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Geomagnetic Secular Variation Based on Spherical Harmonic and Cross Validation Analyses of Historical and Archaeomagnetic Data

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Historical observations of the geomagnetic field have been combined with archaeomagnetic results in order to produce a model of geomagnetic field changes since 1600 A.D. Cross validation of the historical data indicated the most suitable level of truncation of spherical harmonic modelling of the data to be at the fourth degree. Weighted least squares cubic splines have been used to combine our spherical harmonic analyses carried out at 50 year intervals and to model smooth changes of the historical magnetic field throughout the world. Our model is in broad agreement with previous calculations of the historical main field but differs in some of the details of the non dipole field and the axisymmetric coefficients. An area of sustained anticlockwise looping of the local magnetic field vector has been revealed for the first time in the Indian ocean. The present region of strikingly small secular change and low non dipole field in the Pacific appears to have formed during the last 200 years. Changes in local field intensity have also been modelled throughout the world for the last 400 years. Even the largest of these local intensity changes are significantly smaller than the high frequency palaeointensity changes reported by archaeomagnetists. The balance of energy between the magnetic dipole field and the higher order fields suggested to have been occurring since 1900 A.D. is not found from our analyses for before 1800 A.D.

1. Introduction

The geomagnetic field has been of use to navigation for many centuries. Since the 16th century recordings of the declination of the magnetic field have been made at many localities around the world. Inclination measurements became common in the 18th century but intensity measurements were not carried out until the 19th century. It has long been known that the geomagnetic field continuously changes with time. The field variations can be quite rapid and quite large. At London, for example, the compass rotated through 35° in 230 years.

Models of the historical geomagnetic field and its secular change can be constructed by using spherical harmonic analyses. A convenient starting point for analysis of early geomagnetic data is the catalogue of Veinberg and Shibaev (1969). They collected the early historical data into 50 year periods. Their catalogue has been used by a number of investigators to produce spherical harmonic analyses (e.g. Benkova *et al.*, 1970; Braginski and Kulanin, 1971; Barraclough, 1974). Barraclough (1974) described four methods of spherical harmonic analysis which could be used with the historical data. Also he tabulated the mathematical errors associated with the calculations of each spherical harmonic coefficient. These formal errors are very large for the higher degree axisymmetric coefficients of the earlier epochs. The uncertainties in the coefficients become particularly troublesome when the inclination values of early epochs are compared. By combining archaeomagnetic results with the historical observations it is possible to constrain the mathematical solutions and reduce some of the uncertainties (e.g. YUKUTAKE, 1971). This betterment takes place despite the loss of data quality due to the inclusion of archaeomagnetic data with large experimental errors because of the greater benefits of the improved data distribution. We have followed this approach of including archaeomagnetic observations in our spherical harmonic analyses of the historical field. In this investigation we were particularly interested in the longer period changes and so were concerned with the unrealistic high frequency changes which are found in geomagnetic time series based on disjoint spherical harmonic analyses. In an attempt to produce smooth continuous changes we have combined the disjoint spherical harmonic analyses using weighted least squares spline functions. Our final spherical harmonic coefficients are listed in Table 1.

2. Spherical Harmonic Analyses

Spherical harmonic models were derived from the data of VEINBERG and SHIBAEV

Table 1. Spherical harmonic coefficients of the geomagnetic field for eight epochs between 1600 and 1950 A.D. Units: nT.

g, h	*n*	*m*	1600	1650	1700	1750	1800	1850	1900	1950
g	1	0	−35965	−35192	−34419	−33646	−32873	−32222	−31482	−30540
g	1	1	−3527	−3513	−3479	−3385	−3167	−2884	−2555	−2222
h	1	1	2805	2994	3733	4671	5447	5809	5850	5764
g	2	0	508	1111	1052	673	253	−88	−526	−1245
g	2	1	2203	2503	2525	2439	2477	2700	2962	3035
h	2	1	1973	1611	1336	1015	493	−170	−931	−1704
g	2	2	−1901	−2217	−2036	−1475	−671	225	1010	1521
h	2	2	−3177	−2104	−829	346	1133	1382	1124	455
g	3	0	−2297	−720	180	620	795	864	951	1133
g	3	1	−2221	−1084	−583	−533	−730	−1057	−1433	−1840
h	3	1	518	−407	−659	−566	−385	−317	−331	−367
g	3	2	475	502	788	1131	1359	1395	1325	1262
h	3	2	70	86	210	301	274	135	48	139
g	3	3	−240	−178	−207	−205	−51	259	577	805
h	3	3	−102	237	579	833	912	779	475	50
g	4	0	756	892	822	717	694	788	896	949
g	4	1	1988	1573	1349	1261	1173	1022	886	799
h	4	1	−584	−523	−378	−208	−94	5	65	119
g	4	2	585	434	322	285	332	472	585	547
h	4	2	−1888	−1219	−930	−804	−628	−304	−60	−126
g	4	3	96	230	203	85	−102	−315	−461	−459
h	4	3	−483	−355	−321	−328	−330	−291	−196	−54
g	4	4	288	270	53	−175	−230	−53	178	276
h	4	4	−72	−292	−341	−302	−243	−192	−159	−189

(1969) with the addition of archaeomagnetic data. The method used was the iterative method described by CAIN *et al.* (1967) (see also BARRACLOUGH, 1978, Section 6, Method 13). Starting from an approximate model of the geomagnetic field at the epoch concerned, corrections to the coefficients of this model are derived from the differences between the observed values and corresponding values given by the approximate model.

For epochs before 1850, where intensity data are sparse or non-existent, it is only possible to model the pattern of the geomagnetic field; there is no information about the scale of the field. In other words, we can derive only the ratios of the higher order coefficients to the first coefficient, g_1^0. We have attempted to overcome this problem by estimating values of g_1^0 for epochs before 1850 using the expression given by BARRACLOUGH (1974) for the time-dependence of g_1^0:

$$g_1^0(t) = -31110.3 + 15.46(t - 1914.0)$$

where t is the epoch in years A.D. For a given epoch, g_1^0 was assigned the value given by this expression and was held fixed whilst the corrections to the higher-degree coefficients were determined.

The data of VEINBERG and SHIBAEV (1969) were weighted proportionally to the number of original observations that contributed to the mean values in the catalogue. Equal weight is thus given to each original observation. Archaeomagnetic data, at up to 19 localities, were included in the analyses of the earlier epochs and were assigned a weight of between one and four. The models of BARRACLOUGH (1974) were used as the initial approximations. The iterative method showed satisfactory convergence in all cases after four or fewer iterations.

