Review of vibroseis data acquisition and processing for better amplitudes: adjusting the sweep and deconvolving for the time-derivative of the true groundforce

Anton Ziolkowski*

PGS, Geoscience and Engineering, Birch House, 10 Bankhead Crossway South, Edinburgh EH11 4EP, UK

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ABSTRACT

The goal of vibroseis data acquisition and processing is to produce seismic reflection data with a known spatially-invariant wavelet, preferably zero phase, such that any variations in the data can be attributed to variations in geology. In current practice the vibrator control system is designed to make the estimated groundforce equal to the sweep and the resulting particle velocity data are cross-correlated with the sweep. Since the downgoing far-field particle velocity signal is proportional to the time-derivative of the groundforce, it makes more sense to cross-correlate with the time-derivative of the sweep. It also follows that the ideal amplitude spectrum of the groundforce should be inversely proportional to frequency. Because of non-linearities in the vibrator, bending of the baseplate and variable coupling of the baseplate to the ground, the true groundforce is not equal to the pre-determined sweep and varies not only from vibrator point to vibrator point but also from sweep to sweep at each vibrator point. To achieve the goal of a spatially-invariant wavelet, these variations should be removed by signature deconvolution, converting the wavelet to a much shorter zero-phase wavelet but with the same bandwidth and signal-to-noise ratio as the original data. This can be done only if the true groundforce is known. The principle may be applied to an array of vibrators by employing pulse coding techniques and separating responses to individual vibrators in the frequency domain. Various approaches to improve the estimate of the true groundforce have been proposed or are under development; current methods are at best approximate.

INTRODUCTION

The theory of the elastic radiation from a vibrator has been studied extensively for over half a century, following the seminal papers of Bycroft (1956) and Miller and Pursey (1954). Geyer (1989) provided a comprehensive collection of papers on vibroseis. In a landmark paper that provoked much discussion, including the famous comments by Sallas and Weber (1982), Lerwill (1981) compared the amplitude and phase response of a model vibrator with the performance of a real vibrator in the field and came to the conclusion that the main characteristics of the Earth’s response could be obtained by appropriate measurements ‘at the output of the vibrator’. This output, we now understand to be the ‘groundforce’, which is the force of the vibrator baseplate on the ground. It is, in fact, the boundary condition at the Earth’s surface. Its measurement, it turns out, is not simple.

This paper considers the measurements and processing steps that are required to produce seismic reflection data with a known spatially-invariant wavelet, preferably zero phase, such that any variations in the data can be attributed to variations in geology. A major step towards this goal was made by Sallas and Allen (1998a,b) who solved the problem of determining the impulse responses between individual vibrators in a vibrator array and any geophone. The solution requires

*E-mail: anton.ziolkowski@pgs.com
the force of the vibrator plate on the ground, the ‘groundforce’, to be known for each vibrator in the array. Estimation of the groundforce has been studied extensively since the famous ‘weighted sum’ invention of Castenet and Lavergne was published in 1965.

Conventionally the recorded vibroseis data are cross-correlated with the sweep. The control system of the vibrator attempts to make the estimated groundforce equal to the sweep but it is well-known that the control system is imperfect and the groundforce and the sweep are not the same: not only are there non-linearities in the vibrator itself, causing harmonics of the instantaneous frequency to be generated (e.g., Bagaini 2008), the groundforce measurement itself is not perfect, as discussed below. The groundforce varies from place to place, so the result of cross-correlation is to generate a wavelet that depends on the vibrator position. Given the true groundforce at each vibrator, it is possible with deconvolution to produce data in which the wavelet is laterally invariant.

An important issue that appears to have been largely overlooked in the acquisition of vibroseis data is that the downgoing particle velocity wavefield is basically proportional to the time-derivative of the groundforce (Baeten, Fokkema and Ziolkowski 1988). To recover the full bandwidth response of the Earth from the measured data, it is most efficient to illuminate the Earth with a broad bandwidth, preferably white, spectrum. It also follows that the ideal amplitude spectrum of the groundforce should be inversely proportional to frequency. This is counter-intuitive, since it emphasizes the low frequencies rather than the high frequencies and is not in agreement with current practice.

The convolutional model for a single vibrator is considered first to establish the framework for discussion. The cross-correlation process is then compared with deconvolution and a method for designing deconvolution operators is proposed. I discuss sweep design for an amplitude spectrum inversely proportional to frequency. It turns out that physical limitations of the vibrator at low frequencies increase the time that has to be spent beyond what would be expected (Bagaini 2008). The invention of Sallas and Allen (1998a,b) is then briefly described, highlighting the need for estimates of the true groundforce for each vibrator. It turns out that determination of the true groundforce is not solved exactly; but there are several approaches that, combined, may allow us to converge on a satisfactory approximation. Finally, I discuss the consequences for data quality of the proposal for groundforce amplitude to be inversely proportional to frequency.

**CONVOLUTIONAL MODEL FOR ONE VIBRATOR**

Baeten et al. (1988) modelled the wavefield of a vibrator at the surface of a homogeneous isotropic elastic half-space. Because the half-space is elastic, no energy is absorbed. In the far field below the vibrator the surface waves disappear and the downgoing particle displacement components are essentially proportional to the integrated surface traction components, or groundforce. It follows that the far-field downgoing particle velocity components are proportional to the time-derivative of the groundforce. This needs to be put into the formulation of the problem.

