Summary

We present a new and improved non-hyperbolic moveout equation for converted waves (C-waves) propagating in layered media with vertical transverse isotropy (VTI), and demonstrate its application through a case study. The new equation is accurate up to offset-depth ratio \( x/z = 2.0 \), forming the basis for parameter estimation. A new parameter \( \chi \) is introduced to quantify the anisotropic effect on C-wave reflection moveout. With \textit{a priori} knowledge of the vertical velocity ratio and the P-wave stacking velocity, the C-wave stacking velocity and the anisotropic parameter \( \chi \) can be estimated from far-offset C-wave data using a double-scanning procedure.

Introduction

One of the main problems in processing P-SV converted waves (C-waves) in 4C marine seismic data is the wide occurrence of VTI anisotropy (transverse isotropy with a vertical symmetry axis). Tsvankin and Thomsen (1994) derived a generalized form of the C-wave moveout equation based on a Taylor series expansion, but their expression for the quartic coefficient \( A_4 \) is complicated and difficult to use directly for moveout analysis. To overcome this problem, Thomsen (1999) and Cheret et al. (2000) presented simplified forms. However, the expression for \( A_4 \) in Thomsen’s (1999) paper is less accurate and limited to offset-depth ratio of 1.0. In another approach, Li and Yuan (1999) proposed the use of a double-square-root approximation for moveout analysis in VTI media. Here we present an alternative and more accurate approach to utilize the Taylor series expansion of Tsvankin and Thomsen (1994) for moveout analysis in layered VTI media, and demonstrate the methods using both synthetic and field C-wave data.

Method

Consider VTI media with \( n \) layers. The moveout for a C-wave ray \( t_C \), converted at the bottom of the \( n \)-th layer and emerging at offset \( x \) can be written as,

\[
t_C^2 = t_{C0}^2 + \frac{x^2}{V_{C2}^2} + \frac{A_4 x^4}{1 + A_5 x^2},
\]

(1)

where \( V_{C2} \) is the C-wave stacking velocity, \( A_4 \) is the quartic Taylor series coefficient, and \( A_5 \) is introduced to constrain the far-offset moveout signature. Using Thomsen’s (1999) notation and introducing two additional effective anisotropic parameters \( \eta_{\text{eff}} \) and \( \chi_{\text{eff}} \), we obtain simplified expressions for \( A_4 \) and \( A_5 \),

\[
A_4 = \frac{-\left(\gamma_0 \eta_{\text{eff}} - 1\right)^2 + 8\left(1 + \gamma_0 \eta_{\text{eff}}^2 \eta_{\text{eff}}^2 - \xi_{\text{eff}}\right)}{4 \gamma_{\text{eff}}^2 V_{C2}^2 \gamma_0^2 (1 + \gamma_{\text{eff}}^2)}
\]

and

\[
A_5 = \frac{A_4}{1/V_{p0}^2 - 1/V_{C2}^2},
\]

(2)

where \( \gamma_0 \) is the average vertical velocity ratio, \( \gamma_{\text{eff}} \) is the effective velocity ratio,

\[
\eta_{\text{eff}} = \frac{1}{8 \gamma_{p0} V_{p2}^4} \left[ \sum_{i=1}^{n} V_{p2i}^4 \Delta t_{p0}(1 + 8 \eta_i) - t_{p0} V_{p2}^4 \right],
\]

(3)

\[
\xi_{\text{eff}} = \frac{-1}{8 \gamma_{s0} V_{s2}^4} \left[ \sum_{i=1}^{n} V_{s2i}^4 \Delta t_{s0}(1 - 8 \xi_i) - t_{s0} V_{s2}^4 \right],
\]

(4)
\[ \eta_i = (\epsilon_i - \delta_i)/(1 + 2\delta_i), \quad \zeta_i = (\gamma_i^2 / \gamma_0)\eta_i = \gamma_{\text{eff}}\eta_i. \] \hspace{1cm} (5)

\( V_{P2} \) and \( V_{S2} \) are \( P \)- and \( S \)-wave stacking velocities, \( \gamma_2 \) is the stacking velocity ratio, \( \epsilon \) and \( \delta \) are Thomsen (1986) parameters, and \( V_{Ph} \) is the \( P \)-wave horizontal velocity.

Numerical modelling is performed over a three-layer model (Table 1) to verify the accuracy of equations (1)-(4) (Figure 1a). The approximations are accurate up to offset-to-depth ratio of 2.0. For comparison, Figure 1b shows the isotropic equations (letting \( \eta_{\text{eff}} = 0 \) and \( \zeta_{\text{eff}} = 0 \)), which are only accurate to \( x/z \) of 1.0.

