Image Distortions and how to correct them

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If you can not read the red text

By which I mean this bit

then why not move closer to the front?
Distortion of remote-sensed images

There are 2 types of distortion:

- Radiometric distortion
  The brightness of the pixel is affected
  - by differences between sensors
  - by the atmosphere

- Geometric distortion
  - The pixels are different shapes
  - ... and different sizes ...
  - ... and in different places ...
  - ... from what you would naturally assume

Here we mostly consider geometric distortion
Types of geometric distortion

- Earth rotation
- Panoramic distortion
  - Further affected by Earth curvature
- Scan time skew
- Platform variations of
  - Height
  - Velocity
  - Attitude
    * Pitch
    * Roll
    * Yaw
- Aspect Ratio Distortion
Displaying an image
Radiometric Distortion - Striping
Earth Rotation

- Landsat MSS:
  - Standard image is 185 km square
  - and consists of 2340 scan lines
  - Distortion of whole image is about 10 km.
  - Which is about 4.6 m per scan line
Panoramic distortion

\[ p_\theta = \beta h \sec^2 \theta \]
\[ = p \sec^2 \theta \]
Panoramic distortion: S-band effect

Small for SPOT, LANDSAT: $\theta = 7.5^\circ$: edge pixels 2% bigger

Large for AVHRR

Very large for aircraft instruments $\theta = 40^\circ$: edge pixels 70% bigger
Panoramic distortion: Earth curvature

\[ p_c = \beta [h + r_e (1 - \cos \phi)] \sec \theta \sec (\theta + \phi) \]

\[ \phi \rightarrow 0 : p_c \rightarrow \beta h \sec^2 \theta \]

- **NOAA AVHRR**
  - pixels 2.89 times bigger if earth flat
  - pixels 4.94 times bigger if earth round
Scan-time skew

- Landsat MSS:
  - Pixel size 79 m: 6 pixels in swath
  - Linescan skew 213 m
  - About 3 pixels
  - Earth rotation: 27 m per swath
  - 10 km per image

- SPOT
  - Push broom: no skew
Attitude

- Pitch
- Yaw
- Roll
- Forwards Motion
Effects of instant attitude variations

- Velocity Increase
- Altitude Increase
- Roll
- Yaw
- Pitch

Spacecraft motion
Effects of slow attitude variations

- Velocity Increase
- Altitude Increase
- Roll
- Yaw
- Pitch

Spacecraft motion
Scan nonlinearity

Scan Angle

Landsat MSS: 395 m or 5 pixels
Non-square pixels

One Pixel

Scan

Spacecraft Motion
Correcting geometric distortions

For each new pixel \((x, y)\) in the map plane we find the equivalent point \((u, v)\) in the image plane. We need to know the functions \(u(x, y), v(x, y)\) – we can get these in two ways

- Assume functional form and fit parameters
- Understand the nature of the distortion
Interpolating

- Nearest neighbour
  - Use value from pixel nearest to $u, v$
  - That is pixel $f$ in the figure
  - Simple and fast
  - New pixel vectors were in old image
  - Better for classification

- Linear interpolation
  - Use four pixels surrounding $u, v$
  - Interpolate between $e$ and $g \rightarrow p$, $f$ and $h \rightarrow q$, then between $p$ and $q$
  - Smoother result

- Bicubic splines
  - Uses all 16 pixels shown in figure.
  - Even smoother result, but more complex
  - May generate spurious bright and dark pixels
Mapping functions

There are two approaches to the mapping functions.

- **Understand their form**
  - Using the details discussed earlier
  - Good if mapping functions are simple
  - Often done by suppliers before you get the image.

- **Approximate them with polynomials**
  - Coefficients obtained by fitting to control points
  - Useful where many distortion effects are combined
  - So mapping functions are very complex
  - Limitations if distortion is very strong
  - Can be used to register one image to another
Mapping Polynomials

\[ u = a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2 \]

\[ v = b_0 + b_1x + b_2y + b_3xy + b_4x^2 + b_5y^2 \]

We find the \( a \) and \( b \) coefficients by choosing a number of points which we can identify in both the map and the image. These are called CONTROL POINTS.

At each control point we know \( u, v, x \) and \( y \). The first equation above gives one equation with 6 unknown \( a \) coefficients. If we have 6 control points, we have 6 equations with 6 unknown \( a \) coefficients.
Mapping Polynomials

\[
\begin{align*}
u &= a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2 \\
v &= b_0 + b_1 x + b_2 y + b_3 xy + b_4 x^2 + b_5 y^2
\end{align*}
\]

Usually choose more than 6 control points and use least squares solution. The second equation gives another set of equations for the \( b \) coefficients which are solved exactly like the \( a \) set.
Mapping functions: Analytic

Aspect Ratio Distortion

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
1 & 0 \\
0 & \gamma
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
\]

\[
\begin{bmatrix}
u \\
v
\end{bmatrix}
= \begin{bmatrix}
1 & 0 \\
0 & 1/\gamma
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\]

Earth Rotation

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
1 & \alpha \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
\]

\[
\begin{bmatrix}
u \\
v
\end{bmatrix}
= \begin{bmatrix}
1 & -\alpha \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\]
Combining Corrections

Correct aspect ratio, then earth rotation

\[
\begin{bmatrix}
  u \\
  v
\end{bmatrix} = \begin{bmatrix}
  1 & -\alpha \\
  0 & 1
\end{bmatrix} \begin{bmatrix}
  1 & 0 \\
  0 & 1/\gamma
\end{bmatrix} \begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

Note that matrix multiplication is not commutative: The order in which you apply the corrections matters.
Image corrected for Earth Rotation