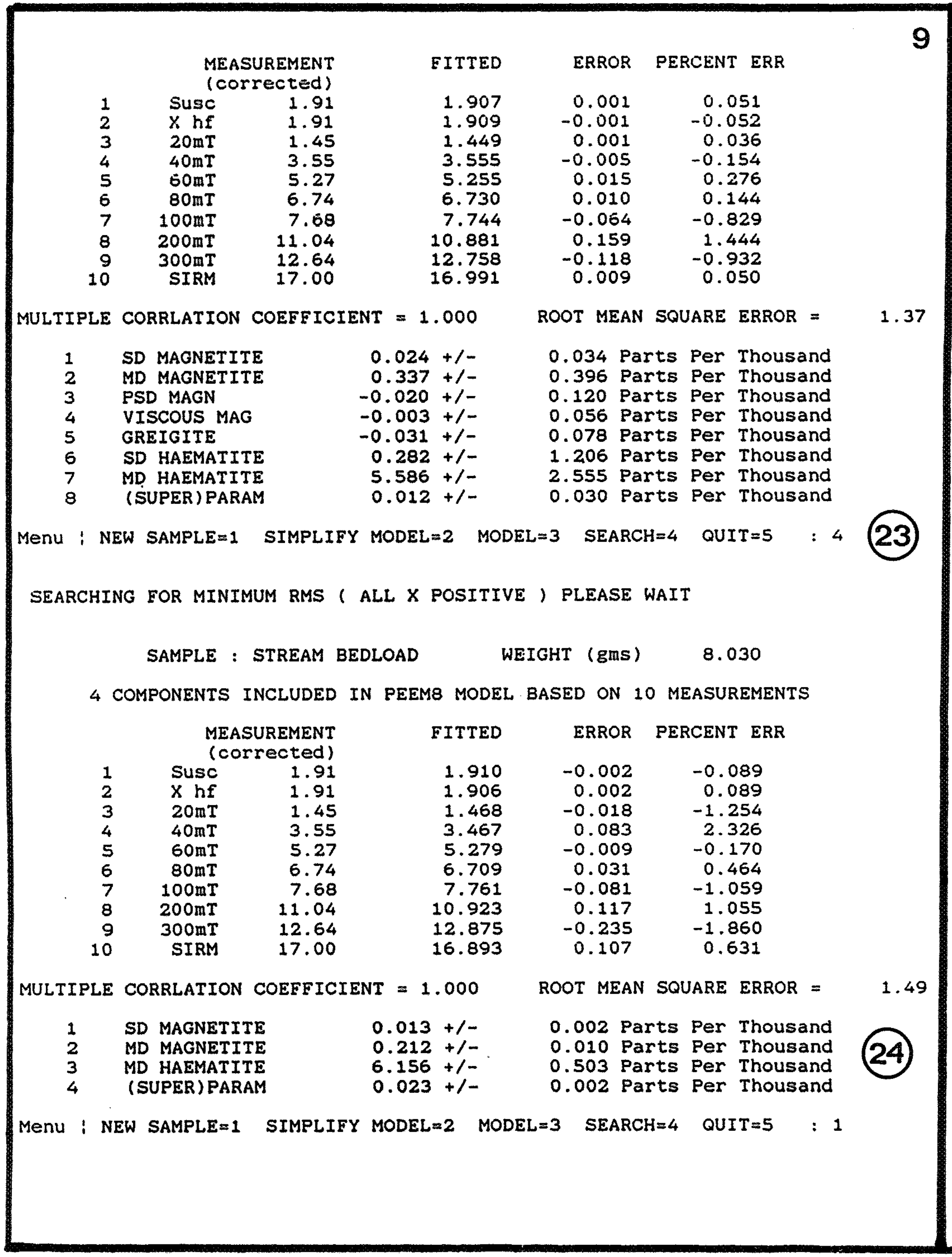
```
Menu ; NEW SAMPLE=1  SIMPLIFY MODEL=2  MODEL=3  SEARCH=4  QUIT=5  : 3
    INCLUDE SD MAGNETITE ?     YES = 0  ;  NO = 1  :0
                     ++++ SD MAGNETITE INCLUDED IN MODEL ++++
    INCLUDE MD MAGNETITE ?     YES = 0  ;  NO = 1  :0
                     ++++ MD MAGNETITE INCLUDED IN MODEL ++++
    INCLUDE PSD MAGN     ?     YES = 0  ;  NO = 1  :0
                     ++++ PSD MAGN     INCLUDED IN MODEL ++++
    INCLUDE VISCOUS MAG  ?     YES = 0  ;  NO = 1  :0
                     ++++ VISCOUS MAG  INCLUDED IN MODEL ++++
    INCLUDE GREIGITE     ?     YES = 0  ;  NO = 1  :0
                     ++++ GREIGITE     INCLUDED IN MODEL ++++
    INCLUDE SD HAEMATITE ?     YES = 0  ;  NO = 1  :0
                     ++++ SD HAEMATITE INCLUDED IN MODEL ++++
    INCLUDE MD HAMATITE  ?     YES = 0  ;  NO = 1  :0
                     ++++ MD HAMATITE  INCLUDED IN MODEL ++++
    INCLUDE (SUPER)PARAM ?     YES = 0  ;  NO = 1  :0
                     ++++ (SUPER)PARAM INCLUDED IN MODEL ++++
          SAMPLE : STREAM BEDLOAD       WEIGHT (gms)     8.030
     8 COMPONENTS INCLUDED IN PEEM8 MODEL BASED ON 10 MEASUREMENTS
```



```
               MEASUREMENT           FITTED       ERROR  PERCENT ERR
               (corrected)
     1     Susc         1.91          1.907       0.001       0.051
     2     X hf         1.91          1.909      -0.001      -0.052
     3     20mT         1.45          1.449       0.001       0.036
     4     40mT         3.55          3.555      -0.005      -0.154
     5     60mT         5.27          5.255       0.015       0.276
     6     80mT         6.74          6.730       0.010       0.144
     7    100mT         7.68          7.744      -0.064      -0.829
     8    200mT        11.04         10.881       0.159       1.444