Some calculations were carried out in order to assess likely errors in the spherical harmonic coefficients due to the distribution of historical observations and the effect of truncation of the expansion at the fourth degree. 'Synthetic' magnetic field data were generated at the sites of the historical data and a fourth degree spherical harmonic model was calculated from the synthetic data. In practice a tenth degree model of the 1980 A.D. magnetic field was used to generate the synthetic data. The calculated coefficients were then compared with the known 1980 A.D. coefficients (Table 2). For the epoch 1750 A.D. sites the differences in the coefficients were reasonable. For all but one coefficient the errors were less than the formal least squares error (l.s.d.) due to the scatter in the historical measurements (Table 2). For details of the formal error associated with the coefficients calculated for early epochs see BARRACLOUGH (1974). For the epoch 1600 A.D. sites the errors associated with truncation and data distribution tended to be larger than those for epoch the 1750 A.D. sites and were comparable with the 1600 A.D. formal errors for the 1600 A.D. data. All but one of the coefficients lay within two standard deviations of the 'true' tenth degree values. The differences between the 10th degree 1980 A.D. coefficients and the fourth degree coefficients at the historical sites (Table 2) give some indication of the likely errors due to truncation and data distribution in estimating the historical field. It has been suggested that spherical harmonic analyses of the 17th and 18th century field are unreliable because of the sparseness and uneven distribution of the available data. However our analyses with this 'synthetic' data suggest that the data distribution and density are adequate for moderately detailed analyses of the 17th and 18th century field.

Table 2. 1980 field coefficients calculated using data at historical sites. Units: nT.

g, h	n	m	1980 A.D. 10th degree	1750 A.D. sites†	1600 A.D. sites†
g	1	0	−29988	−29988	−29988
g	1	1	−1957	−1982	−1807
h	1	1	5606	5660	5192
g	2	0	−1997	−1902	−2668
g	2	1	3028	3241*	2653
h	2	1	−2129	−2103	−1413**
g	2	2	1662	1724	1803
h	2	2	−199	−188	−31
g	3	0	1279	1280	950
g	3	1	−2181	−2210	−1687
h	3	1	−335	−264	−879
g	3	2	1251	1217	1426
h	3	2	271	351	68
g	3	3	833	715	790
h	3	3	−252	−367	−451
g	4	0	938	1274	1742
g	4	1	783	802	1129
h	4	1	212	206	531
g	4	2	398	293	142
h	4	2	−257	−386	−318
g	4	3	−419	−508	−523
h	4	3	53	−26	67
g	4	4	199	183	204
h	4	4	−298	−200	−139

†Fourth degree coefficients calculated after five iterations starting with 1700 A.D. field coefficients.

*1750 A.D. sites coefficient deviating from the 1980 A.D. tenth order model by more than one historical field measurements standard deviation.

**1600 A.D. sites coefficient deviating from the 1980 A.D. tenth order model by more than two historical field measurements standard deviation.

3. Series Truncation Using Cross Validation

Apart from the use of 'intuition and experience' perhaps the simplest method of deciding where to truncate a spherical harmonic series is to examine the residuals to models of increasingly high degree. The spherical harmonic fit will tend to improve rapidly at first and then more slowly. The change in rate of improvement can be taken to indicate a suitable truncation level. A slightly better method is to test statistically the improvement in fit with each increment in complexity and to truncate the spherical harmonic series when no significant improvement occurs. Spherical harmonic geomagnetic field models of degree 4 through 6 have previously been used for the early historical data. The present geomagnetic field has been modelled with series ranging from degree 8 to 25.

Cross validation is another technique which can be applied to experimental data in order to separate the signal under investigation from any random noise. The technique uses the internal consistency of the data to separate the noise and signal. It involves trying spherical harmonic fits of different degrees and judging which is the most appropriate. The basic idea is that a small part of the data is temporarily set aside as a validation sample so that it can be used to judge goodness of fit. A spherical harmonic analysis is carried out on the remaining data and the magnetic field is estimated at the sites of the validation sample. The differences between the estimated field values and the validation sample values are then calculated. Small differences are taken to indicate a good fit of the spherical harmonic series. The validation process is carried out with many different validation samples. In full cross validation a single data point at a time is used as a validation sample. After the residual calculation the single validation point is replaced in the data set and a second point removed. The validation process is repeated taking each data point in turn and a cross validation residual mean square error (CVMSE) is calculated for the whole data set. A CVMSE is produced for each degree of spherical harmonic analysis of interest. With large data sets the time consuming process of full cross validation is not necessary and smaller validation sets can be used. For example only every tenth data point could be used. Another time saving method is to set randomly aside 10% of the data for the validation sample and its associated calculation and to repeat the process ten times. CRAVEN and WHABA (1979) have also described a generalized cross validation procedure.

An example of the results of our cross validation experiments on the historical geomagnetic field is shown in Table 3 for the epoch 1850. The CVMSE is a minimum for the fourth degree model and this level is taken to be the most appropriate level at which to truncate the spherical harmonic analysis in representing the historical geomagnetic field. The minimum occurs in the CVMSE series because lower degree models smooth too much, so that they are poor predictors of the validation data points while higher degree models 'chase' the noise associated with each observation too much so that they also are poor predictors. Our models were calculated with fourth degree series fitted to *all* the original data points.

4. Continuity and Cubic Splines

Our spherical harmonic models can be used to calculate local virtual geomagnetic pole (VGP) positions (Fig. 1). When virtual poles of different epochs, but for just one locality, are plotted they form a polar path (e.g. Fig. 2a). Our virtual poles for certain epochs were clearly aberrant at some localities. For example, the virtual pole of 1750 A.D. for the locality 30°N, 90°W (Fig. 2a) lies out of sequence. In order to form a clear picture of long period geomagnetic changes such aberrant points must be removed. We chose to remove these outliers by emphasizing the continuity of the polar path time series using a robust

Table 3. Cross validation mean square errors for epoch 1850 A.D.