Appendix B of Baeten et al. (1988) defines the far field. There are two components. First, there is a constraint that comes straight from geometry: the far-field is at a distance that is large compared with the source dimensions (equations (B3) and (B4)); for a single vibrator the far-field would be much less than 100 m; for an array of vibrators it would be about 1 km. Second, it is also required that the far field be at a distance greater than a wavelength of both P-waves and S-waves (equation (B9)). Using a maximum velocity of 3000 m/s and a lowest frequency of 5 Hz, gives a far-field of 600 m. Taking the two constraints together, we are in the far field at ranges greater than about 600–1000 m below the source. This is shallower than most targets of interest. That is, most targets of interest are in the far-field.

Figure 1 shows a vertical vibrator source and geophone receivers at the surface of the Earth. Although the geophones measure particle velocity, it is convenient to start with the displacement. The particle displacement at the $k$th geophone

![Figure 1](image_url)
is

\[ u_k(t) = f(t) \ast g_k(t) + m_k(t), \]  

(1)
in which, \( m_k(t) \) is the particle displacement at the geophone, \( f(t) \) is the force of the vibrator on the ground, the asterisk \( * \) denotes convolution, \( g_k(t) \) is the particle displacement response at the receiver to an impulsive force acting vertically downwards at the vibrator position and \( m_k(t) \) is displacement noise at the receiver in the absence of the source. Note that \( g_k(t) \) is not some ‘reflectivity series’; it is the complete impulse response and contains P-waves, S-waves, surface waves including ground roll, reflections, refractions, diffractions, etc.

The particle velocity at the \( k \)th geophone is obtained by differentiating equation (1) to give

\[ v_k(t) = u'_k(t) = f'(t) \ast g_k(t) + n_k(t), \]  

(2)
in which the prime indicates time-derivative and \( n_k(t) \) is the time-derivative of \( m_k(t) \). Note that only one of \( f(t) \) or \( g_k(t) \) are differentiated. We choose to differentiate the groundforce \( f(t) \) because of the result of Baeten et al. (1988), resulting in a better description of the convolutional model in the far field. Krohn (2006) noted in paragraph 45: “Use of the time derivative of the ground force signal results in less noise and artifacts and less phase distortion than using the ground force signal itself.”

Equation (2) is the starting point for the subsequent discussion. We will be concentrating on trying to obtain the best possible estimate of \( g_k(t) \) from the measurement of the particle velocity \( v_k(t) \). We are particularly interested in the far field of the source where our targets are. The noise \( n_k(t) \) is the noise we would measure in the absence of our signal and has the normal characteristics of seismic noise measured with geophones. We will make no assumptions about this noise; we note that it is a problem for the deconvolution and that it is probably not white.

**CROSS-CORRELATION, DECONVOLUTION AND THE SPECTRUM OF THE SWEEP**

Cross-correlation

In conventional vibroseis processing, the data are cross-correlated with the sweep \( s(t) \). Cross-correlation is convolution with the time-reverse, so this can be expressed as

\[ s(-t) \ast v_k(t) = s(-t) \ast f'(t) \ast g_k(t) + s(-t) \ast n_k(t). \]  

(3)
The wavelet in the data is now

\[ p(t) = s(-t) \ast f'(t). \]  

(4)
which is the cross-correlation of the sweep with the time-derivative of the groundforce. The vibrator control system is usually designed to make the estimate of the groundforce equal to the sweep. Normally, however, the true groundforce is not exactly equal to the sweep. Even if it were, the wavelet would not be zero phase because of the time-derivative operator.

To obtain a better approximation to a zero-phase wavelet in the far-field it would be better to cross-correlate with the time-derivative of the sweep, which can be expressed as

\[ s'(-t) \ast v_k(t) = s'(-t) \ast f'(t) \ast g_k(t) + s'(-t) \ast n_k(t). \]  

(5)
The wavelet in the data is now

\[ p'(t) = s'(-t) \ast f'(t). \]  

(6)
If the groundforce is in-phase with the sweep, this wavelet will be more nearly zero phase. We recognize that although this makes sense for the far-field data, it is less good for the shallow near-field data.

Ideally, we should like to know the groundforce. If \( f(t) \) is known, the deconvolution is straightforward.

**Deconvolution**

The objective of deconvolution is to replace the source signature with an impulse or delta function \( \delta(t) \). First transform equation (2) to the frequency domain

\[ V_k(\omega) = -i\omega F(\omega) G_k(\omega) + N_k(\omega), \]  

(7)
in which \( \omega \) is angular frequency, \( V_k(\omega) \), \( F(\omega) \), \( G_k(\omega) \) and \( N_k(\omega) \) are the Fourier transforms of \( v_k(t) \), \( f(t) \), \( g_k(t) \) and \( n_k(t) \), respectively, the time-derivative becomes \(-i\omega \) and the convolution becomes a multiplication. The Fourier transform of \( \delta(t) \) is 1, so signature deconvolution in the frequency domain is simply division by the Fourier transform of the signature, which in this case is \(-i\omega F(\omega)\), yielding

\[ \frac{-V_k(\omega)}{i\omega F(\omega)} = G_k(\omega) - \frac{N_k(\omega)}{i\omega F(\omega)}. \]  

(8)
in which the Fourier transform of the Earth’s impulse response \( G_k(\omega) \) is recovered without contamination by any wavelet but the noise blows up at frequencies where \(|i\omega F(\omega)|\) becomes close to zero. Since the groundforce function \( F(\omega) \) is band-limited, \(|i\omega F(\omega)|\) is zero, or close to zero, at frequencies outside this band.

