Marine seismic sources: QC of wavefield computation from near-field pressure measurements

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Abstract

A commercial marine seismic survey has been completed with the wavefield from the \( n \)-element (single guns and clusters) airgun array measured for every shot using an array of \( n+2 \) near-field hydrophones, \( n \) of which were required to determine the source wavefield, the remaining two providing a check on the computation. The source wavefield is critical to the determination of the seismic wavelet for the extraction of reflection coefficients from seismic reflection data and for tying the data to wells.

The wavefield generated by the full array of interacting airguns can be considered to be the superposition of \( n \) spherical pressure waves, or notional source signatures, the \( n \) hydrophone measurements providing a set of \( n \) simultaneous equations for each shot. The solution of the equations for the notional source signatures requires three ingredients: the geometry of the gun ports and near-field hydrophones; the sensitivity of each hydrophone recording channel; and the relative motion between the near-field hydrophones and the bubbles emitted by the guns. The geometry was measured on the back deck using a tape measure. A calibration data set was obtained at the approach to each line, in which each gun was fired on its own and the resulting wavefield was measured with the near-field hydrophones and recorded. The channel sensitivities, or conversion from pressure at the hydrophones to numbers on the tape, were found for each near-field hydrophone channel using the single gun calibration data, the measured geometry, and the peak pressure from each gun, known from the manufacturer’s calibration. The relative motion between the guns and hydrophones was obtained from the same calibration data set by minimizing the energy in the computed notional source signatures at the guns which did not fire. The full array data were then solved for the notional source signatures, and the pressure was computed at the two spare hydrophones and compared with the actual recordings. The rms errors were 5.3% and 2.8% and would have been smaller if the hydrophone channel sensitivities had been properly calibrated beforehand and if the movement of the guns with respect to the hydrophones had been more restricted.
This comparison of the predicted and measured signatures at spare hydrophones can, in principle, be done on every shot and we recommend that this be implemented as a standard quality control procedure whenever it is desired to measure the wavefield of a marine seismic source.

Introduction

The goal of seismic data processing is to render the data interpretable and to reveal not only the geological structure, but also the stratigraphy, including the reflection coefficients at the interfaces. The variation of reflection coefficient with angle of incidence, or AVO (amplitude-versus-offset), which can only be estimated prestack, has been an extremely valuable hydrocarbon indicator in a number of fields. The determination of reflection coefficients from seismic data, and tying wells to the data, requires knowledge of the seismic wavelet, which is intimately related to the source time function, the response of the recording system and the absorption in the earth. This wavelet is usually estimated from the data using statistical methods, but it is not necessary to do this. One can now use measurements to get at the wavelet rather than rely on statistical assumptions of dubious validity, as pointed out by Ziolkowski (1991). For the first time, a commercial marine seismic survey has been completed with the source wavefield measured for every shot and with a continuous check on both the quality of the measurements and the reliability of the computed wavefield. We present the method, which is valid for water of any depth exceeding about 10 m, and demonstrate the quality control aspects.

Several methods have been proposed for the determination of the wavefield of an airgun array using pressure measurements made in the vicinity of the array, rather than in the far-field. Ziolkowski et al. (1982) argued that the array of interacting airgun bubbles can be treated as \( n \) independent non-interacting monopole sources (single bubbles or clusters), each generating a spherical pressure wave. The total wavefield thus generated is simply the superposition of the \( n \) spherical waves plus their reflections in the sea-surface. Ziolkowski et al. (1982) proposed the use of an array of \( n \) independent near-field hydrophones to measure the wavefield, the pressure at each hydrophone being a linear superposition of the \( n \) spherical pressure waves. The \( n \) source time functions, and hence the total wavefield, can be found by solving the \( n \) linear simultaneous equations provided by the hydrophone data. In this method it is essential to know the positions of all the source elements and all the hydrophones, and to know the relative sensitivities of the hydrophones. The ability of the method to predict the far-field signature was demonstrated using near-field and far-field pressure measurements of a single airgun array. Parkes et al. (1984) showed that the method of Ziolkowski et al. (1982) allows the far-field signature, measured in deep water, to be determined very precisely from the near-field measurements, particularly if the motion of the rising bubbles with respect to the towed hydrophones is taken into account. The method has been used in two experimental seismic lines in water less than 100 m deep in the North Sea, both using four airgun subarrays: one line was acquired by Seismic