Degree	Number of coefficients	CVMSE $10^{-12}\ T^2$
1	3	18
2	8	11
3	15	4.6
4	24	2.7
5	35	7
6	48	11

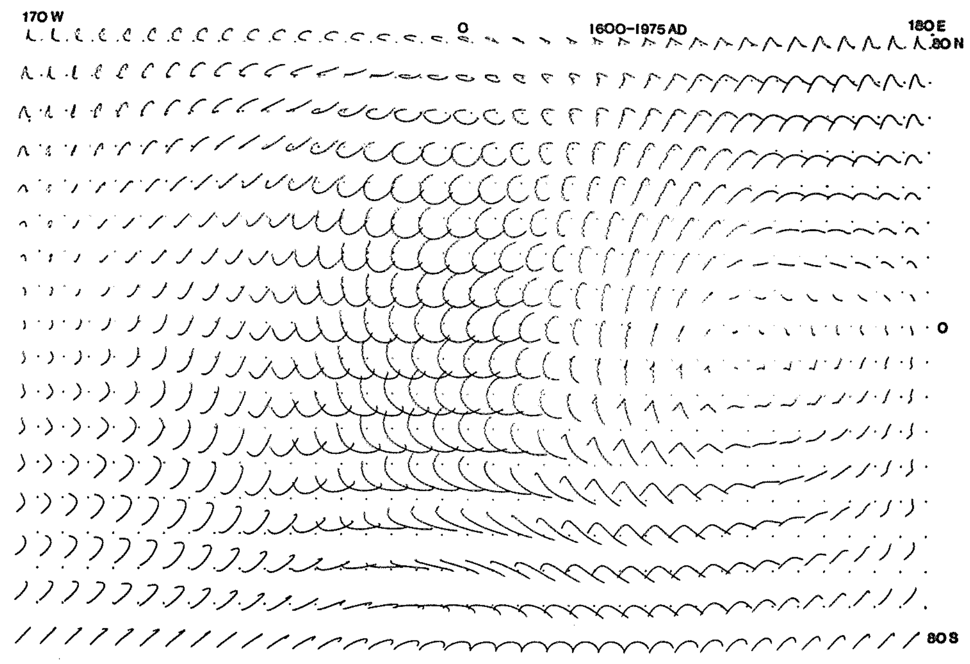

Fig. 1. Illustration of virtual geomagnetic pole (VGP) paths for 1600 to 1975 A.D. on a 10° by 10° grid. Paths based on least squares cubic splines. The well known open clockwise looping for Europe can be seen in the centre of the figure. All the paths show clockwise motion except those in the southern Indian ocean which loop in an anticlockwise direction.

smoothing procedure. In practice we fitted weighted least squares cubic spline functions to the virtual pole paths using the procedures described by THOMPSON and CLARK (1981).

Examples of the smooth paths resulting from this robust curve fitting exercise are shown in Fig. 2. Spline functions were fitted to all the 612 virtual polar paths on a 10° by 10° grid (Fig. 1). The directional data of all these spline functions were used to calculate another series of fourth degree spherical harmonic coefficients. The coefficients of this series of geomagnetic models at 50 year intervals are tabulated in Table 1.

The full procedure of our analyses is summarized in the flow diagram of Fig. 3. As a first step, cross validation indicated that fourth degree models would be appropriate in our analyses. CAIN *et al.*'s (1967) method of spherical harmonic analysis was used to calculate coefficients up to the fourth degree. The coefficients were scaled using g_1^0 values taken from BARRACLOUGH (1974). Normally analysis of historical data would be stopped at an equivalent stage to this and the resulting spherical harmonic coefficients used to calculate the properties of the ancient field, for example, to plot maps of declination (BARRACLOUGH, 1974) or the vertical component of the non dipole field (YUKUTAKE, 1971). We have added an extra step in order to improve the continuity between our 50 year epochs (Fig. 3). The extra procedure involves fitting robust cubic splines on the unit sphere to virtual pole paths and using the fitted values of polar longitude and latitude (or equivalently declination (D_0) and inclination (I_0)) to calculate new spherical harmonic coefficients. These new coefficients are listed in Table 1 and the associated virtual pole paths are shown in Fig. 1.

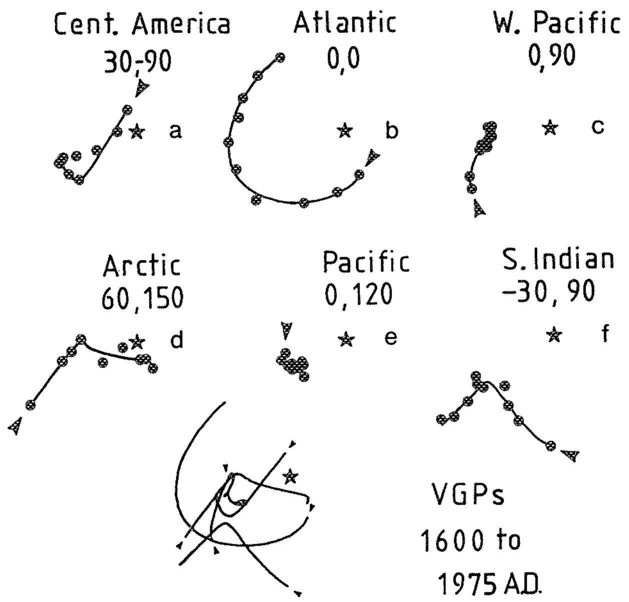

Fig. 2. Six examples of cubic spline virtual geomagnetic pole paths from 1600 A.D. to the present day. An arrow marks the start of each path in 1600 A.D. The data points, solid circles, which were used in constructing the splines, were derived from fourth degree spherical harmonic analyses of epochs 1600, 1650, 1700, 1750, 1800, 1850, 1890, 1910, 1942, and 1975 A.D. The geographic pole is shown by a star and the latitude and longitude of the locality of each VGP path site is noted. The continuous curves are weighted least squares spline approximations to the paths. At the bottom of the figure the paths from the six localities are compared.

Clear differences between the VGP paths can be seen around the world (Figs. 1 and 2). The extents of geographic regions within which the VGP paths are roughly similar has been estimated from a cluster analysis of the paths. Boundaries between clusters of similar VGP paths are drawn in Fig. 4.

The paths of Fig. 1 are enlarged in Fig. 5 for the North Atlantic and European region. The amount of detail in our fourth degree models can be assessed by comparing our curve for 50°N 0°E (Fig. 5) with the recent compilation of historical field measurements for London (MALIN and BULLARD, 1981, Fig. 4). The inverted triangles, on our paths in Fig. 5, lie at 25 year intervals running from 1600 A.D. to 1975 A.D.