A stabilized deconvolution of equation (7) for \( G_k(\omega) \) can be performed as described by Baeten and Ziolkowski (1990, Ch. 7). Consider the filter \( z(t) \) with Fourier transform \( Z(\omega) \)
defined as

\[ \frac{D(\omega)}{i\omega F(\omega)} \approx \frac{i \tilde{F}(\omega) D(\omega)}{\omega |F(\omega)|^2 + \varepsilon} = Z(\omega), \]

(9)
in which \( \tilde{F}(\omega) \) is the complex-conjugate of \( F(\omega) \), \( \varepsilon \) is a small positive constant and \( D(\omega) \) is the spectrum of some short desired wavelet \( d(t) \). Appendix A discusses the design of this wavelet: it is preferably zero phase and as close to a delta-function as is possible within the available bandwidth. Its amplitude spectrum \( |D(\omega)| \) is smooth and, in this case, approximately equal to \( |i\omega F(\omega)| \). Therefore \( |Z(\omega)| \) is a constant over the entire bandwidth. Multiplication of the numerator and denominator by the complex conjugate of the spectrum of the wavelet \( i\omega \tilde{F}(\omega) \) makes the denominator real and positive and the small positive constant \( \varepsilon \) prevents overflow when \( |F(\omega)| \) is very small (Stoffa and Ziolkowski 1983). Multiplication of (7) by this filter gives

\[ Z(\omega) V_k(\omega) = -Z(\omega) i\omega F(\omega) G_k(\omega) + Z(\omega) N_k(\omega) \approx D(\omega) G_k(\omega) + Z(\omega) N_k(\omega). \]

(10)
The result is to replace the time-derivative of the groundforce \( f'(t) \) with the short desired zero-phase wavelet \( d(t) \), essentially without affecting the noise, since \( |Z(\omega)| \) is essentially a constant. This is the best we can do. Clearly, it is critical to know the groundforce.

Since the objective of the deconvolution is to replace \( f'(t) \) with an impulse, which has a flat amplitude spectrum, we conclude that it is desirable that \( f'(t) \) has a flat spectrum. Therefore \( f(t) \) should have an amplitude spectrum that decays inversely with frequency; that is,

\[ |F(\omega)| \approx \frac{A}{\omega}, \]

(11)
in which \( A \) is a constant. Equation (11) has implications for sweep design and vibroseis operations that are considered below.

**Adjusting the sweep**

A sweep can be described as

\[ s(t) = e(t) \sin (\theta(t)). \]

(12)
where \( e(t) \) is an envelope function and \( \theta(t) \) is the instantaneous phase. The derivative of the phase \( \theta'(t) \) is the instantaneous angular frequency \( \omega(t) \) and the second derivative \( \theta''(t) \) is the sweep rate. Rietveld (1977) showed that the amplitude spectrum of the sweep is related to the sweep rate as

\[ |S(\omega)| = \frac{1}{2} e(t) \left[ \theta''(t) \right]^{\frac{1}{2}}. \]

(13)

As shown in Appendix B, defining \( |S(\omega)| = A/\omega \) in equation (13), where \( A \) is a function of the sweep parameters and with the envelope function equal to one, yields the following sweep

\[ s(t) = \sin \left( \theta(0) + \frac{\omega_1 \omega_2 T}{(\omega_1 - \omega_2)} \ln \left[ 1 + \frac{(\omega_1 - \omega_2)T}{\omega_2 T} \right] \right), \]

(14)
where \( \theta(0) \) is the phase at \( t = 0 \) and \( \omega_1 \) and \( \omega_2 \) are the instantaneous angular frequencies at \( t = 0 \) and \( t = T \), respectively. Figure 2 shows this sweep, its autocorrelation function and its amplitude spectrum, for an instantaneous frequency that varies from \( \omega_1/(2\pi) = 5 \) Hz to \( \omega_2/(2\pi) = 80 \) Hz in a time \( T = 2 \) s. The ripples in the amplitude spectrum are the well-known Gibbs’ phenomenon caused by the sharp corners of the envelope function. A taper is usually applied at each end of the envelope to reduce these ripples (e.g., Bagaini 2008). Andersen (1994) developed a method for designing sweeps with simple autocorrelation functions having minimal side lobe energy. This was achieved partly by extending the bandwidth to frequencies as low as 1 Hz. Andersen’s (1994) method did not address the issue of harmonics introduced by non-linearities in the vibrator, bending of the baseplate and variable ground coupling.