     9    300mT        12.64         12.758      -0.118      -0.932
    10     SIRM        17.00         16.991       0.009       0.050

MULTIPLE CORRLATION COEFFICIENT = 1.000     ROOT MEAN SQUARE ERROR =      1.37

    1     SD MAGNETITE           0.024 +/-     0.034 Parts Per Thousand
    2     MD MAGNETITE           0.337 +/-     0.396 Parts Per Thousand
    3     PSD MAGN              -0.020 +/-     0.120 Parts Per Thousand
    4     VISCOUS MAG           -0.003 +/-     0.056 Parts Per Thousand
    5     GREIGITE              -0.031 +/-     0.078 Parts Per Thousand
    6     SD HAEMATITE           0.282 +/-     1.206 Parts Per Thousand
    7     MD HAEMATITE           5.586 +/-     2.555 Parts Per Thousand
    8     (SUPER)PARAM           0.012 +/-     0.030 Parts Per Thousand

Menu ¦ NEW SAMPLE=1  SIMPLIFY MODEL=2  MODEL=3  SEARCH=4  QUIT=5    : 4   ㉓

 SEARCHING FOR MINIMUM RMS ( ALL X POSITIVE ) PLEASE WAIT

          SAMPLE : STREAM BEDLOAD       WEIGHT (gms)      8.030

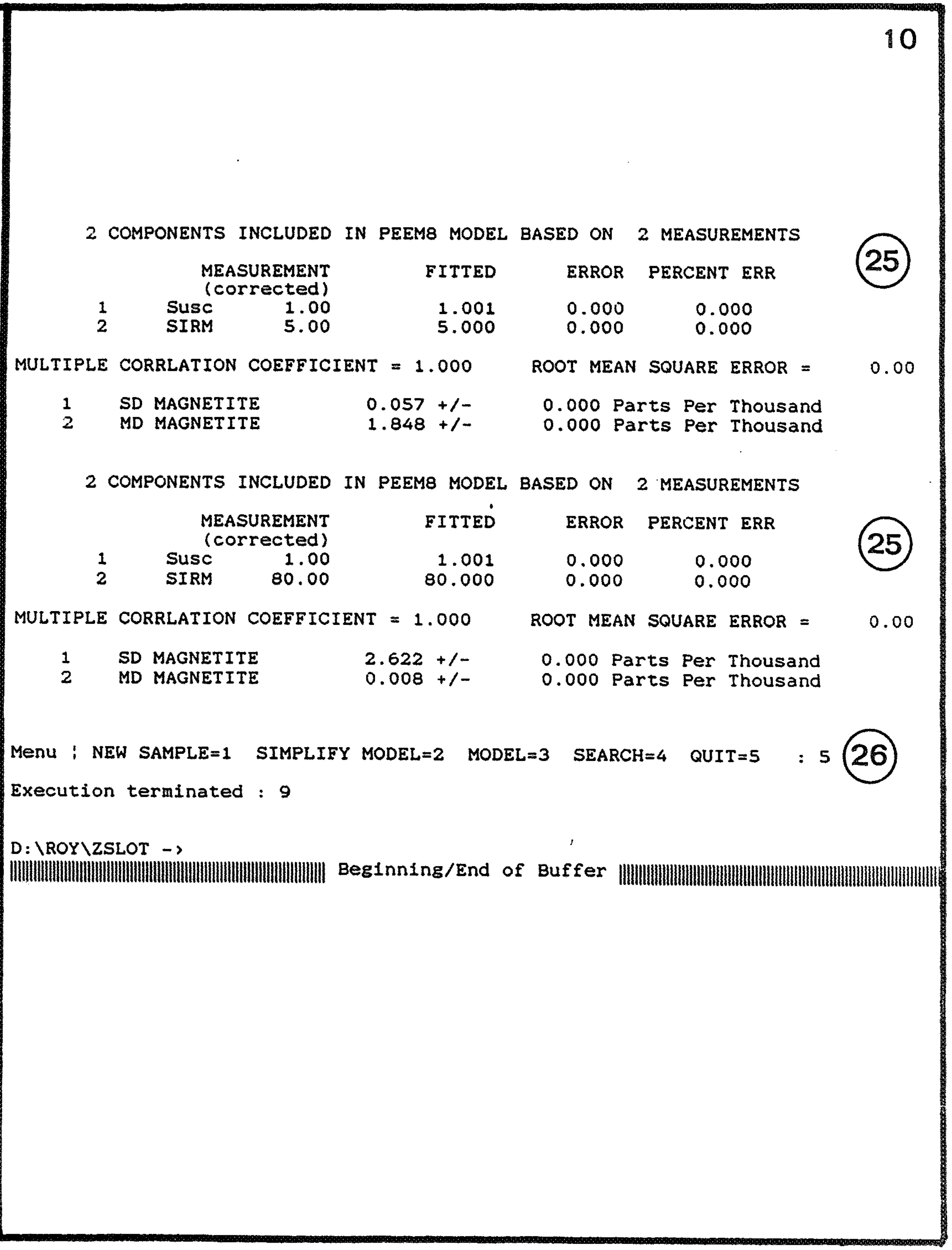
      4 COMPONENTS INCLUDED IN PEEM8 MODEL BASED ON 10 MEASUREMENTS

               MEASUREMENT           FITTED       ERROR  PERCENT ERR
               (corrected)
     1     Susc         1.91          1.910      -0.002      -0.089
     2     X hf         1.91          1.906       0.002       0.089
     3     20mT         1.45          1.468      -0.018      -1.254
     4     40mT         3.55          3.467       0.083       2.326
     5     60mT         5.27          5.279      -0.009      -0.170
     6     80mT         6.74          6.709       0.031       0.464
     7    100mT         7.68          7.761      -0.081      -1.059
     8    200mT        11.04         10.923       0.117       1.055
     9    300mT        12.64         12.875      -0.235      -1.860