**Parameter estimation**

Similar to Alkhalifah (1997) for \( P \)-waves in VTI media, we use a double-scan semblance analysis to determine the \( C \)-wave moveout velocity and anisotropic parameters for a given event. A combined parameter \( \chi_{\text{eff}} \) is used for this purpose,

\[ \chi_{\text{eff}} = \eta_{\text{eff}} \gamma_0 V_{P2}^2 - \zeta_{\text{eff}}, \] \hspace{1cm} (6)

which is the total influence of VTI and layering on the \( C \)-wave moveout signature. In these formulations, the \( C \)-wave moveout is controlled by four parameters, \( V_{C2}, \gamma_0, \gamma_{\text{eff}} \) and \( \chi_{\text{eff}} \) whereas the \( P \)-wave moveout signature is controlled only by two parameters \( V_{P2} \) and \( \eta_{\text{eff}} \). Assume that \( \gamma_0 \) can be obtained by correlating \( P \)- and \( C \)-wave stacked sections, and \( \gamma_{\text{eff}} \) can be obtained by

\[ \gamma_{\text{eff}} = \frac{V_{P2}^2}{V_{C2}^2(1 + \gamma_0) - V_{P2}^2}, \] \hspace{1cm} (7)

where the \( P \)-wave moveout velocity \( V_{P2} \) can be determined from \( P \)-wave data. Thus equations (1), (2) and (7) can be used for double scanning to determine \( V_{C2} \) and \( \chi_{\text{eff}} \). Assumptions also have to be made to constrain the \( P \)-wave horizontal velocity \( V_{Ph} \). Empirically, after various numerical tests, we find that,

\[ V_{P2} = V_{P2} \sqrt{1 + 2\eta} = V_{P2} \sqrt{1 + \frac{2\chi_{\text{eff}}}{(\gamma_0 - 1)\gamma_{\text{eff}}^2}}, \] \hspace{1cm} (8)

is a good approximation for moveout analysis, although it is only strictly valid for a single VTI layer.

**Results**

Figure 2a is a synthetic gather calculated by ray tracing for the three-layer model in Table 1. The semblance analyses for \( V_{C2} \) and \( \chi_{\text{eff}} \) are shown in Figure 3 for the different reflectors, using a priori information about \( V_{P2} \) and \( \gamma_0 \). The offset range is limited to \( x/z \) of 2.0. The circles are the exact values of \( V_{C2} \) and \( \chi_{\text{eff}} \). The semblance spectra show good resolution and good inversion results for \( V_{C2} \) and \( \chi_{\text{eff}} \). Figures 2b and 2c are the results using isotropic (letting \( \chi_{\text{eff}} = 0 \)) and anisotropic moveout corrections, respectively. They demonstrate the efficiency and necessity to perform anisotropic moveout correction.

The \( C \)-wave anisotropic velocity analysis and moveout correction are applied to Guillemot data from the North Sea. Figure 4a shows a \( C \)-wave CCP gather. Three events at 1520ms, 2050ms and 2570ms, are used for double scanning (Figure 5) after the \( P \)-wave moveout velocity and \( \gamma_0 \) are determined. Good resolution of \( V_{C2} \) and \( \chi_{\text{eff}} \) is obtained. Anisotropic moveout correction (Figure 4c) gives much better results than isotropic correction only (Figure 4b).

**Conclusions**

We have derived simplified and accurate expressions for calculating \( C \)-wave moveout in layered VTI media. An anisotropic velocity analysis method is presented to estimate the strength of VTI anisotropy. It requires knowledge of \( P \)-wave moveout velocity and \( \gamma_0 \). Synthetic and field examples demonstrate the feasibility and accuracy of the method.

**Acknowledgements**

We thank Shell Expro UK for providing the field data. We thank Leon Thomsen for discussions. This work was jointly funded by the University of Edinburgh industry-sponsored project “Processing 3-C
marine seismic data” and by the Edinburgh Anisotropy Project (EAP) of the British Geological Survey (BGS). The work was carried out when Jerry Yuan, was a PhD student at the University of Edinburgh and EAP, and is published with the approval of all partners, and the Director of BGS (NERC).

References


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Table 1: Parameters for a three-layer model with VTI. The parameters are from Thomsen (1986). Each layer is 500m thick.

Figure 1: Accuracy of $t_c$ approximations for the three-layer model (Table 1). (a) The anisotropic equations (1)-(4), and (b) the isotropic equations with $\eta_{eff}=0$ and $\zeta_{eff}=0$.

Figure 2: (a) Synthetic gather for the model in Table 1, (b) isotropic and (c) anisotropic NMO correction.
Figure 3: Semblance analysis of the CCP gather in Figure 2a for $V_{C2}$ and $\chi_{eff}$ using priori information for $V_{P2}$ and $\gamma_0$. The circles are expected values of $V_{C2}$ and $\chi_{eff}$ in Table 1. (a) First reflector, (b) second reflector, and (c) third reflector.

Figure 4: (a) Field CCP gather, (b) isotropic and (c) anisotropic moveout correction.

Figure 5: Semblance analysis of the field data in Figure 4a for $V_{C2}$ and $\chi_{eff}$ using a priori information for $V_{P2}$ and $\gamma_0$. Events (a) $t_{C0} = 1520$ms, (b) $t_{C0} = 2050$ms, and (c) $t_{C0} = 2570$ms.