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Profilers A/S for Britoil in 1981 (Ziolkowski 1984, ch. 7), and a second was acquired in 1984, again by Seismic Profilers, for a consortium of oil industry sponsors (Amoco, BP, Britoil, Chevron, Mobil, Merlin Geophysical, Seismic Profilers, Statoil, TNO, Western Geophysical) in the Delft Air Gun Experiment (Ziolkowski 1987). In the second experiment the line was shot twice, once with the tuned airguns firing within $\pm 0.5$ ms of each other, and once with the guns detuned, firing within $\pm 50$ ms of each other. The near-field hydrophone measurements allowed the far-field signature to be computed for both data sets and the data shot with the detuned array were then deconvolved to recover essentially the same section as was obtained with the tuned array. This method of determining the far-field signature from near-field measurements is covered by a patent (Ziolkowski et al. 1984) owned by Schlumberger.

Hargreaves (1984) introduced a wavefield extrapolation method that uses a mini-streamer, or line of hydrophones, towed below the airgun array. The method relies on the streamer measuring the full downgoing wavefield from the source. Since the measurements would be made with a single line of hydrophones towed behind the vessel, it assumes that the wavefield is invariant in the horizontal direction perpendicular to the ship’s track, or, in other words, that the source array consists of line sources perpendicular to the ship’s track rather than monopoles. The finite size of the true source array can be handled with Hargreaves’s (1984) method only by using a large 2D array of hydrophones, or numerous mini-streamers towed beneath the airgun array. This method essentially requires the downgoing wavefield to be known on an entire plane beneath the source array. We are not aware of any published data demonstrating that Hargreaves’s (1984) method works in practice.

Hargreaves’s idea of the mini-streamer was resurrected by Landrø and Sollie (1992), who proposed that the wavefield of the airgun array could be modelled as an interacting array of oscillating bubbles, using the known geometry of the airgun array, the known gun volumes, and an equation of motion for each bubble that has two unknown empirical factors that must be determined for each gun. Their method is to calculate the pressure that would be measured by every hydrophone in the mini-streamer, incorporating the theory proposed by Ziolkowski et al. (1982) to model the interaction between bubbles, to superpose the spherical waves generated by each modelled bubble, and then to search for a combination of model parameters that minimizes the sum of the squares of the errors, for all time samples and for all hydrophones. The reflection coefficient at the sea surface is regarded as unknown, as is the exact configuration of the mini-streamer, which is, however, assumed to be in the vertical plane containing the airgun subarray. The method is entitled “Source-signature estimation by inversion”, but it is really source-signature estimation by iterative forward modelling: the accuracy of the solution is limited not only by the precision with which the geometry and hydrophone sensitivities are known, it is also constrained by the precision with which the forward model is able to generate the true wavefield. Landrø and Sollie (1992) demonstrated their method on synthetic data with and without noise. It is not clear how many hydrophones are required. In their synthetic data example Landrø and Sollie used 100 hydrophones of known sensitivity over a length of 64m. We see no reason why
more than one hydrophone in an approximately known position should be required, provided the number of significant data in a single hydrophone measurement exceed the number of parameters that need to be determined. According to our reading of the paper, this should amount to \(2n+4\) parameters, where \(n\) is the number of guns (2 unknown damping constants for each gun, one sea surface reflection coefficient, and three coordinates for the hydrophone position). The methodology of this paper is in principle quite different from that of Ziolkowski et al. (1982), as it relies on a forward model to obtain the solution, rather than strict inversion of the measured propagating wavefield.

Amundsen (1993) uses exactly the same formulation for the wavefield generated by an interacting array of airguns that was proposed by Ziolkowski et al. (1982). That is, the array is treated as an equivalent array of non-interacting monopoles, but Amundsen limits its application to the mini-streamer hydrophone geometry of Hargreaves (1984) and Landrø and Sollie (1992), and does not include the relative motion of the rising bubbles to the towed hydrophones. Amundsen’s (1993) method is thus a special case of the more general method proposed by Ziolkowski et al. (1982). The information required to solve for the source functions is the same as for Ziolkowski et al. (1982). The direct inversion of the hydrophone measurements proposed by Ziolkowski et al. (1982) and by Parkes et al. (1984) is converted by Amundsen (1993) into an over-determined optimization problem by adding at least one more hydrophone. The source functions are determined in the frequency domain by minimizing the sum of the squares of the errors between the measured and computed pressures for all the hydrophones, frequency by frequency. The method was demonstrated with synthetic data without noise.