5. Opening of the Pacific Window

At present the non dipole field is strikingly smaller than average in the Pacific hemisphere. This region in which the dipole field can be clearly observed due to the almost total absence of any non dipole field has been referred to as 'the Pacific window'. FISK (1931) pointed out that the amplitude of secular change is also lower than average in the Pacific region. COX and DOELL (1964) suggested that this low rate of secular change may have persisted in the Pacific region for several million years. Such persistence could, for example, explain why they found thick sequences of lava flows through which the remanent

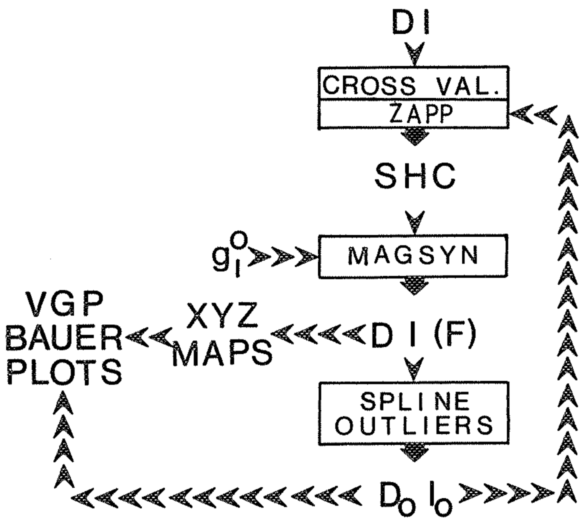

Fig. 3. Flow diagram of analysis procedure. Declination and inclination data are subjected to spherical harmonic analysis using subroutine Zapp, the maximum degree of the coefficients calculated is determined by cross validation. The resulting spherical harmonic coefficients are scaled using an estimated value of the axial dipole field and passed through subroutine Magsyn in order to calculate gridded magnetic field parameters. The resulting regular global grid of declination and inclination results is smoothed using a weighted least squares spline fitting programme which produces new declination and inclination data which can either be displayed directly as VGP paths or else be returned for the final calculation of a new series of spherical harmonic coefficients.

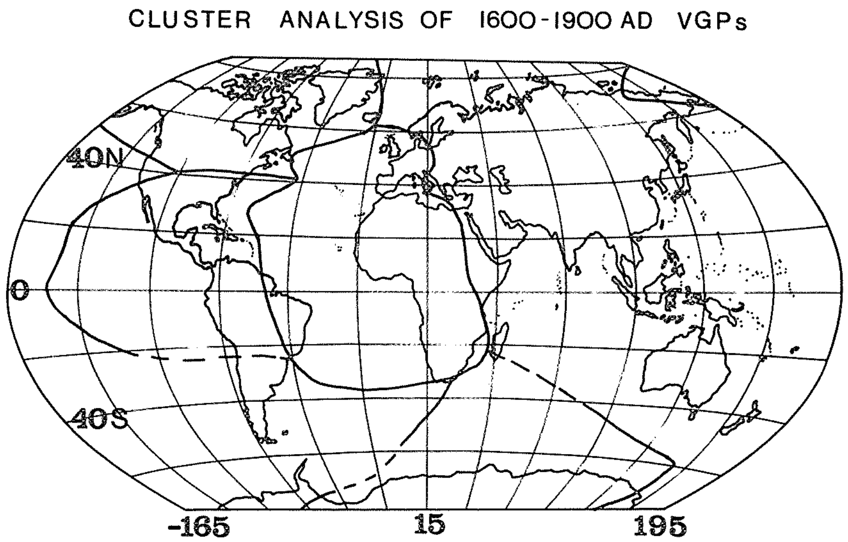

Fig. 4. Boundaries of regions exhibiting similar patterns of historical secular change as determined by a cluster analysis of 1600 to 1900 A.D. virtual geomagnetic pole positions. Boundaries marked by dashed lines are not as well determined as those indicated by solid lines.

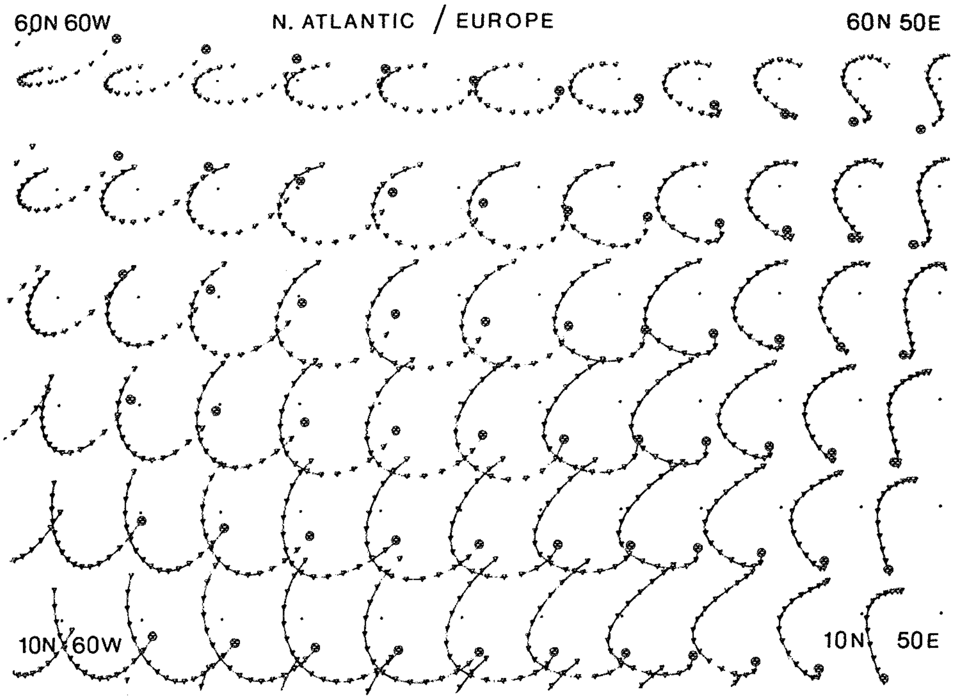

Fig. 5. North Atlantic/European region VGP paths on the 10° by 10° grid. The inverted triangles mark the position on the spline paths at 25 year intervals. All paths start at 1600 A.D. and end at 1975 A.D. The position of the geographic north pole is marked for each locality by a small circle. The solid circles mark the start of each VGP path. The site latitudes and longitudes are noted for the paths in the four corners of the diagram.

magnetization directions exhibit little change. DOELL and COX (1971) went on to suggest that long term geographical variations in behaviour of the global geomagnetic field might indicate that some type of inhomogeneity near the base of the lower mantle, such as a temperature change, has existed beneath the Pacific and suppressed the generation of non dipole fields in the Earth's core. Further studies of the magnetization of Pacific lava flows have been reviewed by COX (1975). These studies reveal that on average the remanence inclination is a few degrees lower than that of a centred axial dipole. This long-term time average non dipole field component has been found to reverse direction when the main dipole field changes polarity and to persist for at least 5 m.y. WILSON (1970, 1971) has also noticed such persistent non dipole fields and showed that they can be modelled by an axial dipole, offset a few hundred kilometres from the Earth's centre. COX (1975) shows that long term ordering of non dipole field anomalies could also account for this type of bias in palaeomagnetic remanence measurements. For example, he demonstrates that if non dipole vertical field foci of one sign were to occur preferentially at certain latitudes, as in the present day field, and to drift longitudinally, then averaging over a long time period would produce a similar persistent non dipole field pattern to that of an offset axial dipole. COX (1975) suggests that this type of field behaviour might have produced the palaeomagnetic records found on the volcanic islands of the northern Pacific. It is appealing that the proposed persistent global palaeomagnetic non dipole fields have the same sign as the present non dipole field and also a similar magnitude.

