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An alternative method of calculating finite plate rotations

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Finite plate rotation angles and poles for the Mesozoic opening of the north America–Africa plate pair have been calculated using a fast direct analytical technique based on a maximum likelihood estimator rather than the usual trial and error search methods. Good agreement is found between the two methods. The maximum likelihood method involves matching paired locations of fracture zone magnetic lineation junctions.

1. Introduction

Finite rotations of plate pairs are commonly estimated by an iterative search approach. Following Morgan (1968) and le Pichon (1968) the observed azimuths of transform faults are compared with the calculated azimuths of circles concentric about trial poles. An optimization procedure is used to locate the 'best' position of the plate rotation pole by finding the minimum of a function based on the differences between the calculated and observed azimuths. The variation of sea-floor spreading rate with the sine of the angular distance from the rotation pole may also be used to help in locating the 'best' pole position.

Determination of errors associated with these iterative methods poses some difficulties. Commonly, so-called confidence limits are estimated instead although there is usually no probabilistic reason for this nomenclature. A trial and error search method is again used to investigate the structure of the azimuth difference function around the 'best' rotation pole position. The 'confidence limits' are often placed where the azimuthal difference function has increased 10% above the lowest value discovered. Elliptical limits are found elongated along the general direction perpendicular to the transform faults of the plate pair under study. The magnitudes of the plate rotations are estimated from the superimposition of sea-floor spreading isochron anomalies, again commonly through a trial and error search technique.

As an alternative to trial and error approaches we have investigated a method which involves firstly combining fracture zone traces and marine magnetic anomaly lineations to form sets of paired locations. These matched pairs can then be used to calculate finite plate rotations and their errors directly. The basis of the method is that pairs of latitudes and longitudes from the two plates are formed from anomaly/fracture zone meeting points which originally lay together on the plate boundary. The matched pairs are aligned using a maximum likelihood solution (Moran, 1976) for the three-dimensional finite plate rotation.

As an example of the approach we have used the matched pair method to calculate the relative rotations of the African–North American plates for six Mesozoic anomalies.

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2. Estimation of the rotation

Moran (1976) has suggested a simple and elegant algorithm for estimating a three-dimensional rotation which appears to be not widely known among earth scientists concerned with plate tectonics and continental drift.

Suppose for a given pair of magnetic anomalies that n such matched pairs are obtained, and that the data are expressed in direction cosine form as $\underline{x}_i = (x_{i1}, x_{i2}, x_{i3})^T$, $i = 1, 2, \ldots, n$ for the African plate, and correspondingly $\underline{y}_i = (y_{i1}, y_{i2}, y_{i3})^T$ for the North American plate, where $\underline{x}_i^T x_i = 1$ and $\underline{y}_i^T \underline{y}_i = 1$. If $\underline{x}_1, \ldots, \underline{x}_n$ are regarded as fixed and known exactly, if $\underline{y}_1, \ldots, \underline{y}_n$ are supposed subject to measurement error, and if the usual laws of plate tectonics are obeyed strictly, then Moran suggests the following statistical model. In the absence of measurement error, $\underline{y}_i = H^T \underline{x}_i$, where H is a proper 3×3 rotation matrix. Suppose that $\underline{y}_i$ is a realization of the Fisher distribution with unknown concentration parameter K, and with mean direction $H^T \underline{x}_i$; the matrix H is to be estimated. Thus, if $t_i = \underline{x}_i^T H \underline{y}_i$ the cosine of the angle between the African direction x_i and the North American direction rotated back into Africa. Hy_i, then we assume that the t_i are a random sample from the probability density $(K/2 \sinh K)\, e^{Kt}\, dt$, $-1 \leqslant t \leqslant 1$.

The method suggested by Moran is a special case of the classical Procrustes rotation problem in multivariate analysis (Mardia et al., 1979, p. 416, see also Stephens, 1979) wth point estimates expressed in terms of the singular value decomposition of the cross-product matrix S defined below. For present purposes the following alternative method is more convenient in that interval estimation is relatively straightforward. Moran shows that the maximum likelihood estimate of H given the above assumptions is relatively easily calculated, provided that the matrix H is parameterized using a unit quaternion, or equivalently by using the symmetric Euler parameters of the rotation (le Pichon et al., 1973, Appendix, Whittaker, 1944, p. 8). If H represents a clockwise rotation through an angle ψ, $0 \leqslant \psi \leqslant \pi$, about the unit 3-direction $\underline{v} = (v_1, v_2, v_3)^T$, $\underline{v}^T \underline{v} = 1$, then H may be expressed simply in terms of the *unsigned* unit 4-direction $u = (u_0, u_1, u_2, u_3)^T$, $\underline{u}^T \underline{u} = 1$, where $u_0 = \cos \frac{1}{2}\psi$ is taken here to be non-negative, and $u_j = v_j \sin \frac{1}{2}\psi$, $j = 1, 2, 3$ (le Pichon et al., 1973, Appendix, Prentice, 1986). Thus $v_3 = \cos\theta$, $v_2 = \sin\theta \cos\phi$, and $v_1 = \sin\theta \sin\phi$, where $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$ and $-\pi < \phi \leqslant \pi$. Moran (1976) showed that for the spherical regression model described above with homoscedastic Fisher errors, the maximum likelihood estimate $\hat{\underline{u}}$ of $\underline{u}$ was the dominant eigenvector v_0 of the 4×4 symmetric matrix.