Laws, Landrø and Amundsen (1995) presented an experimental comparison of the methods of Ziolkowski et al. (1982), Landrø and Sollie (1992) and Amundsen (1993), in which a single airgun subarray fired into a far-field hydrophone in deep water. The subarray was equipped with near-field hydrophones, according to the method of Ziolkowski et al. (1982), and a mini-streamer of additional near-field hydrophones was towed beneath the subarray according to the methods of Landrø and Sollie (1992) and Amundsen (1993). All three methods were able to compute the far-field signature satisfactorily.

The method of Ziolkowski et al. (1982) and Parkes et al. (1984) has been implemented by Schlumberger Geco-Prakla within their Trisor system on a number of vessels. The Trisor system computes and stores the monopole source time functions and the vertical far-field signature on a shot-by-shot basis. Lunde, Peebles and Walker (1995) showed once again that this method enables the vertical far-field signature of a single airgun subarray to be computed from near-field measurements.

The point of these methods is not, however, to show that a far-field measurement made in deep water can be predicted from near-field measurements. The point is to be able to use the near-field measurements to compute the signature in the far-field in any direction on a shot-by-shot basis, independent of the depth of the water, as this information is extremely valuable in the preservation of true amplitudes in the
processing of the seismic data, as already demonstrated in the Delft Air Gun Experiment (Ziolkowski 1987). In much of the continental shelf the water is too shallow to make a far-field measurement. These methods are designed to overcome this difficulty. They also permit the data processing to compensate for shot-to-shot variations in the source wavefield.

This paper demonstrates a method for determining the quality of computation using data from a commercial survey. The survey was a speculative well-tie seismic survey shot in October 1992 by the Seismograph Service Limited (SSL) Seisventurer in the Inner Moray Firth of the North Sea. We show that additional near-field hydrophones which were available on the SSL Seisventurer were able to provide a continuous check on the quality of the measurements and on the reliability of the wavefield computed with the method of Ziolkowski et al. (1982) and Parkes et al. (1984). We conclude that this check can be made routinely on a shot-by-shot basis.

We first review the theory, including the argument that the array of interacting oscillating airgun bubbles can be treated as an array of non-interacting monopole sources. We also discuss the positioning of the hydrophones, the determination of the relative sensitivities of the uncalibrated hydrophones to within about 2%, the approximate determination of their absolute sensitivities, and the determination of the relative velocities between the rising bubbles and towed hydrophones. We then describe the application of the theory in detail for the case of the measurements made on the Seisventurer, and demonstrate the check on the computation using two spare independent hydrophones.

**Theory**

*The pressure wave from a single oscillating bubble*

A single airgun emits an air bubble which oscillates approximately radially and has a diameter, at normal operating depths of 5–10 m, of about 1 m. Since the bubble is small compared with the wavelength of pressure waves it generates (300–15 m for a frequency range of 5–100 Hz), the pressure radiation has radial symmetry and can be considered to be generated by an equivalent spherical bubble (Ziolkowski 1970). As shown in the Appendix, the pressure field contains two terms: a linear term that propagates at the speed of sound in the water and decays inversely as the distance from the centre of the bubble, and a non-linear *afterflow* term caused by the motion of the water close to the bubble, which also propagates at the speed of sound, but decays approximately as $1/r^2$. This non-linear term becomes negligible at about 1 m from the centre of the bubble, for bubbles generating pressure waves in the frequency range 5–100 Hz, as shown in the Appendix. Thus, at distances $r$ of 1 m or more, the pressure field of a single oscillating airgun bubble may be written in the form

$$p(r, t) - p_\infty = \frac{1}{r} s \left( t - \frac{r}{c} \right),$$

(1)
in which \( p(r, t) \) is the pressure in the water at a distance \( r \) and time \( t \), \( p_h \) is hydrostatic pressure, \( c \) is the speed of sound in water, and \( s(t) \) is the source function. A hydrophone measures the difference between the time-varying pressure and hydrostatic pressure, that is, in the absence of the water surface, it measures \( p(r, t) - p_h \).