We have used the spherical harmonic coefficients of Table 1 to model the geomagnetic field in the Pacific over the last 350 years. The use of virtual geomagnetic pole positions eases comparison with the palaeomagnetic results. Details of the changes in local VGP are shown in Fig. 6 for part of the Pacific. The regions in which the angular deviation between the geomagnetic pole and virtual geomagnetic pole has been less than 3° have been plotted in Fig. 7 at 100 year intervals. Cox (1962, Fig. 1) plots this region, or Pacific window, for the 1945 A.D. geomagnetic field. Our models indicate that the present region of low non dipole field components has formed during the last 200 years. The only area displaying low non dipole components for over 300 years appears to have been a restricted region near North West Australia. Inspection of the individual local VGP paths of Fig. 6 reveals that, at many localities in the Pacific region, moderate changes in VGP location have been replaced by slower changes as the Pacific window has opened. Some localities e.g. near 20°S 160°E display a decrease in rate of VGP movement followed by an increase. In summary our models suggest that regions of low secular change and low non dipole field components, such as the present day Pacific, have life times of only a few hundred years.

6. Anticlockwise Looping

BAUER (1896) showed for the first time that the sense of motion of the geomagnetic field vector had been the same at over 100 observation points around the earth and concluded,

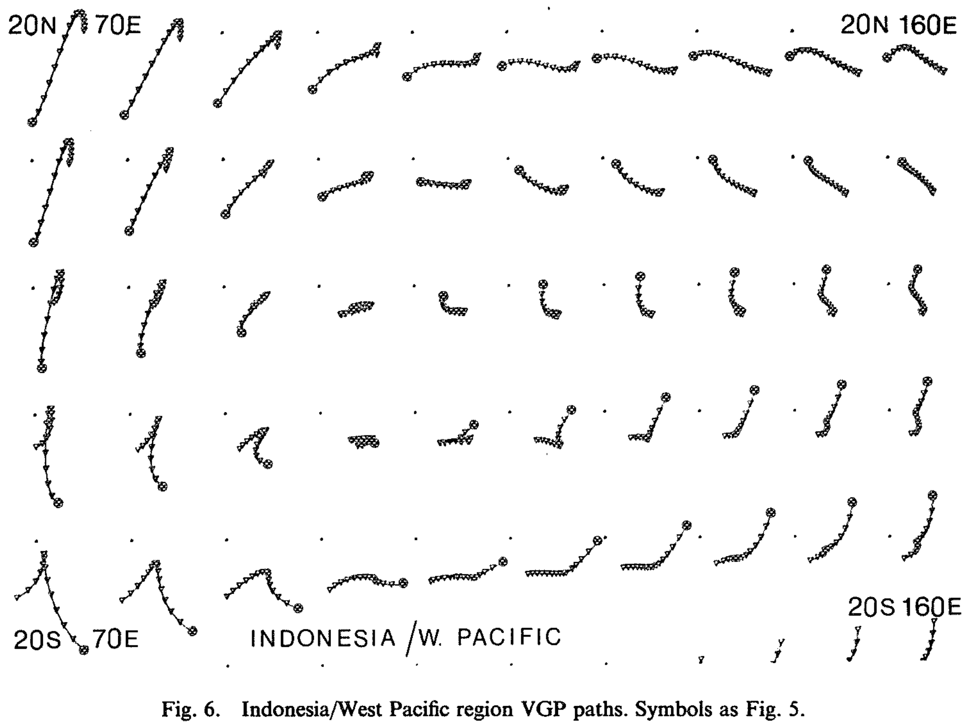

Fig. 6. Indonesia/West Pacific region VGP paths. Symbols as Fig. 5.

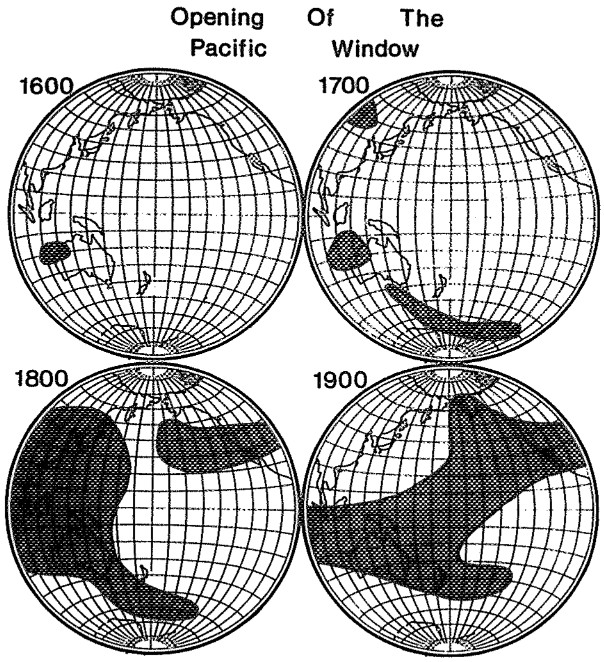

Fig. 7. Opening of the Pacific Window between 1600 and 1900 A.D. Regions within which the angular deviation between the local VGP and the geomagnetic dipole was less than 3° are shaded.

"In consequence of the secular variation of geomagnetism the north end of a freely suspended magnetic needle viewed from the center of suspension of the needle, moves on the whole earth in the direction of the hands of a watch". This general clockwise form of motion has been explained in terms of the westward drift of the geomagnetic field (RUNCORN, 1959; SKILES, 1970). No unambiguous evidence for sustained counter clockwise motion of the present field vector has previously been found (e.g. AS, 1967; DODSON, 1979). Examination of Figs. 1 and 8 however, reveals a large area of the earth which is dominated by anticlockwise looping. This region, centred on the Indian ocean, has presumably not been noticed before, firstly because it used to be of smaller extent, and secondly, because no permanent observatories have been sited in the region which could provide long series of observations. The anticlockwise looping becomes apparent in our models as they combine results from different parts of the Indian Ocean and its neighbouring areas. Figure 9 diagrammatically illustrates the expansion of the region of anticlockwise looping as judged by the curvature of VGP paths since 1600 A.D. The anticlockwise motion has a time scale of several hundred years. Superimposed on this long term behaviour are higher frequency changes which can exhibit clockwise motion, as seen for example near locality 30°S 60°E (Fig. 8).