The autocorrelation of the sweep of equation (14) is not very compact but this is to be expected since the amplitude spectrum is not flat. The time-derivative of this sweep,

\[ s'(t) = \cos \left( \theta(0) + \frac{\omega_1 \omega_2 T}{(\omega_1 - \omega_2)} \ln \left[ 1 + \frac{(\omega_1 - \omega_2)T}{\omega_2 T} \right] \right) \times \frac{\omega_1 \omega_2 T}{\omega_2 T - (\omega_1 - \omega_2)t}, \]

(15)
however, does have a flat amplitude spectrum, apart from Gibbs’ phenomenon ripples, as shown in Fig. 3. It also has a more compact autocorrelation function, as expected.

As mentioned above, if the data are cross-correlated, rather than deconvolved, it makes more sense for identification of far-field targets to cross-correlate with the time-derivative of the sweep rather than with the sweep itself. Even so, the wavelet in the data will not be zero phase because the groundforce is not equal to the sweep and varies from place to place. To achieve correlated data that have a spatially-invariant wavelet, preferably zero phase, it is necessary to perform the deconvolution after cross-correlation.

**Deconvolution after cross-correlation**

After cross-correlation with the sweep, the wavelet is given by equation (4); after cross-correlation with the time-derivative.
Figure 2  a) Sweep 5–80 Hz, computed with 2 ms sampling interval, b) autocorrelation of (a), c) instantaneous frequency for sweep (a) and d) amplitude spectrum of sweep (a).

of the sweep, as proposed here, the wavelet is given by equation (6). In either case the wavelet is different for every vibrator position but is known if the groundforce is known. Letting this wavelet be \( h(t) \), with Fourier transform \( H(\omega) \), the deconvolution filter would be \( y(t) \), with Fourier transform

\[
Y(\omega) = \frac{H(\omega)D(\omega)}{|H(\omega)|^2 + \epsilon}
\]

in which \( |D(\omega)| \) is fixed for the survey and is essentially a smooth version of \( |H(\omega)| \), as described in Appendix A. A new filter \( y(t) \) should be calculated for every sweep at every vibrator position. Deconvolution of the correlated data with this filter will replace the wavelet with the fixed desired wavelet \( d(t) \).

Practical issues at low frequencies
Creating a vibroseis sweep with an amplitude spectrum inversely proportional to frequency is more difficult in practice than in theory. Limits to the vibrator reaction mass maximum displacement and pump flow rate both impede generation of low-frequency energy (Bagaini 2007; Wei 2008). For frequencies below about 8 Hz, the envelope function \( e(t) \) cannot be a constant: it must decrease with decreasing frequency.

This problem has been recognized for a number of years. Independent of the argument presented here, the need for more low-frequency vibroseis energy has been noted for example by Andersen (1994) and Jeffries and Martin (2003). Bagaini (2007, 2008) and Bagaini et al. (2008) developed an approach to vibrator sweep design that accommodates the physical limitations at low frequencies. It requires lower sweep rates because the amplitude is limited by the reaction mass stroke at low frequencies. If this is combined with the approach proposed here it will lead to even lower sweep rates at low frequencies than are shown in Fig. 2(c). Some days the sun does not shine.
The low-frequency limitations of vibrators have not been ignored by manufacturers and there is at least one vibrator on the market that is designed specifically for low frequencies (but can also operate up to 100 Hz), with a reaction mass stroke of plus or minus 20 cm and an output of 4,500 kg force (10 000 lbf) at 1 Hz (E. Christensen, pers. comm. 2009).

An important consideration when working in built-up areas is property damage. If more time and effort is spent at low frequencies the risk of damage may increase. The risk may be mitigated by using pseudo-random sweeps instead of a swept-frequency sine wave. The damage is caused by exciting resonance of the building. A pseudo-random sweep has a randomly-varying instantaneous frequency, so in a given time window only a small percentage of the energy is near the resonance frequency. In a swept-frequency sine wave the energy in a corresponding time window is always concentrated around a narrow band of frequencies and, if the resonance frequency is within the bandwidth, there is greater potential to excite the resonance. An early use of pseudo-random vibroseis sweeps (but for a different purpose) was described by Payton, Waters and Goupillaud (1979).

Normally pseudo-random sweeps and pseudo-random binary sequences have flat amplitude spectra over a defined bandwidth. In our application we need an amplitude spectrum that is inversely proportional to frequency. The problem of producing pseudo-random sequences with desired spectra has been solved. Tough and Ward (1999), for example, developed a procedure that produces a non-Gaussian random sequence with a desired autocorrelation function by a non-linear transformation of a Gaussian process.

ARRAY OF VIBRATORS

Model of the seismogram

Normally an array of vibrators is used, rather than a single vibrator. Figure 4 shows an array of $M$ vibrators and an array of geophones. Sallas and Allen (1998a,b) invented a method known as high fidelity vibratory source or HFVS that solves the intra-array vibrator interaction problem and permits the responses to individual vibrators in an array to be extracted from the data. In the following, the equations of Sallas and
Allen are modified to include the time-derivative operator described above.

Let the displacement impulse response of the Earth for the ith vibrator and the kth geophone be \( g_{ik}(t) \). The response at the kth geophone to all M vibrators is a superposition of convolutions, similar to the convolution in equation (2), plus noise,

\[
v_k(t) = \sum_{i=1}^{M} f'_i(t) * g_{ik}(t) + n_k(t),
\]

in which \( f'_i(t) \) is the groundforce of the ith vibrator and the prime indicates the time-derivative. Clearly we need to find the \( g_{ik}(t) \).