**Interaction among oscillating bubbles**

It has been known for many years that the far-field signature of an airgun array is not equal to the sum of the far-field signals of the individual guns fired on their own. In order to determine the far-field signature of an array it was necessary to measure it. For this measurement to be made a hydrophone had to be put in the far-field of the array, at a range greater than about \( D^2/\lambda \), where \( D \) is the length of the array and \( \lambda \) is the wavelength of the radiation. To obtain a representative measurement of the far-field signature of the array, including the source ghost, the far-field hydrophone is normally put 100 m below the array. The measured pulse is typically 300 ms long, and can be measured without contamination by the reflected wave from the sea-floor, provided the water is at least 325 m deep. The water is normally much shallower than this in the survey area, so this far-field measurement cannot normally be made in the place where the survey is to be conducted.

Ziolkowski et al. (1982, 1984) devised a method to solve this problem. Using pressure measurements made close to the guns, they provided a method to enable the whole source wavefield to be calculated even in very shallow water. Using a conservation of energy argument, they showed that interaction between oscillating bubbles must occur whenever the distance between bubbles is small compared with a wavelength. The interaction occurs as changes in the water pressure on the outside of the bubble. Thus a bubble oscillating near to another bubble does not oscillate in the same way as it would if the other bubble were not there and, therefore, the pressure wave it generates is different from the wave it generates when it is oscillating on its own. In fact, the oscillations are coupled. In an array of airgun bubbles, each bubble oscillates in the pressure field of the other bubbles, and any change in one bubble affects all the others. Ziolkowski et al. (1982) argued that the effect on the bubble oscillations of the dynamic interaction pressure on the outside of each bubble could be made by an equal and opposite pressure on the inside of the bubble, this notional bubble oscillating in water of hydrostatic pressure, and therefore behaving independently of the other bubbles. Since all these notional bubbles are still small compared with a wavelength, the pressure wave generated by each bubble is spherical. The pressure field in the water is then simply the superposition of the spherical waves from all the bubbles, plus the reflections of these waves from the water surface. These reflected waves can be treated as spherical waves originating at virtual sources above the water surface. (The method of Landrø and Sollie (1992) attempts to model all this.)

**Superposition of spherical waves**

Ziolkowski et al. (1982) and Parkes et al. (1984) proposed that the superposition of the
direct and reflected waves from the $n$ sources (single guns or clusters) would combine to create the following pressure at a point $(x, y, z)$ in the water:

$$p(x, y, z, t) - p_\infty = \sum_{k=1}^{n} \left[ \frac{1}{r_k(t)} s_k \left( t - \frac{r_k(t)}{c} \right) - \frac{1}{R_k(t)} s_k \left( t - \frac{R_k(t)}{c} \right) \right],$$  

(2)

in which $s_k(t)$ is the source time function of the $k$th source, $r_k(t)$ is the distance from the $k$th source to the point $(x, y, z)$ and is time-dependent because the buoyant bubble rises as it oscillates, and $R_k(t)$ is the time-dependent distance from the virtual bubble above the water surface to the point $(x, y, z)$. A hydrophone of sensitivity $h$ at this point in the water would give the following voltage output:

$$v(t) = h \cdot (p(x, y, z, t) - p_\infty) = h \sum_{k=1}^{n} \left[ \frac{1}{r_k(t)} s_k \left( t - \frac{r_k(t)}{c} \right) - \frac{1}{R_k(t)} s_k \left( t - \frac{R_k(t)}{c} \right) \right].$$  

(3)

If the voltage, the sensitivity and all the distances $r_k(t)$ and $R_k(t)$ are known, there are in this equation $n$ unknown source functions $s_k(t), k = 1, 2, \ldots, n$. This equation is simply a linear superposition of the unknown source time functions, weighted and delayed according to the distances from the sources to the point of interest. As this point of interest is moved, so are the weights and delays. By choosing appropriate positions for the measurement of the wavefield, a sufficient number of simultaneous equations may be obtained which may be solved for the unknown source functions. Thus Ziolkowski et al. (1982) and Parkes et al. (1984) proposed that $n$ hydrophones be placed in appropriate positions, as close to the guns as possible, but no closer than about 1 m to avoid the influence of the afterflow term. The resulting equations to be solved are

$$v_i(t) = h_i \sum_{k=1}^{n} \left[ \frac{1}{r_{ik}(t)} s_k \left( t - \frac{r_{ik}(t)}{c} \right) - \frac{1}{R_{ik}(t)} s_k \left( t - \frac{R_{ik}(t)}{c} \right) \right], \quad i = 1, 2, \ldots, n, \quad (4)$$

in which $v_i(t)$ is the output from the $i$th hydrophone. In practice the hydrophones are moving through the water, so both the bubbles and hydrophones are moving relative to a frame fixed to the earth, as shown in Fig. 1.