$$M = \begin{pmatrix} (S_{11}+S_{22}+S_{33}), & (S_{32}-S_{23}), & (S_{13}-S_{31}), & (S_{21}-S_{12}) \\ & (S_{11}-S_{22}-S_{33}), & (S_{12}+S_{21}), & (S_{31}+S_{13}) \\ & & (S_{22}-S_{11}-S_{33}), & (S_{23}+S_{32}) \\ & & & (S_{33}-S_{11}-S_{22}) \end{pmatrix}$$

where (S_{jk}), $1 \leqslant j, k \leqslant 3$ is the 3×3 cross-product matrix $n^{-1} \sum_{i=1}^{n} \underline{y}_i x_i^T$. Also, if λ_0, $0 \leqslant \lambda_0 \leqslant 1$, is the corresponding dominant eigenvalue then the maximum likelihood estimate of K is the solution of $K^{-1} - \coth K + \lambda_0 = 0$, or $\hat{K} = (1 - \lambda_0)^{-1}$ approximately, if $1 - \lambda_0$ is small, as it is in all examples considered below. The maximum likelihood estimate of H is

$$\hat{H} = \begin{pmatrix} (\hat{u}_0^2 + \hat{u}_1^2 - \hat{u}_2^2 - \hat{u}_3^2), & 2(\hat{u}_1\hat{u}_2 - \hat{u}_0\hat{u}_3), & 2(\hat{u}_0\hat{u}_2 + \hat{u}_1\hat{u}_3) \\ 2(\hat{u}_0\hat{u}_3 + \hat{u}_1\hat{u}_2), & (\hat{u}_0^2 + \hat{u}_2^2 - \hat{u}_1^2 - \hat{u}_3^2), & 2(\hat{u}_2\hat{u}_3 - \hat{u}_0\hat{u}_1) \\ 2(\hat{u}_1\hat{u}_3 - \hat{u}_0\hat{u}_2), & 2(\hat{u}_0\hat{u}_1 + \hat{u}_2\hat{u}_3), & (\hat{u}_0^2 + \hat{u}_3^2 - \hat{u}_1^2 - \hat{u}_2^2) \end{pmatrix}$$

Moran referred to the possibility of finding the variance–covariance matrix of the estimated $\hat{\underline{u}}$ and $\hat{K}$, but did not explicitly evaluate the second derivatives of log L, the log-likelihood function. It is straightforward to verify that $\partial^2 \log L/\partial \underline{u} \partial \underline{u}' = 2nK(M - \lambda_0 I_4)$ so that if $\hat{\underline{u}}$ has variance matrix Σ_u then in large samples $n\Sigma_u$ is approximately

$$W = (2K)^{-1} \sum_{j=1}^{3} (\lambda_0 - \lambda_j)^{-1} \underline{v}_j \underline{v}_j^T,$$

where

$$W = (w_{jk}),\ 0 \leqslant j,\ k \leqslant 3,$$

and where $\underline{v}_1$, $\underline{v}_2$, $\underline{v}_3$ and λ_1, λ_2, λ_3 are the other eigenvectors and corresponding eigenvalues of the matrix M. The large-sample variance matrix of (ψ, θ, ϕ), where ψ is the rotation angle, $u_0 = \cos \frac{1}{2}\psi$, θ is the colatitude of the pole of rotation, $u_3 = \sin \frac{1}{2}\psi \cos \theta$, and ϕ is the longitude of the pole of rotation, $u_1/u_2 = \tan \phi$ may be found by transforming W by the Jacobian

$$J = \begin{pmatrix} -2(1-u_0^2)^{-1/2} & 0 & 0 & 0 \\ u_3 u_0 (1-u_0^2)^{-1} (u_1^2+u_2^2)^{-1/2} & 0 & 0 & (u_1^2+u_2^2)^{-1/2} \\ 0 & u_2(u_1^2+u_2^2)^{-1} & -u_1(u_1^2+u_2^2)^{-1} & 0 \end{pmatrix}$$

Thus $(\hat{\psi}, \hat{\theta}, \hat{\phi})$ has large-sample covariance matrix $n^{-1}W^*$ where $W^* = JWJ^T$; this may be used to construct approximate confidence intervals for each of ψ, θ, ϕ and confidence regions for (θ, ϕ) for example, or (ψ, θ, ϕ), provided that the intervals, ellipses or ellipsoids constructed are relatively small and nowhere near the North or South Poles. Note that well-determined estimates are obtained only if the rotation angle ψ is not too near 0, else the vector v is not well defined. Also, if the data are too closely clustered then the eigenvalues of M coalese and the variance matrix W becomes very large. Chang (1986) has developed the large sample theory of 'spherical regression' as he calls it for the general case of $p \times p$ rotations, without the assumption of Fisher errors, and gives explicit forms for very general hypothesis tests and also confidence regions which are independent of the coordinate system chosen. We are grateful to a referee for drawing this to our attention but do not develop this approach further here; we are content for present purposes to have easily computable approximate confidence regions rather than exact rotationally invariant regions with boundaries not so easily traced out. The work of Chang (1986) will be applied to an extension of the current work for use at triple points (Prentice and Thompson, 1987) with the usual constraint on the three angular velocity vectors.