Separation of responses to individual vibrators in an array

Silverman (1979) began the investigation of the problem of separating the responses to individual vibrators in an array by considering the case of two vibrators A and B and two sweeps. In the first sweep the geophone measures the response to A+B. Then, in the second sweep, the polarity of the second vibrator is reversed so that the geophone measures the response to A-B. Adding the two measurements gives twice the response to A; subtracting the second measurement from the first gives twice the response to B. Following Silverman, there were a number of patent applications on phase encoding, including Landrum (1987).

Sallas and Allen (1998a,b) recognized that phase encoding is very important but it is not sufficient for complete separation of individual responses because it relies on the response of a vibrator being repeatable from sweep to sweep. This is rarely the case. Nevertheless, Sallas and Allen (1998a,b) recognized that the impulse responses \( g_{ik}(t) \) could be found by encoding the M vibrator signals, using M or more sweeps, and transforming the data to the frequency domain. In the time domain the measurements at the kth geophone are

\[
v_{ik}(t) = \sum_{j=1}^{M} f'_{ij}(t) * g_{ik}(t) + n_{ik}(t), \quad j = 1, 2, \ldots, M \text{ or more},
\]

in which \( j \) is the number of the sweep and \( f'_{ij}(t) \) is the groundforce of the ith vibrator in the jth sweep. Transforming (18) to the frequency domain converts the convolutions to multiplications and gives

\[
V_{jk}(\omega) = -i\omega \sum_{i=1}^{M} F_{ji}(\omega) G_{ik}(\omega) + N_{jk}(\omega),
\]

\[
j = 1, 2, \ldots, M \text{ or more},
\]

which is M or more simultaneous equations for each frequency. In the absence of noise only M sweeps are required and, for a given frequency, the equations may be written in the matrix form

\[
\begin{bmatrix}
V_{1k} \\
V_{2k} \\
\vdots \\
V_{Mk}
\end{bmatrix} =
-i\omega
\begin{bmatrix}
F_{11} & F_{12} & \cdots & F_{1M} \\
F_{21} & F_{22} & \cdots & F_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
F_{M1} & F_{M2} & \cdots & F_{MM}
\end{bmatrix}
\begin{bmatrix}
G_{1k} \\
G_{2k} \\
\vdots \\
G_{Mk}
\end{bmatrix}.
\]

Given the measurements \( V_{jk}(\omega) \) and the groundforce coefficients \( F_{jk}(\omega) \), the equations can be solved for the \( G_{jk}(\omega) \) for each frequency. Adding more sweeps gives an over-determined system of equations and allows the transfer functions \( G_{jk}(\omega) \) to be recovered in the presence of noise with errors that can be estimated.

Transforming the \( G_{jk}(\omega) \) to the time domain yields the desired impulse responses \( g_{ik}(t) \). The solution of equations (20) is a frequency domain deconvolution and \( G_{jk}(\omega) \) should be multiplied by \( D(\omega) \) before transformation back to time. This will ensure that the wavelet is known, is the same on all traces and has a bandwidth consistent with the source.

Note that each vibrator has a separate source static, which can be estimated using a common vibrator gather: which is, keeping \( j \) fixed and varying \( k \). Unfortunately, the situation is often more complicated than described here. The ground often compacts in response to the vibrations of the vibrator, so the impulse responses \( g_{ik}(t) \) depend not only on the source and receiver positions but also on the sweep number \( j \). So solution of equations (20) gives a sort of average impulse response from the M or more sweeps.
In the formulation of Sallas and Allen (1998a,b) there is a period of listening time after each sweep, so that the measurements are complete convolutions, as described by equation (18). Krohn and Johnson (2006) noticed that it is possible to save time by cascading the different sweeps together into a continuous sequence for each vibrator and having only one period of listening time at the end. Provided the impulse responses are shorter than the individual sweeps, it is possible to sort out the data in processing and still recover the $g_{ik}(t)$.

**GROUNDFORCE**

In all the foregoing we see that it is essential to know the groundforce. Castenet and Lavergne (1965) invented a method to estimate the groundforce using the simplified model of the vibrator shown in Fig. 5 in which the baseplate has mass $M_p$, the reaction mass has mass $M_r$, the vertical upwards displacements of the two masses are $u_p$ and $u_r$, respectively and $w(t)$ is the force applied by the vibrator drive system to the two masses. The groundforce is $f(t)$. The force of the plate acting downward is equal to the force of the ground acting upward, by Newton’s third law. So the groundforce in Fig. 5 is the same force as that shown in Fig. 1.

Applying Newton’s second law of motion to the reaction mass and the baseplate yields

$$M_r u_r''(t) = w(t),$$  \hspace{1cm} (21)

$$M_p u_p''(t) = f(t) - w(t).$$  \hspace{1cm} (22)

Adding these two equations gives the famous ‘weighted sum’ estimate of the groundforce:

$$M_r u_r''(t) + M_p u_p''(t) = f(t).$$  \hspace{1cm} (23)

That is, the sum of the reaction mass and the baseplate, each weighted by its vertical upwards acceleration, is equal to the groundforce. The two masses $M_r$ and $M_p$ are fixed and known; the two accelerations $u_r''(t)$ and $u_p''(t)$ need to be measured. This method is still widely used today to estimate the groundforce. Bagaini (2007, 2008) showed clearly that the groundforce signal estimated in this way has clear harmonics that are not present of course in the sweep; that is, the groundforce is not equal to the sweep. The harmonics are attributed to non-linearities in the hydraulics of the vibrator, bending of the baseplate and to the different response of the soil to loading and unloading.