If the motion of the bubbles relative to the hydrophones is neglected, equations (4) reduce to

$$v_i(t) = h_i \sum_{k=1}^{n} \left[ \frac{1}{r_{ik}(t)} s_k \left( t - \frac{r_{ik}(t)}{c} \right) - \frac{1}{R_{ik}(t)} s_k \left( t - \frac{R_{ik}(t)}{c} \right) \right], \quad i = 1, 2, \ldots, n. \quad (5)$$

Using the Fourier transform

$$S_k(\omega) = \int_{-\infty}^{\infty} s_k(t) \exp \{i\omega t\} \, dt, \quad (6)$$

(5) may be written as

$$V_i(\omega) = h_i \sum_{k=1}^{n} S_k(\omega) \left[ \frac{1}{r_{ik}} \exp \left\{ i\omega \frac{r_{ik}}{c} \right\} - \frac{1}{R_{ik}} \exp \left\{ i\omega \frac{R_{ik}}{c} \right\} \right], \quad i = 1, 2, \ldots, n. \quad (7)$$

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These are the equations that Amundsen (1993) solved, with the minor modification that the reflection coefficient at the sea surface is allowed to be a variable, rather than equal to $-1$, and more hydrophones are added to make the problem over-determined. This approach allows the problem to be solved frequency by frequency, but neglects the relative motion of the bubbles and hydrophones which Parkes et al. (1984) showed was important.

**Positioning of the independent hydrophones**

The closer the near-field hydrophones are to the guns, the more robust is the solution of the equations. There are a number of reasons for this. First, consider the signal-to-noise ratio. The amplitude of the signal from any element in the source array decreases inversely with the distance from that element. If the noise in the water is random, the signal-to-noise ratio is best where the measurements are closest to the guns. Secondly, consider the signal separation. If, at one extreme, the guns in the array are, for example, horizontally 3 m apart, and the hydrophones are placed 3 m apart and 1 m above the guns, the pressure at each hydrophone is dominated by the signal from the nearest gun, due to spherical divergence, and the signals at the hydrophones contain different information. If, at the other extreme, the hydrophones are less than 1 m apart, as proposed by Landrø and Sollie (1992) and Amundsen (1993), and placed farther away from the guns, the signal information they contain is similar, the signal-to-noise ratio is reduced, and the degree to which they may be regarded as independent is reduced. Thirdly, consider the positioning. The guns are supported in a frame which is fairly rigid. The positioning of the hydrophones fixed to this frame can be done with great precision, which is not the case with the towed mini-streamer. The mini-streamer is free to move with respect to the airgun array. Therefore, any mini-streamer method...
must introduce the coordinates of the hydrophones as unknown parameters to be estimated from the data. Finally, consider the discrimination against sea-bottom reflections. All the methods acknowledge that the sea-surface reflection will be measured with these near-field hydrophones; they all also neglect the influence of reflections from the sea-floor and lower interfaces. Because of the spherical divergence of the waves, the amplitudes of these reflections are inversely proportional to the distances from the airguns to the sea-floor and back to the hydrophones. The amplitudes of the source signals are inversely proportional to the distances from the guns to the hydrophones. The best discrimination against these sea-floor reflections is obtained by placing the hydrophones as far from the sea-floor as possible, and as close to the guns as possible. This is achieved by placing the hydrophones above the guns and about 1 m away, as first recognized by Haugland, in his original configuration, first implemented on the Nina Profiler in 1981. The mini-streamer geometry only works if the water is deep enough, as recognized by Amundsen (1993). Haugland’s configuration will work in water as shallow as 10 m and was used by Ziolkowski et al. (1982), Parkes et al. (1984), Ziolkowski (1984, 1987). It was also used aboard the SSL Seisventurer, as described below, and is implemented in Geco-Prakla’s Trisor system.

**Computation of the signal at a point in the water using the source time functions**

Equations (4) must be solved for the $s_k(t)$ for all $k$. There is more than one way to solve (4), given the sensitivities $h_i$ and the distances $r_{ik}(t)$ and $R_{ik}(t)$. The method of successive approximations of Parkes et al. (1984) allows a previous solution to be used as a first approximation to the current problem and is easy to implement for computing shot-to-shot variations. This is the method used here.

