On the Fuzzy Distance Between Fuzzy Geographical Objects

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Summary The distance between geographical entities is an important geographical measurement. The geographical distance between two Boolean entities is found with the Pythagorean equation, but the distance between fuzzy geographical entities is a more complex phenomenon. Distance is no longer a single number but has to be modelled as a set of values, and modelling this set as a fuzzy number or numbers has the advantage that fuzzy mathematics can be applied to the resulting values. The basic approach to doing this is explored in this paper.

Keywords Fuzzy sets, Fuzzy Distance, Euclidean Distance, Fuzzy Numbers

1. The distance between two fuzzy spatial entities.

The geographical distance between two places is one of the fundamental metric relations between any two different locations (Gatrell, 1983). Within a Boolean concept of space, the distance between two locations is simple and well understood, but if the model of the objects is changed from Boolean to fuzzy then the problem of identifying the distance between them is vastly complicated. If we model locations in space as fuzzy sets or entities, then any location can belong to a set (a place) to a degree, recorded as a real number between 0 and 1 (Fisher, 2000; Zadeh, 1965). This paper addresses the issue of the distance between two such fuzzy geographical entities.

The issue of fuzzy distances has been explored in the past. Altman (1994), for example, presented a fuzzy distance function, but his function yielded a non-fuzzy number, with which it is hard to perform subsequent mathematical operations. In the fuzzy set literature, however, Rosenfeld (1985), Voxman (1998), Bloch (1999) and Guha and Chakrborty (2010), among others have examined the issue. Most of these extend the concept of distance to subsets of a metric space, and they argue for the representation of fuzzy distance as fuzzy numbers. These studies tend to suggest many potential applications for different areas, including pattern recognition, image processing, robotics, computer graphics and engineering. However, few if any make use of geographical distance or link fuzzy distance to real world phenomena although Guesgen and Hertzberg (2001), Guesgen et al. (2003) and Fisher and Almadani (2010) have all looked at the related topic of fuzzy buffering.

One important concept of the fuzzy set literature, which is extensively exploited in this paper, is the α-cut. An α-cut set is a Boolean (or classic) set, where an object or location belongs to a particular α-cut of a fuzzy set if its membership in that set is greater than or equal to α. It therefore follows that any fuzzy set can be divided into a large number of α-cuts or, more correctly, α-cut sets. Further, from a group of α-cuts of a particular fuzzy set, it is possible to recreate the original fuzzy set; indeed, this is one one method for generating fuzzy sets in the first place (Pedrycz and Gomide, 1998, p20).

2. The distance between two Boolean geographical objects

If we have two Boolean objects, A and B, modelled as point locations (for the present), then both A and B are conceptualised to occupy infinitesimally small areas at particular x, y locations, x_a, y_a and x_b, y_b. The distance between them can be found very simply by the usual Pythagorean equation (1).

$$DIST(x,y) = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2} \qquad (1)$$

This equation is a special case of the more general Minkowski distance (Pedrycz and Gomide, 1998, p49), where the power term can attract any value and its reciprocal. The Boolean case means that the distance between the two locations is a single value. For locations A and B in this study their separation when modelled in this way is 150 distance units.

3. The distance between a Boolean and a fuzzy geographical object

When one of the two locations is modelled as a fuzzy entity, then the situation is complicated. There are number of possible distances between the Boolean object and the fuzzy object, which can be modelled as a fuzzy number. The situation is illustrated in Figure 1, and the distances for each α-cut through the fuzzy sets illustrated in Figure 2, giving rise to a fuzzy number (Table 1) illustrated in Figure 3.