The weighted-sum method assumes the baseplate is rigid. In fact it is not and Sallas, Amiot and Alvi (1985) were the first to show that the bending of the plate causes a loss of force, especially at high frequencies. There are three known approaches to this problem: 1) make the baseplate stiffer, 2) model the baseplate deformation to obtain the stress distribution at the interface between the baseplate and the earth and 3) measure the stress at the interface between the baseplate and the earth.

The first of these is rather obvious. Work is currently being done on this, for example Wei (2008). The boundary condition that the displacement directly under the baseplate is uniform has been studied extensively (Bycroft 1956; Awojobi and Grootenhuis 1965; Robertson 1966; van Onselen 1982; Tan 1985): it requires a stress singularity at the edge of the plate, which is impossible in practice. In theory, then, no amount of engineering is ever going to make equation (23) exact. In practice, the stiffer the plate, the better approximation equation (23) becomes at higher frequencies. Sallas et al. (1985) did a series of experiments that showed how the modulus and phase of both the acceleration and the traction varied under the baseplate. Figures 3.3.1 and 3.3.2 in Baeten and Ziolkowski (1990) showed some of these results.

Baeten (1989) was the first to attempt to model the baseplate deformation analytically: his model predicted the stress underneath the plate from the measurement of the vertical acceleration at the centre of the plate, assuming perfect contact with the ground and no horizontal stresses. This method, known as the ‘flexural rigidity method’ requires a detailed model of the baseplate. It successfully predicts the attenuation of high frequencies as determined by stress measurements under the plate.
John Sallas (1984) proposed measuring the stress at the baseplate-ground interface using ‘load cells’. I like this approach the best because it gets to the heart of the problem: it measures the boundary condition. Although these stress measurements have been made in controlled test-bed conditions and have produced excellent results, they have not been adopted by the industry for vibroseis surveys. I understand from an anonymous reviewer that there appear to be two reasons for this: the cost of the baseplates (which is not great) and the added mass to the baseplate. Perhaps industry should reconsider the option to use load cells to measure the stress and obtain the true groundforce.

There is more and more focus on the use of accelerometer measurements on the baseplate to determine the stress under the plate. This is essentially an inverse problem and is therefore likely to be difficult to solve. The forward problem is to compute the accelerations on the surface of the baseplate from applied stresses and a model of the plate. The inverse problem is to compute the applied stresses (and thus the groundforce) given the model of the baseplate and measured accelerations on the surface of the plate.

**DISCUSSION: WILL THE DATA BE BETTER?**

What I am proposing is expensive: it will take more time to acquire the data, especially with most existing vibrators. Will the seismic data be better if the sweep has an amplitude spectrum inversely proportional to frequency instead of a conventional flat spectrum? If the data are better, is the increased cost justified? What assumptions are we making about the noise? And what about intrinsic attenuation?

The argument for the amplitude spectrum being inversely proportional to frequency is straightforward. It is well-known that for a homogeneous isotropic unbounded elastic medium the far-field particle displacement is in-phase with the applied force: Aki and Richards (1980) said on p. 73 that Stokes first derived this in 1849. There is, therefore, really nothing remarkable about the result of Baeten et al. (1988): it is consistent with and rigorously based on, classical elasticity theory. The time-derivative comes in because we measure particle velocity, not displacement. Since our target is, in general, in the far field, the spectrum of the groundforce is deficient in low-frequency energy if the groundforce has a flat spectrum. The response $g_k(t)$ we are trying to measure at the $k$th geophone is unknown but contains all the frequencies we are able to generate. It is well-known from linear system theory that the recovery of the system response is best achieved with an input signal that has a flat spectrum. To make the particle velocity spectrum flat in the far field, the spectrum of the force must be of the form $A/\omega$.

Suppose we measure particle acceleration instead of particle velocity, which we do, sometimes. Should we make the spectrum of the force of the form $A/\omega^2$?

The answer is no. Take the time-derivative of equation (2):

$$v_k(t) = u_k'(t) = f'(t) \ast g_k'(t) + n_k'(t).$$

(24)

where $v_k(t)$ is the particle acceleration, $n_k(t)$ is the time-derivative of the noise and $g_k'(t)$ is the time-derivative of $g_k(t)$. Only one of the two functions in the convolution $f'(t) \ast g_k'(t)$ is differentiated. We differentiate the impulse response, not the source, because we are still focusing on the far field. In going from equation (1) to equation (2), we were accounting for propagation effects from the source to the far field. In equation (24) we simply have the time-derivative of equation (2): particle acceleration instead of particle velocity, particle acceleration noise instead of particle velocity noise and the time-derivative of the Earth’s impulse response instead of the Earth’s impulse response. After deconvolution we would recover the time-derivative of the Earth’s impulse response, convolved with the desired wavelet $d(t)$. To recover the Earth’s impulse response, we would need to integrate. This is all straightforward.