Once the $s_k(t)$ are known, the pressure of the incident field at a fixed point $(x, y, z)$ in the water can be calculated according to (2). We propose that the calculation be performed for a spare hydrophone used purely for checking the quality of the measurements and the computations. Equation (2) then becomes

$$p_i(t) - p_o = \sum_{k=1}^{n} \left[ \frac{1}{r_{ik}(t)} s_k \left( t - \frac{r_{ik}(t)}{c} \right) - \frac{1}{R_{ik}(t)} s_k \left( t - \frac{R_{ik}(t)}{c} \right) \right],$$

in which $r_{ik}(t), k=1, 2, \ldots, n$, are the time-dependent distances from the moving bubbles to the moving spare hydrophone, and $R_{ik}(t)$ are the corresponding distances for the virtual bubbles. Only $n$ hydrophones are required to determine the source signatures $s_k(t)$ from (1). According to our method, spare hydrophones are used to provide independent measurements of the pressure wavefield of the airgun array. The $s_k(t)$ already determined can be substituted into (8) to predict the response at each spare hydrophone. These predictions may be compared with the actual measurements, to determine the error and provide a check on the method. This can be done for each shot, if required.

Determination of hydrophone sensitivities

When only one gun fires, the signal from the gun is measured on all hydrophones. For example, when only the $k$th gun fires, the signal at the $i$th hydrophone is

$$v_i(t) = h_i \left[ \frac{1}{r_{ik}(t)} s_0^k \left( t - \frac{r_{ik}(t)}{c} \right) - \frac{1}{R_{ik}(t)} s_0^k \left( t - \frac{R_{ik}(t)}{c} \right) \right], \quad i = 1, 2, \ldots, m,$$

in which $s_0^k(t)$ is the source time function of the $k$th gun firing alone, and is different from $s_k(t)$ because of the mutual interaction between the bubbles, as described above. Equation (9) describes the arrival of the direct wave and the wave reflected off the sea surface, where the sea surface is assumed to be planar for distances of the order of the source array (20 m), and the reflection coefficient is $-1$. The direct wave arrives first; that is, the distance $r_{ik}(t)$ is always smaller than $R_{ik}(t)$. The initial peak of the direct wave arrives before any reflected signal and has an amplitude $s_0^k(t_{\text{peak}})$. Neglecting the very small change in the source–hydrophone distance that occurs in the time it takes the initial peak to reach the hydrophone after the gun fires, the peak voltage output of the $i$th hydrophone is given by

$$v_{i,\text{peak}} = h_i \frac{1}{r_{ik}(0)} s_0^k(t_{\text{peak}}),$$

in which $s_0^k(t_{\text{peak}})$ is normally known, at least approximately, for all the guns in the array, $v_{i,\text{peak}}$ is the measured peak output from the hydrophone, $r_{ik}(0)$ and is known. Hence $h_i$.

The sensitivities are independent of which gun is fired. Therefore they may be checked by firing any gun on its own. To obtain a data set for the minimization of the errors in the determination of these relative sensitivities takes only a few minutes: each gun is fired while the vessel is approaching the start of the line.

In fact, it is not really the hydrophone sensitivities that are required. What is required is the conversion from pressure at the hydrophone to numbers on the recorded data tape, including the hydrophone sensitivity and the analogue-to-digital converter (ADC) for the appropriate recording channel. That is exactly what these single gun data provide.

Each hydrophone is a linear transducer that converts the change in pressure to volts. The sensitivity of the hydrophone is typically quoted in volts/MPa and is the same for all frequencies in the bandwidth of interest. There is a decay in response at low frequencies, of course, but this is usually below about 3 Hz, and is outside the bandwidth of energy generated by most airguns. The output volts from the hydrophone are often first conditioned by, say, a charge amplifier, to bring the voltage down to a level that can be accepted by an ADC, which then converts the volts to numbers on tape. The maximum input voltage to the ADC is typically $\pm 0.5$ volts which will give numbers on tape to a maximum that is determined by the dynamic range of the system (for example, $\pm (2^{15} - 1)$ for a 16-bit recording system).