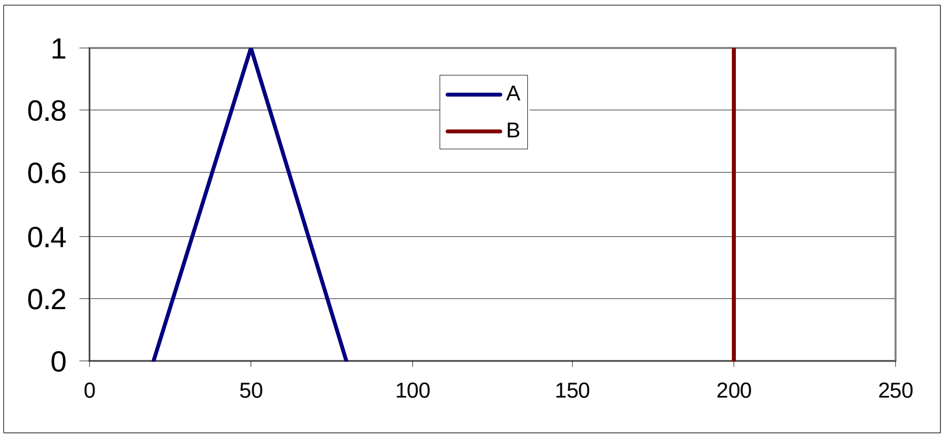

Figure 1. Modelling Entity A as a fuzzy set and Entity B as a Boolean point.

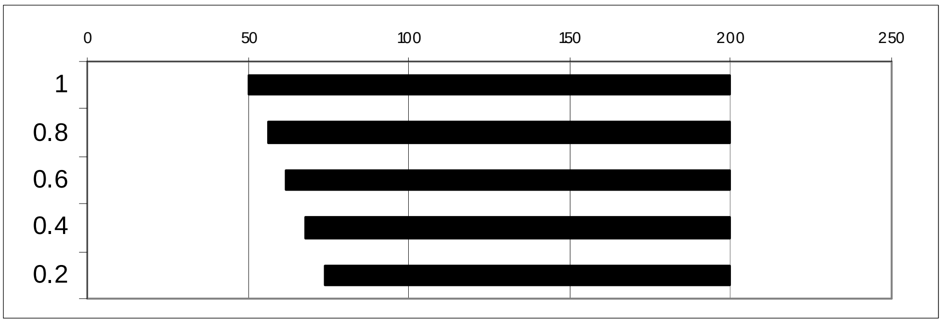

Figure 2. The α-cut distances between the fuzzy entity and the Boolean point

Table 1. α-cut distances between fuzzy A and Boolean B

α-cut	Distance
1	150
0.8	144
0.6	138
0.4	132
0.2	126

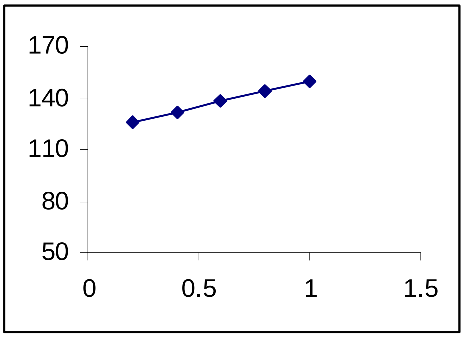

Figure 3. A graphic illustration of the fuzzy number of the distance between fuzzy A and Boolean B.

4. The distance between two Fuzzy geographical objects

When both A and B are modelled as fuzzy sets, the situation is complicated still further. In the first place, the model can be illustrated as in Figure 4.

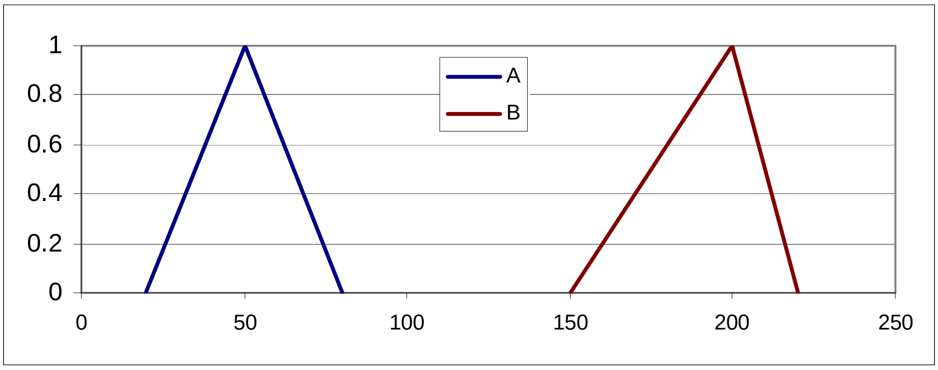

Figure 4. The models of both A and B as fuzzy sets.

The distance can then be determined at each α-cut value of both A and B (Figure 5) and can be represented in graphic form in Figure 6.