Theoretically, as argued above, the data should be better if we use a force with amplitude spectrum inversely proportional to frequency. Will the data actually be better? Or will there be ‘a nice lot of ground roll and very little reflection data’ as one reviewer has suggested?

We can do two experiments to find out. The first, less costly, experiment would be to compute synthetic seismograms (using, for example, a 3D finite-difference elastic modelling code) for a force source at the surface of an elastic Earth and look at the particle velocity seismograms generated at the Earth’s surface. We could convolve the calculated Earth’s impulse responses with a normal linear sweep, add real noise and process the resulting data as described above. We could also convolve the same Earth’s impulse responses with the sweep of equation (14), add the same noise, process the data and compare the result with the linear sweep. If the results look promising we could do the second experiment, which would be field tests with real vibrators.

If the real data are better, the issue of cost then needs to be considered and there will be immediate pressure for vibrators to deliver more power at low frequencies.

I am making no assumptions about the noise. The signature deconvolution described above is essentially a multiplication.
in the frequency domain by an operator with an amplitude spectrum that is essentially a constant. It increases the resolution of the data by compressing the wavelet in time but leaves the spectrum of the uncorrelated noise unchanged. This is easily seen in equation (10). The ground roll is part of the impulse response \( g_k(t) \). As the low-frequency content of the reflection data is increased, the amplitude of the low-frequency ground roll is increased. All the methods we use in acquisition and processing to enhance reflections and reduce ground roll will work just as well as they do with existing data but the amplitudes of both reflections and ground roll at low frequencies will be greater.

If the far-field input signal has a flat spectrum, as I propose, the effect of constant-Q intrinsic attenuation is to apply a linear negative slope to the logarithmic amplitude as a function of frequency. I expect this effect to be more easily observable with a \( A/\omega \) groundforce spectrum.

**CONCLUSIONS**

The goal of vibroseis data acquisition and processing is to produce seismic reflection data with a known spatially-invariant wavelet, preferably zero phase, such that any lateral variations in the data can be attributed to variations in geology.

The control system of the vibrator is usually designed to make the estimated groundforce signal equal to the predetermined sweep and the resulting particle velocity data are usually cross-correlated with the sweep. Since the downgoing far field particle velocity signal is proportional to the time-derivative of the groundforce, it makes more sense to cross-correlate with the time-derivative of the sweep. It also follows that the ideal amplitude spectrum of the groundforce should be inversely proportional to frequency.

Because of non-linearities in the vibrator, bending of the baseplate and variations in ground coupling, the true groundforce is not equal to the pre-determined sweep and varies not only from vibrator point to vibrator point but also from sweep to sweep at each vibrator point. The wavelet in the data therefore varies from sweep to sweep, both before and after the cross-correlation process. So we do not want to stack yet. To achieve the goal of a spatially-invariant wavelet, these wavelet variations should be removed by signature deconvolution, converting the wavelet to a much shorter zero-phase wavelet but with the same bandwidth and signal-to-noise ratio as the original data. This can be done if the true groundforce is known.

A deconvolution study presented in Appendix A shows that there is plenty of scope for improving the resolution of vibroseis data without altering the bandwidth of the source.

If the sweep is designed to give the groundforce an amplitude spectrum that is inversely proportional to frequency, the data ought to be better but the acquisition costs will go up because the sweep time will be longer. The ability to generate low-frequency energy is also limited in most vibrators by the pump flow rate and the reaction-mass stroke, so this further increases the sweep time. There is scope to reduce the low-frequency limitations on the vibrator.

Sallas and Allen (1998a,b) solved the problem of determining the impulse responses between individual vibrators in a vibrator array and a given geophone for the case in which the groundforce under each baseplate is known and there is no ground compaction between sweeps. In practice the groundforce is not known exactly and there is often compaction that is variable from place to place. Load cells can measure the groundforce directly but are not used. The reasons for not using load cells are weak. A combination of stiffer baseplates and solving the inverse problem for the determination of the stresses on the plate from measured surface accelerations might lead to significantly better groundforce estimates.

We do not know how good our current estimates of the groundforce are. To find out we must compare current estimates with true groundforce measurements. That is, we need to make the load cell measurements.

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APPENDIX A: DESIGN OF THE DESIRED WAVELET

Trantham (1993) gave a general method for constructing minimum uncertainty filters from analytical functions. The uncertainty of a filter, or wavelet, is dimensionless: it is the product of its length in time and bandwidth in frequency. Trantham (1993) said “Note that data processing objectives may require a specific deconvolution impulse response that is not minimum uncertainty”. That is the case here. I describe here an approach I have used for many years (for example, Baeten and Ziolkowski 1990, Ch. 7; Ziolkowski, Sneddon and Walter 1999) to design a desired wavelet. The approach of Krohn (2006) is similar.

If the amplitude spectrum of a wavelet has a lot of detailed variation, many samples are required to define it in the frequency domain and there must be at least as many corresponding samples in the time domain. The number of time-domain samples also depends on the phase. If the wavelet is zero...
phase the number of time samples is minimum and the zero-
phase wavelet is the shortest wavelet with a given amplitude
spectrum (Berkhout 1974). If the phase is zero and the ampli-
tude spectrum is smooth, few samples are required to specify
the spectrum and there are few corresponding samples in the
time domain. In other words, the wavelet is short.