Thus the numbers on tape are related to the voltage by some conversion factor which
may be different for each recording channel:

\[ n_i(j) = C_i \int_{-\infty}^{\infty} v_i(t) \delta(t - j \Delta t) \, dt, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, N, \]  

(11)

in which \( n_i(j) \) are the numbers on tape for the \( i \)th channel, \( \Delta t \) is the sample interval, \( j \) is the sample number, the integral is the analogue-to-digital conversion, and \( C_i \) is the conversion factor for the \( i \)th channel. Combining (9) with (11) yields

\[ n_i(t) = Ch_i \int_{-\infty}^{\infty} \left[ \frac{1}{r_{ik}(t)} \left( t - \frac{r_{ik}(t)}{c} \right) - \frac{1}{R_{ik}(t)} \left( t - \frac{R_{ik}(t)}{c} \right) \right] \delta(t - j \Delta t) \, dt, \]

\[ i = 1, 2, \ldots, m, \]  

(12)

in which

\[ Ch_i = C_i h_i \]  

(13)

is the channel sensitivity and has units of (pressure)\(^{-1}\).

The peak is close to the largest number on the tape, which occurs a few milliseconds after the gun fires. The value of the true peak \( n_{i\text{peak}} \) can be found from the digital samples by sinc-function interpolation. Equation (9) then becomes

\[ n_{i\text{peak}} = Ch_i \frac{1}{r_{ik}(0)} \delta_k(t_{\text{peak}}). \]  

(14)

### Determination of relative velocities

In practice both the hydrophones and the sources are moving. Let the \( k \)th source fire at time \( t_k \), so \( s_k(t) = 0 \) for \( t < t_k \). The airgun releases a bubble of air which is buoyant and has a bubble-rise velocity \( -v_z \) which is here assumed to be constant and the same for all bubbles. This is not strictly true: the buoyancy of the bubble decreases with its density, which is time-variant, due to its expansion and contraction, and the drag on the bubble, reducing its vertical motion, increases with the surface area of the bubble. The vertical-rise velocity is therefore not constant, and varies with the size of the bubble. A rough estimate of the bubble-rise velocity can be obtained by standing on the back deck of the vessel and counting the seconds for the bubbles to reach the surface after the guns fire. For guns fired at a depth of 5 m it takes approximately 5 seconds for the bubbles to reach the surface.

The hydrophones are attached to the vessel which is moving forward relative to the water. When the guns fire, each bubble is dragged backwards, relative to the vessel. Let the horizontal velocity of any bubble relative to any hydrophone have components \( v_x \) and \( v_y \). The distances to the \( i \)th hydrophone at time \( t \) from the \( k \)th bubble and its virtual image are

\[ r_{ik}(t) = \left[ (x_i - x_k)^2 + (y_i - y_k)^2 + (z_i - z_k)^2 \right]^{1/2}, t < t_k, \]  

(15)

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Typically, \( v_x \) and \( v_z \) are of the order of 1 or 2 m/s and \( v_y = 0 \), while the speed of sound in water \( c \) is about 1500 m/s, so the relative velocity of the bubble to the hydrophone is very small compared with the speed of sound. The Doppler effects represented by the time-dependent delays \( r_{ik}(t) = c \) and \( R_{ik}(t) = c \) are therefore negligible, as noted by Parkes et al. (1984). The time-dependent amplitude scaling factors \( 1/r_{ik}(t) \) and \( 1/R_{ik}(t) \) vary significantly during the full duration of the signal, which is normally about 0.5 s, and these factors cannot be considered constant. (Amundsen (1993) assumes they are constant. The error he makes decreases as the mini-streamer depth increases. On the other hand, the discrimination against sea-floor reflections decreases as the depth of the mini-streamer increases.)

The velocities \( v_x \) and \( v_z \) are assumed to be the same when the guns fire on their own and can then be determined from the same single-gun data set used to determine the relative sensitivities. When the \( k \)th gun fires, the source time function from each of the other guns is zero. The velocities can be found by a search for the solution to (3), as described by Parkes et al. (1984), in which the signals from the guns that do not fire are minimized.

**Application of the method in practice**

**Definition of the gun–hydrophone geometry**

The airgun array used in this survey consisted of 38 guns in two 3-string subarrays, symmetrically disposed with respect to the ship’s track, as shown in Fig. 2. The geometry of the hydrophones and gun ports was measured with a tape measure on deck with the configuration shown in Fig. 3. Each gun string was hanging from a rail on the back deck and was stretched out to adopt the configuration it would follow in the water. Each gun was hung by chains from its harness, and the gun was thus free to move relative to the hydrophones. It is known from underwater high-speed films that the guns jump when they are fired. At the instant of firing, however, the guns are hanging vertically. The motion of the gun, after it fires, is irrelevant to our calculations, as we are interested in following the motion of the air bubble emitted by the gun. The positions of the hydrophones and guns were measured relative to the deck, and errors in each of the two horizontal and the vertical coordinates of about 1% could be incurred. In operation the gun harnesses were supported by ropes attached to
Norwegian buoys to keep them at a depth of 6 m in the water. To correct the coordinates of the measurements made on deck we shifted our horizontal reference plane (the gun deck) upwards by 6.45 m (to the sea surface). The final set of coordinates for the guns and hydrophones is given in Table 1.