This approach of describing geomagnetic field changes in terms of the sense of looping is of importance in geophysics on account of the relationship between the sense of longitudinal drift and the sense of looping. RUNCORN (1959) pointed out that for a wide range of spherical harmonic coefficients, including all the sectorial harmonics, clockwise

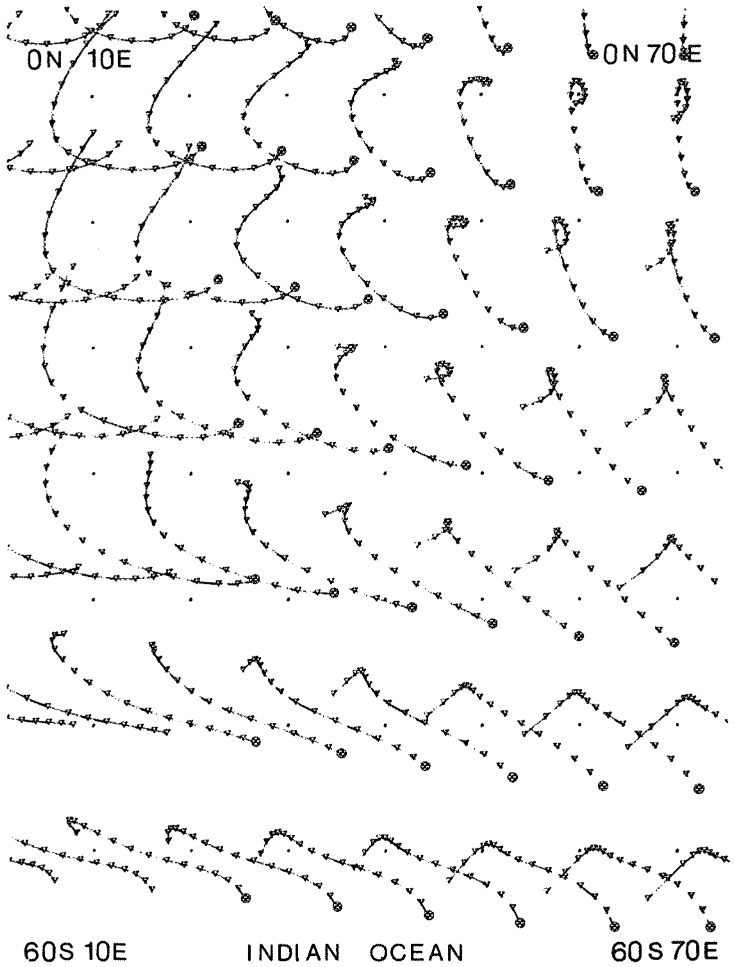

Fig. 8. Indian Ocean region VGP paths. Symbols as Fig. 5.

looping results from westward drift and anticlockwise looping from eastward drift. SKILES (1970) elaborated on Runcorn's work and investigated the effects of drifting satellite dipoles. DODSON (1979) pointed to particular geometries of radial satellite dipoles and observatory sites which could lead to discrepancies between the general sense of looping and sense of drift. The anticlockwise motion of the geomagnetic vector in the Indian ocean is being investigated in more detail to see if it has been produced by an easterly moving source in the Southern Ocean or by one of Dodson's exceptions to the general rule.

As (1967) has suggested that in the southern hemisphere clockwise looping is only in its initial stages and that pronounced field motions can be expected in the future. However, our model suggests if the present trends continue that the large motions, observed for example near Cape Town, will not continue.

7. Virtual Dipole Intensities

Past changes in field intensity of a factor of about two have been deduced from archaeomagnetic measurements (e.g. SMITH, 1967). The measurements reveal a global increase in field strength which reached a maximum around 2000 years BP following a broad minimum between 7500 and 5500 BP (MCELHINNY and SENANAYAKE, 1982). These global variations are taken to reflect changes in the dipole moment. Archaeomagnetic investigations of well dated shards (WALTON, 1979) and adobe bricks (GAMES, 1980) suggest that higher frequency (<200 years) changes of a similar magnitude to the long period field changes have also occurred. The question of whether such changes are global or local is of current interest to archaeomagnetists. Such rapid changes in field intensity are to be found in the behaviour of the geomagnetic field over the last 30 years and these are taken to lend credence to the archaeomagnetic results (GAMES, 1980). In principle our coefficients of Table 1 can be used to investigate such intensity changes over the last 350 years. However, observations of field intensity only began 150 years ago (GAUSS, 1833). This means that although we can calculate the shape of the 17th and 18th century geomagnetic field from historical observations we do not know its magnitude. Our coefficients have been calculated using BARRACLOUGH's (1974) linear extrapolation of the moment of the axial dipole as derived from intensity measurements made during the last 150 years. Thus it is possible that our estimates of past intensity changes should follow a more complex pattern. YUKUTAKE (1971) used the then available archaeomagnetic palaeointensity measurements to determine the past changes in field intensity since 1550 A.D.. A more recent compilation of archaeomagnetic results (BURLATSKAYA and NACHASOVA, 1977)

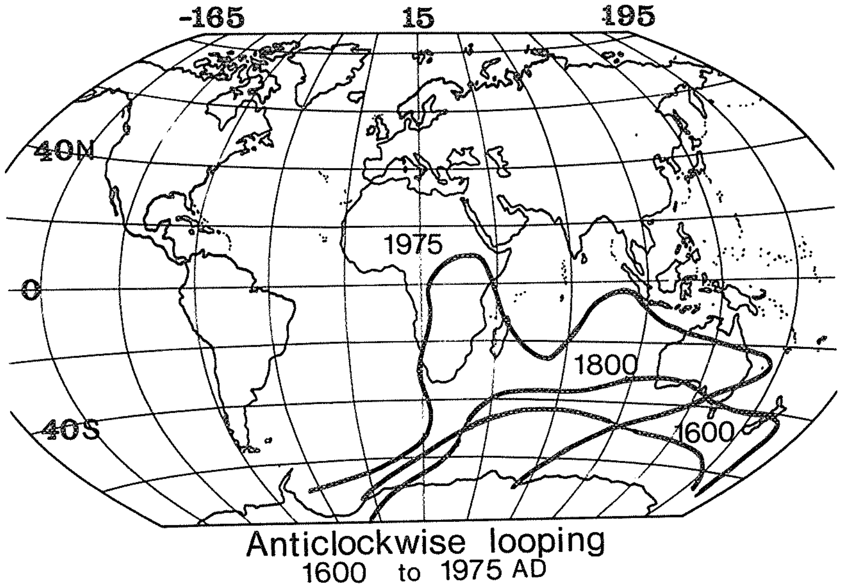

Fig. 9. Schematic diagram illustrating the increase in extent of the geographic region exhibiting anticlockwise looping of the historical geomagnetic field vector. The approximate boundary separating anticlockwise from clockwise looping is shown for epochs 1600, 1800, and 1975 A.D.

unfortunately presents very scattered results for the 17th to 18th centuries. These results are not in particularly good agreement with YUKUTAKE's (1971) estimates. Palaeointensity measurements are better grouped in the 15th and 16th centuries when they agree well with Barraclough's linear extrapolation. On balance the archaeomagnetic data indicate that a linear decrease is not an unreasonable model of the past change in g_1^0.