3. Data

Mesozoic magnetic lineations have been charted off Northwest Africa by, for example, Hayes and Rabinowitz (1975) and off Southeast North

TABLE I

Mesozoic North American–African finite rotations

Anomaly	Estimated age [a] (Ma)	No. of paired meeting points	Rotation angle (°)	Latitude of rotation pole (°N)	Longitude of rotation pole (°W)	Estimated concentration parameter
M0	119	37	55.33 (0.28)	65.39 (0.22)	20.67 (0.30)	111000
M4	127	37	57.14 (0.28)	65.46 (0.22)	19.90 (0.30)	112000
M11	135	14	57.62 (0.30)	66.07 (0.30)	18.33 (0.52)	235000
M17	146	21	60.37 (0.22)	65.50 (0.18)	18.79 (0.29)	381000
M22	156	24	62.37 (0.31)	66.05 (0.21)	17.09 (0.37)	200000
M25	161	21	65.30 (0.52)	66.32 (0.32)	14.85 (0.67)	75000

* Harland et al. (1982).
Standard errors in parentheses.

America by, for example, Vogt and Einwich (1979). An excellent compilation of these and other geophysical central Atlantic data is provided by the TAG atlas of Rona (1980). Digitization of the meeting points of Mesozoic fracture zones and magnetic lineations in the TAG atlas provided us with six sets of paired latitudes and longitudes. Obvious inaccuracies in digitization or magnetic anomaly identification were removed by discarding meeting point pairs with unusually large discrepancies when rotated back to their supposedly coincident Mesozoic ridge crest positions. In this way we obtained six data sets of between 20 and 40 data pairs (Table I) on which to try out the maximum likelihood approach.

We have chosen to treat the paired latitudes and longitudes as independent rotation estimates and to calculate 90% confidence regions. Engebretson et al. (1984) discussed an alternative method of weighting such data based on the number of fracture zones rather than the number of meeting points under analysis.

4. Results

The finite rotation poles for anomalies M0, M4, M11, M17, M22 and M25 as calculated from the matched magnetic lineations and fracture zone meeting points are listed in Table I along with

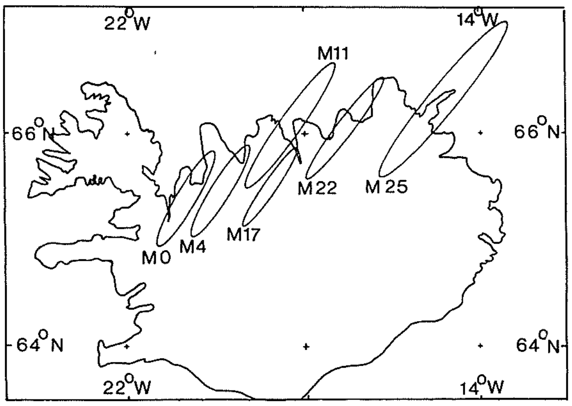

Fig. 1. 90% confidence regions around the maximum likelihood finite rotation poles of the Mesozoic North American–African anomalies M25, M22, M17, M11, M4 and M0.

their standard errors, in parentheses, and their associated Fisherian concentration parameters. The six rotation poles all lie close together in the region of Iceland (Fig. 1). Our Mesozoic poles can be compared with the total opening pole of 67.6° N 14° W of Bullard et al. (1965) and the quiet zone fit of 67.0° N 13.7° W of le Pichon et al. (1977). The general agreement is good as all the poles lie in the same part of the world. Closer inspection reveals our Mesozoic poles moving towards the total opening pole and quiet zone pole as we go backwards in time through the Mesozoic. Such a progression is not unexpected and could have been caused either by the instantaneous rotation pole changing position throughout the Mesozoic or else by a more recent jump of the instantaneous rotation pole to a new location. Preliminary differencing and differentiation calculations indicate that the instantaneous rotation pole was moving throughout the Mesozoic.

Our error limits, calculated using large-sample statistical theory, are much smaller than those based solely on fracture zone data, and they do not substantially overlap. Their small size suggests that with a slightly better series of matched pair derived rotations, obtained for example through analysis of original rather than charted and digitized data, from additional magnetic anomaly isochrons or from improved fracture zone location using SEASAT data, it should be possible to estimate the movements of instantaneous rotation poles.

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