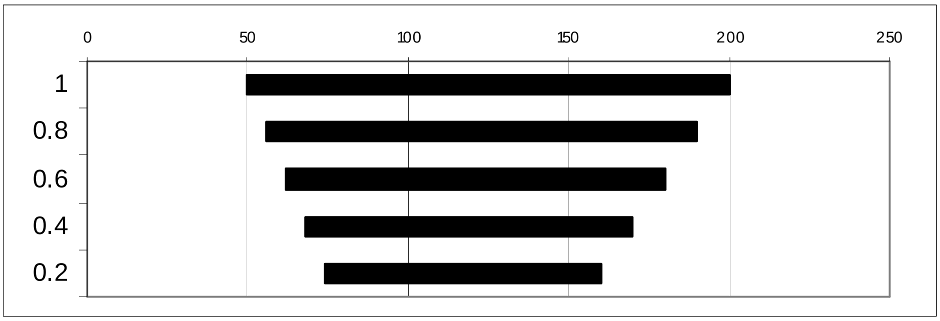

Figure 5. Distances between fuzzy A and fuzzy B as a set of α-cut values.

Table 2. α-cut distances between fuzzy A and fuzzy B

α-cut	Distance
1	150
0.8	134
0.6	118
0.4	102
0.2	86

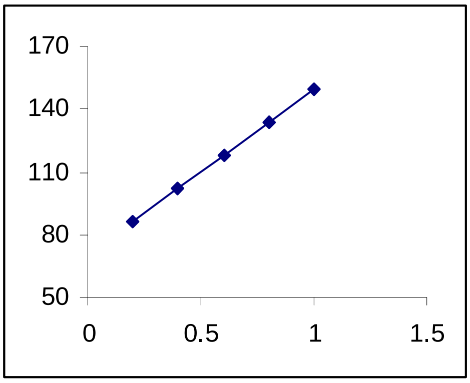

Figure 6. The distance between fuzzy A and fuzzy B as a fuzzy number.

Table 3. The full development of the fuzzy distance between fuzzy A and fuzzy B for α-cut intervals of 0.2.

A \ B	1	0.8	0.6	0.4	0.2
1	150	144	138	132	126
0.8	140	134	128	122	116
0.6	130	124	118	112	106
0.4	120	114	108	102	96
0.2	110	104	98	92	86

However, there is no reason that the distance should be measured from A(α=0.9) to B(α=0.9). Rather, there are a range of possible distances for A(α=0.9) to B(α=0.1), and A(α=0.1) to

B(α=0.9) and all possible distances between. Thus, the fuzzy number can now be approximated as a two dimensional array (Table 3) in which the α-cuts form the two dimensions.

5. Fuzzy distance operators

The Distance Operation available in some raster GIS determines the shortest distance at each grid intersection to a set of target locations. In the interpretation of distance presented here, this distance operation should return not a single number for each grid intersection as in all standard GIS, but rather a fuzzy number for each grid intersection. Because the grid intersection is a clearly defined location, itself infinitesimally small in extent, the interpretation of distance presented here in Figure 2 and 3, and Table 1 is applicable. This can be approximated by determining the distance to all locations for each α-cut of the target fuzzy object, resulting in as many distance maps as there are α-cuts being considered.

A number of spatial statistics are dependent on the determination of the distance between objects, and from this work follows the clear suggestion that when the objects are conceptualized as fuzzy objects the result of spatial statistical analysis should be a fuzzy number. Fuzzy clustering of statistical cases is well established (Bezdek, 1981; Bezdek et al., 1984) in remote sensing applications, based on the idea that observations are Boolean point objects. However, this notion of distance between fuzzy objects is novel in a spatial analytical setting. It is clear that the similarity matrix for such geographical objects, conceptualized as having fuzzy extent within the n-dimensional attribute space, has numerous implications. Future work will explore these implications.

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8. Biography

Peter Fisher is Professor of Geographical Information at the University of Leicester. He has researched many aspects of uncertainty in geographical information.

Firdos Almadani is a post graduate student at the University of Leicester, and a Teaching Assistant in the Geography Department at King Abdul-Aziz University, Jeddah, Saudi Arabia. Work reported here relates to her PhD research.

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