Figure 10 plots the changes in field intensity calculated using our coefficients in Table 1. The changes are plotted in terms of virtual dipole field intensities on the same 10° by 10° grid used in Fig. 1. The scales at the left hand side of Fig. 10 show the corresponding total field intensities for sites on the equator and at a latitude of 80°. In Fig. 11 three sites have been taken from Fig. 10 which illustrate extremely large intensity changes during the last 350 years and are compared with WALTON's (1979) archaeomagnetic palaeointensity results from Greece. A smooth freehand curve has been drawn through Walton's data and scaled to facilitate comparison with the historical field intensity changes. The dashed lines in Fig. 11 show the contribution from the change in the dipole field. The gradual decreases reflect the linear decay of g_1^0, and the wobbles reflect the changes in geomagnetic latitude due to the slow movement of the dipole axis over the last 350 years. The differences between the continuous and dashed curves arise from the non dipole field contributions. Comparing all the curves in Fig. 10 one can see that the high (~200 year) frequency changes deduced from the archaeomagnetic records are not present in our geomagnetic model. This absence

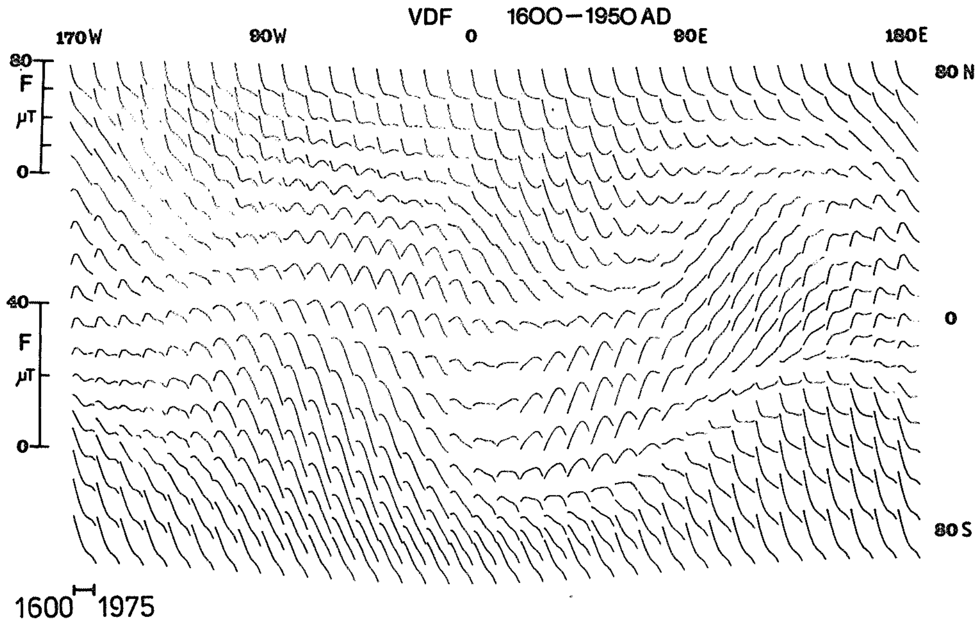

Fig. 10. Virtual dipole field intensity variations from 1600 to 1975 A.D. are shown for 612 localities arranged on the regular 10° latitude and longitude grid of Fig. 1. At each locality the time scale runs from 1600 A.D. on the left to 1975 A.D. on the right and field intensity increases towards the top of the diagram. The intensity scales have been adjusted according to latitude in order to facilitate comparison between localities around the world. Scales for the local total field intensity (F) at localities at 80°N and on the equator are shown at the left hand side of the diagram.

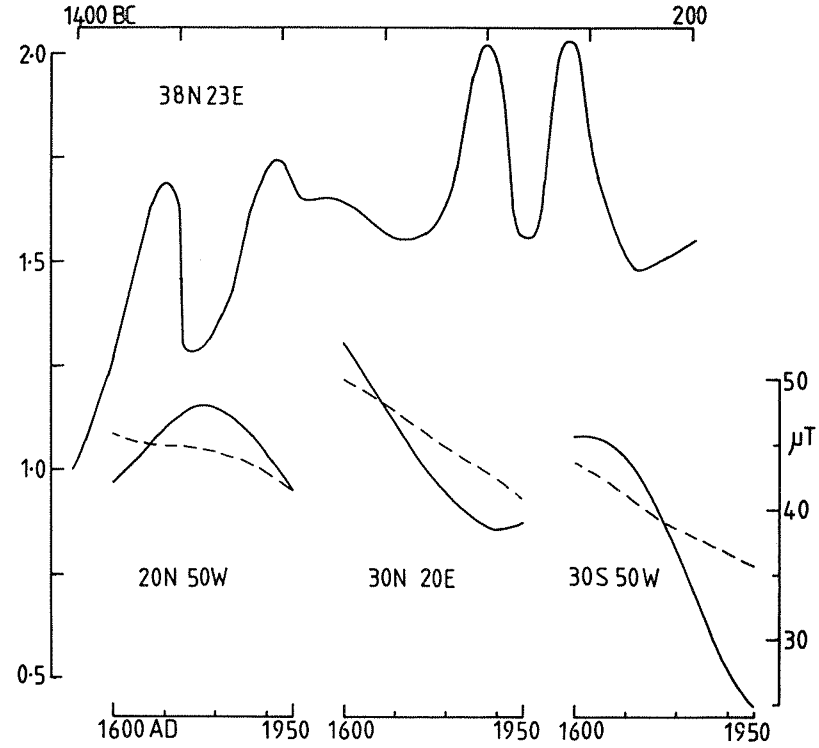

Fig. 11. Comparison of three large virtual dipole field intensity changes from Fig. 10 with archaeomagnetic palaeointensity records for ancient Greece (Walton, 1979). The 'archaeomagnetic intensity record from 38°N 23°E runs for 1200 years from 1400 to 200 years B.C. and is shown by the upper curve. Historical field intensity changes are illustrated by the three lower solid curves. The dashed curves show the historical field changes at the three sites caused by the geomagnetic dipole variation, the general declines being due to the decrease in dipole intensity and the gentle oscillation due to the change in orientation of the dipole axis between 1600 and 1950 A.D. The historical field intensity variations are plotted on a scale to facilitate comparison with the archaeomagnetic record. Note that although the historical curves were chosen from localities exhibiting particularly large intensity variations they do not show as pronounced or rapid changes as are contained in the archaeomagnetic study.

arises because very rapid field intensity changes have not been maintained for hundreds of years but have had lifetimes of only tens of years.