I illustrate my trial-and-error approach to the design of a
desired wavelet for signature deconvolution using the auto-
correlation function shown in Fig. 3(c). It is reproduced again
in Fig. 6(a) but plotted from $-1$ to $+1$ seconds. Its ampli-
tude spectrum is shown in Fig. 6(b). This is the power spec-
trum of the sweep shown in Fig. 3(a) or the square of the

Figure 6 Elements of the design of a desired wavelet: a) same as Fig. 3(a) but plotted $-0.1$ to $+0.1$ s, b) amplitude spectrum of (a), c) desired zero-phase wavelet, d) amplitude spectrum of (c), e) comparison of input wavelet and desired wavelet plotted $-0.1$ to $+0.1$ s and f) comparison of amplitude spectra of input wavelet and desired wavelet.
amplitude spectrum shown in Fig. 3(b). This spectrum has a lot of wiggles, so it needs many samples to specify it in the frequency domain. We see the corresponding wiggles in the time domain in Fig. 6(a). We need a desired amplitude spectrum that is smooth and which is close to the amplitude spectrum of Fig. 6(b), such that the ratio of the amplitudes is of the order of 1. A possible design is shown in Fig. 6(d), with the corresponding zero-phase time-domain wavelet shown in Fig. 6(c). To achieve this, I set the flanks of the desired spectrum to be the same as those of the input spectrum, from 0–5.5 Hz and from 80.0–250.0 Hz (the sampling interval was 2 ms) and I set two control points with amplitudes of 15 000 at 15 Hz and 55 Hz. In-between the ratio is of the order of 1–perhaps varying between 0.3–3.0. We could make the ratio closer to 1 by making the desired spectrum flatter in the middle but this would introduce shoulders in the spectrum and force extra oscillations in the time domain.

Figure 6(e) shows the original wavelet and the desired wavelet superposed: the desired wavelet is much shorter and smoother than the original.

The wavelet I used for this illustration is much shorter than wavelets we will measure if we try to measure the true ground-trum to be the same as those of the input spectrum, from 0–5.5 Hz and from 80.0–250.0 Hz (the sampling interval was 2 ms) and I set two control points with amplitudes of 15 000 at 15 Hz and from 80.0–250.0 Hz (the sampling interval was 2 ms) and I set two control points with amplitudes of 15 000 at 15 Hz and 55 Hz. In-between the ratio is of the order of 1–perhaps varying between 0.3–3.0. We could make the ratio closer to 1 by making the desired spectrum flatter in the middle but this would introduce shoulders in the spectrum and force extra oscillations in the time domain.

Appendix B: Sweeps

Following Rietsch (1977), a sweep $s(t)$ can be defined as

$$s(t) = e(t) \sin(\theta(t)),$$

where $e(t)$ is an envelope function and the phase function $\theta(t)$ is given by

$$\theta(t) = \theta(0) + \int_0^t \theta'(\tau) \, d\tau,$$  \hspace{0.5cm} (B2)

where $\theta(0)$ is the phase at $t = 0$ and $\theta'(\tau) = \omega(\tau)$ is the instantaneous angular frequency at time $\tau$. The instantaneous frequency changes with time at a rate known as the sweep rate which, in general, is frequency-dependent:

$$\omega'(t) = \theta''(t) = a(\omega).$$  \hspace{0.5cm} (B3)

Rietsch (1977) showed that the amplitude spectrum of the sweep $|S(\omega)|$ is related to the sweep rate as

$$|S(\omega)| = \frac{1}{2} e(t) [\theta''(t)]^{-\frac{1}{2}}.$$  \hspace{0.5cm} (B4)

We require $|S(\omega)|$ to be of the form $B/\omega$, where $B$ is a constant. We let the envelope function $e(t) = 1$. Then, from equation (B4), we have

$$\theta''(t) = \omega'(t) = A \omega^2,$$  \hspace{0.5cm} (B5)

where $A = 1/(4B^2)$ is a constant that depends on the sweep parameters. We rewrite equation (B5) as

$$\frac{d\omega(t)}{\omega^2} = Adt.$$  \hspace{0.5cm} (B6)

Integrating equation (B6) leads to

$$\theta'(t) = \omega(t) = \frac{\omega_1 \omega_2 T}{\omega_2 T - (\omega_2 - \omega_1) t},$$  \hspace{0.5cm} (B7)

where $\omega(0) = \omega_1$, $\omega(T) = \omega_2$ and, therefore, $A = (\omega_2 - \omega_1)/(T \omega_2 \omega_1)$.

Integrating equation (B7) gives

$$\theta(t) = \int_0^t \omega(\tau) d\tau = \frac{\omega_1 \omega_2 T}{(\omega_1 - \omega_2)} \ln \left[ 1 + \frac{(\omega_1 - \omega_2) t}{\omega_2 T} \right]$$  \hspace{0.5cm} (B8)

and the sweep is

$$s(t) = \sin \left( \theta(0) + \frac{\omega_1 \omega_2 T}{(\omega_1 - \omega_2)} \ln \left[ 1 + \frac{(\omega_1 - \omega_2) t}{\omega_2 T} \right] \right).$$  \hspace{0.5cm} (B9)