    10     SIRM        17.00         16.893       0.107       0.631

MULTIPLE CORRLATION COEFFICIENT = 1.000     ROOT MEAN SQUARE ERROR =      1.49

    1     SD MAGNETITE           0.013 +/-     0.002 Parts Per Thousand
    2     MD MAGNETITE           0.212 +/-     0.010 Parts Per Thousand   ㉔
    3     MD HAEMATITE           6.156 +/-     0.503 Parts Per Thousand
    4     (SUPER)PARAM           0.023 +/-     0.002 Parts Per Thousand

Menu ¦ NEW SAMPLE=1  SIMPLIFY MODEL=2  MODEL=3  SEARCH=4  QUIT=5    : 1
```



```
      2 COMPONENTS INCLUDED IN PEEM8 MODEL BASED ON  2 MEASUREMENTS

               MEASUREMENT           FITTED       ERROR  PERCENT ERR    ㉕
               (corrected)
     1     Susc         1.00          1.001       0.000       0.000
     2     SIRM         5.00          5.000       0.000       0.000

MULTIPLE CORRLATION COEFFICIENT = 1.000     ROOT MEAN SQUARE ERROR =      0.00

    1     SD MAGNETITE           0.057 +/-     0.000 Parts Per Thousand
    2     MD MAGNETITE           1.848 +/-     0.000 Parts Per Thousand

      2 COMPONENTS INCLUDED IN PEEM8 MODEL BASED ON  2 MEASUREMENTS

               MEASUREMENT           FITTED       ERROR  PERCENT ERR
               (corrected)                                                ㉕
     1     Susc         1.00          1.001       0.000       0.000
     2     SIRM        80.00         80.000       0.000       0.000

MULTIPLE CORRLATION COEFFICIENT = 1.000     ROOT MEAN SQUARE ERROR =      0.00

    1     SD MAGNETITE           2.622 +/-     0.000 Parts Per Thousand
    2     MD MAGNETITE           0.008 +/-     0.000 Parts Per Thousand

Menu ¦ NEW SAMPLE=1  SIMPLIFY MODEL=2  MODEL=3  SEARCH=4  QUIT=5    : 5 ㉖

Execution terminated : 9

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