There are thus difficulties in explaining the archaeomagnetic data in terms of present field behaviour. Furthermore simply increasing the relative importance of the non dipole field for long periods in the past does not appear to be possible as McElhinny and Senanayake (1982) have shown from the distribution of field directions over the last few thousand years that "variations in the non dipole field . . . remain in the same proportions to the dipole field irrespective of its magnitude".

8. Balance of Coefficients

Merrill *et al.* (1979) discussed the evidence for long term asymmetries in the earth's magnetic field as recorded by palaeomagnetic data. They concluded that the time averaged palaeomagnetic field deviates slightly, but significantly, from a geocentric axial dipole field. They found the main deviations to be in the axisymmetric terms, particularly the axial quadrupole. It is argued that it is even possible to demonstrate a significant difference in the time average axial quadrupole coefficient between normal and reverse polarity states of the geomagnetic field. They discuss possible physical causes for the departures from a centred dipole field and for the differences between polarity states. COX (1975) also discusses the failure of the axial dipole model to account completely for the time average properties of the geomagnetic field and again points out that the mean palaeofield has generally been found to differ slightly but significantly from the field of an axial dipole. THOMPSON and CLARK (1982) describe difficulties in mathematically matching apparent polar wander paths over a period of 10^8 years using the assumption of a centred dipole. COX (1975) discusses persistent components of the time average non dipole field of several thousand nanoteslas. As described in Section 5 COX (1975) has shown that the averaging effect of longitudinal drift of ordered non dipole field anomalies of the same magnitude, as found in the 1965 IGRF, would produce a persistent g_2^0 field of the same sign as the g_1^0 field and that this averaging could explain the above palaeomagnetic anomalies. An alternative explanation is that the g_2^0 field corresponds to a global standing field.

Our coefficients (Tables 1 and 4) suggest that the axial quadrupole changed sign in the mid 19th century and that when averaged over the last 350 years g_2^0 is close to zero. It would seem that in order to produce a suitable palaeomagnetic time average field with a significant positive g_2^0/g_1^0 ratio that the axial quadrupole field must at times be considerably more important than at present.

MCDONALD and GUNST (1968) have suggested that the decrease in energy of the dipole field since 1900 A.D. is being transformed into neighbouring higher degree fields due to a change in the pattern of fluid motions in the earth's core. Models of geomagnetic polarity transitions have been calculated using such a balance of energies. VEROSUB and COX (1971) have shown that conservation of the total magnetic energy stored outside the earth's core is not warranted over periods of the order of 10^3 to 10^4 years by palaeomagnetic analyses. Nevertheless they suggest that the energy transfer behaviour is likely to be of relevance during rapid polarity reversals. We find from our analyses of the historical field that the

Table 4. The ratio g_2^0/g_1^0.

Epoch	$g_2^0/g_1^0 \pm$ sd	Source
1600 A.D.	-0.014 ± 0.016	This paper
1650 A.D.	-0.032 ± 0.014	"
1700 A.D.	-0.031 ± 0.008	"
1750 A.D.	-0.020 ± 0.004	"
1800 A.D.	-0.008 ± 0.003	"
1850 A.D.	$+0.003 \pm 0.002$	"
1900 A.D.	$+0.016 \pm 0.001$	"
1950 A.D.	$+0.041 \pm 0.001$	"
Normal	$+0.050$	MERRILL *et al.* (1979)
Reverse	$+0.083$	"
Present	$+0.063$	"
1980 A.D.	$+0.067 \pm 0.001$	BARRACLOUGH (1981)

balancing or transformation of energy between neighbouring modes did not hold before 1800 A.D. Our calculations, summarized in Table 5, taken in conjunction with Verosub and Cox's analysis of palaeomagnetic data, would suggest that the relationship observed by McDonald and Gunst is likely to be only coincidental and that balancing the energy between neighbouring modes is an unnecessary restriction in modelling polarity transitions.

9. Summary

Our model of historical field behaviour has been compared with the field properties expected by interpolating between the time average properties of the palaeomagnetic field and the instantaneous properties of the present day field. Our model is in broad agreement with earlier estimates of the historical field. For example our declinations are very similar to those of BARRACLOUGH (1974) and maps of the non dipole field computed from our coefficients are similar to those of YUKUTAKE (1971). The main differences involve the axisymmetric field coefficients and localized details of the non dipole field for individual epochs. Further palaeomagnetic work is needed before reliable determinations of past changes in field intensities can be made. We have computed field intensity changes using as simple a model as possible consistent with the available palaeointensity data.

Using a robust weighted least squares method we have produced a continuous series of spherical harmonic coefficients which facilitate comparison of the historical field with ancient palaeomagnetic fields. The plots of changes in local historical field direction can be used to date palaeomagnetic records from recent sediments anywhere in the world. Changes in local field direction in the Indian Ocean have traced out anticlockwise loops with a time scale of hundreds of years. The particularly low non dipole field in the Pacific area appears to have formed since 1800 A.D. and so does not seem to be due to a permanent or standing feature of the geomagnetic field. Many of the anticipated features of the historical field which have been inferred by investigators comparing time average palaeomagnetic data with the present day geomagnetic field are not revealed in our models.

Cross validation analysis produces a clear minimum in the optimization parameter (CVMSE) and so can be used to truncate 'objectively' the spherical harmonic analysis series.

Table 5. Total mean square contribution to the vector field from all harmonics of degree n.

Epoch	Degree			
	1 $10^{-11}\,T^2$	2 $10^{-11}\,T^2$	3 $10^{-11}\,T^2$	4 $10^{-11}\,T^2$
1600	263	6.8	4.3	4.6
1650	252	5.8	0.9	2.8
1700	242	4.2	0.7	1.9
1750	233	2.9	1.2	1.6
1800	224	2.5	1.6	1.3
1850	216	2.9	1.8	1.1
1900	206	3.7	2.2	1.1
1950	194	4.9	2.8	1.1

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