Separation of signal and noise by signature deconvolution
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Summary
This paper proposes a method for signature deconvolution that also separates signal from noise and provides a measure of the quality of the deconvolution. Classical signature deconvolution provides no measure of the quality of the result. We here formulate recovery of the earth impulse response as the calculation of a Wiener filter in which the estimated source signature is the input and the measured response is the output. Convolution of this filter with the estimated source signature is the component of the measured response that is correlated with the estimated signature. Subtraction of this correlated component from the measured response yields the uncorrelated component: the estimated noise. If the estimated source signature contains errors, the estimated earth impulse response is incomplete, and the estimated noise contains signal, recognizable as trace-to-trace correlation.

We first review the convolutional model for the measured data and the classical inverse-filter approach to signature deconvolution. Then we present the optimization approach based on Wiener’s (1949) theory and finally show how this method works on real data.

Convolutional model and classical signature deconvolution

We consider a geophysical experiment to investigate the earth using an active source and passive receivers. For instance, this could be a seismic experiment or a transient electromagnetic experiment. Let the measurement at one receiver be

\[ x_t = s_t \ast g_t + n_t, \]

where \( s_t \) is the source signature, \( g_t \) is the earth impulse response, the asterisk \( \ast \) denotes convolution, and \( n_t \) is the noise and is what would be measured if there had been no geophysical experiment; the subscript \( t \) denotes the time sample. The source signature is not known exactly. An estimate \( \hat{s}_t \) of the signature is obtained by measurements, by modeling, or
Separation of signal and noise by signature deconvolution

by some other means (e.g. Osman and Robinson, 1996) such that

$$\hat{s}_t = s_t - e_{st},$$

(2)

where $e_{st}$ is the unknown error in the source signature estimate. Classical deconvolution (e.g. Robinson and Osman, 1996) finds an approximate inverse filter $f_t$ of $s_t$ such that

$$f_t * \hat{s}_t = d_t,$$

(3)

where $d_t$ is a known band-limited impulse. Signature deconvolution is then the result of the convolution of this approximate inverse filter with the measurement:

$$f_t * \hat{s}_t = d_t,$$

(4)

The first term on the right-hand side of equation (4) is the required result: it is the true, unknown, impulse response convolved with the known bandlimited impulse; the second term is an error caused by uncertainty in the source signature estimate; the third term is the convolution of the approximate inverse filter with the noise. It is well known (e.g. Robinson and Osman, 1996) that this noise term is dependent on the design of the bandlimited impulse $d_i$; for example, if $d_t = \delta_t$, a perfect digital impulse, the noise blows up at frequencies where the estimated signature $\hat{s}_t$ has little or no energy. It is not known how big the second and third terms are compared with the desired first term. That is, we are not able to distinguish these terms in the deconvolved data.

Wiener optimization approach

We desire to find an estimate $\hat{g}_t$ of $g_t$, with error $e_{g} = g_t - \hat{g}_t$, such that

$$\sum_k (x_t - \hat{s}_t * \hat{g}_t)^2$$

is a minimum. This problem was solved by Wiener (1949). The equations to be solved for $\hat{g}_t$ are the well-known normal equations

$$\begin{bmatrix} a(0) & a(1) & \cdots & a(n) \\ a(1) & a(0) & \cdots & a(n-1) \\ \vdots & \vdots & \ddots & \vdots \\ a(n-1) & a(n) & \cdots & a(0) \end{bmatrix} \begin{bmatrix} \hat{g}_0 \\ \hat{g}_1 \\ \vdots \\ \hat{g}_n \end{bmatrix} = \begin{bmatrix} b(0) \\ b(1) \\ \vdots \\ b(n) \end{bmatrix}$$

(5)

where $a(\tau)$ is the autocorrelation of $\hat{s}_t$, and $b(\tau)$ is the cross-correlation of $x_t$ with $\hat{s}_t$:

$$a(\tau) = \sum_k \hat{s}_t \hat{s}_{t-\tau},$$

(6)

$$b(\tau) = \sum_t x_t \hat{s}_{t-\tau}.$$  

(7)

Convolution of $\hat{s}_t$ with $\hat{g}_t$ gives the best least-squares match to the data, by definition, and is

$$y_t = \hat{s}_t * \hat{g}_t$$

$$= (s_t - e_{st}) * g_t * e_{g}$$

$$= s_t * g_t - s_t * e_{g} + e_{st} * g_t - e_{st} * e_{g}.$$  

(8)

Subtracting (8) from (1) yields an estimate of the noise, which is

$$x_t - y_t = \hat{n}_t = n_t + s_t * e_{g} + e_{st} * g_t + e_{st} * e_{g}$$

$$= n_t + s_t * e_{g} + e_{st} * g_t + e_{st} * e_{g}.$$  

(9)
Separation of signal and noise by signature deconvolution

The terms on the right-hand side of (9) are all unknown. The first term is the true noise; the second term is an error caused by the limitations of the least-squares process and will look like geophysical data; the third term is caused by a systematic error in the source signature estimate and will look like geophysical data; the fourth term is a second-order error caused by the source signature error and the limitations of the least-squares process.

A measure of the quality of the deconvolution is the ratio of the energy in \( y_i \) divided by the energy in the original data \( x_i \); that is, the quality factor \( Q \) may be defined as

\[
Q = \frac{\sum y_i^2}{\sum x_i^2}.
\]

This is a number less than unity, but the closer it is to unity, the better is the result.

Examples

Figure 1 shows an application of the method to marine transient electromagnetic data (Ziolkowski et al., 2011) in which the source is an electric current dipole, the signature is one period of a pseudo-random binary sequence, and the receiver is an in-line voltage dipole. The \( Q \) factor for this example is 0.946.

The estimated noise for the 48 traces in the line is displayed in Figure 2. There is no obvious correlation of the noise from trace to trace, indicating that the source signature estimate is excellent, as one would expect from the direct measurement of the source current.

Figure 1: (a) Measured source signature (Amp); (b) measured receiver response at 2025 m offset (V/m); (c) Wiener impulse response derived from (a) and (b) plotted on same time scale; (d) blue curve is the part of (b) correlated with (a), red curve is response (b) minus the blue curve.
Separation of signal and noise by signature deconvolution

A seismic example is shown below. Figure 3 shows the estimated deghosted signature from a marine air gun array and Figure 4 shows the result of deconvolving one trace of a shot gather with this signature. The quality factor is 0.98, so the correlated part of the signal, shown in blue in Figure 4(c) is almost identical with the seismogram, 4(a). The deconvolution has increased the resolution considerably. The noise, shown in red in Figure 4(c) is not random.

Conclusions

Classical signature deconvolution produces deconvolved data with a known wavelet, provided the source signature is known precisely. Systematic errors in the estimate of the source signature lead to an additional noise term that is indistinguishable from seismic data and is unquantifiable. By reformulating deconvolution as the problem of finding the optimum estimated earth impulse response, given the estimated source signature and measured output signal, we have separated out the portion of the signal that is correlated with the estimated signature and have also obtained an estimate of the noise, which may contain remains of undeconvolved seismic data. We have applied the method to seismic data, and to transient electromagnetic data, described more fully in Ziolkowski et al. (2011), with excellent results.

Acknowledgements

I gratefully thank PGS for supporting the research and for permission to show the data.
EDITED REFERENCES

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REFERENCES