There is always some uncertainty about the depths of the guns, particularly with the conventional Norwegian buoy suspension used in this survey. The drag on the buoys varies with the ship’s speed and the height of the waves, and the angle to the vertical of the suspension ropes increases with this drag. It is possible that the depth of the guns is in error by 0.5 m. The absolute depths of the guns and hydrophones in Table 1 is

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Figure 2. Plan of the full airgun array (not to scale). The gun volumes in cubic inches are indicated on each gun.

subject to this uncertainty. The relative positions of the guns and hydrophones is of course unaffected by it. In determining the notional source signatures, the effect of the ghosts is therefore not well controlled, but these are small effects, and the notional source signatures can be well-determined, as we show below. The uncertainty in the

Figure 3. Geometry of the hydrophones and airgun ports for the three port strings (schematic only). The gun volumes in cubic inches are indicated on each gun.
depth is much more important for the calculation of the far-field signature. For example, the ghost notch in the vertical direction moves from 100 Hz to 107 Hz when the gun depth moves from 7.5 m to 7 m. To overcome this uncertainty we recommend that the depth be measured, as it is in the latest gun monitoring systems (e.g. Lunde et al. 1995).

Definition of the absolute sensitivities of the near-field hydrophone recording channels

There were not enough available channels to enable both port and starboard subarrays to be fully instrumented, but, since they were virtually identical, we decided that one of the arrays should be instrumented properly, and selected the port array. Figure 4 shows the measurements on the hydrophones on one string for one gun firing. The initial peak is easily seen at about 133 ms on each of the hydrophones in the string. From such measurements and the geometry, the relative sensitivities of the hydrophone channels can be estimated.

Equation (14) gives the peak output at the $i$th hydrophone when the $k$th gun fires. At the $i$th hydrophone the corresponding peak is

$$n_{\text{peak}} = C h_{i} \frac{1}{r_{ik}(0)} s_{k}(t_{\text{peak}}). \quad (19)$$

Eliminating the common factor $s_{k}(t_{\text{peak}})$ between (14) and (19) yields the relative

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sensitivities of the two hydrophones:

\[ \frac{C_{h_i}}{C_{h_l}} = \frac{n_{peak,r_{ik}(l)}}{n_{peak,r_{il}(l)}} \]  

(20)

Using each gun and (20), six or seven estimates of the ratio of any two hydrophone sensitivities can be obtained, depending on the gun string. The relative sensitivities, which are then known to about ±2%, can be converted to absolute sensitivities if one of the hydrophone channels has a known sensitivity, or if the value of \( s_0(t_{peak}) \) in (14) and (19) is known.

Consider the two 150 cu. in. guns at the centre of the port middle array. These were both G.Guns, manufactured by Seismic Systems, Inc. in Houston, Texas. The technical specifications for these guns give a measured peak pressure of 2.6 bar referred to 1 m, for an operating pressure of 2000 psi and depth of 6.0 m and recording filters of 0–128 Hz/72 dB. In the survey, the operating depth and firing pressure of all the guns were the same as the conditions under which this calibration was done. The recording filter was 3–218 Hz and the data were recorded at 2 ms intervals. We removed any d.c. component from the data before analysis. The low-cut filter at 3 Hz has a negligible effect on the amplitude of the peak because these airguns generate very little energy below about 5 Hz, but the high-cut filter obviously allows more high-frequency energy to contribute to the recording and exaggerates the peak relative to the calibration. To enable our data to be calibrated, we filtered the near-field hydrophone recordings with a 0–128 Hz/72 dB filter. The amplitude of the peak \( s_0(t_{peak}) \) in (10), (14) and (19) was then known to be 2.6 bar-m for the 150 cu.in. G.Gun.

Figure 4. Recording of the signal of each hydrophone of the port outer string when only the fifth gun fired.
The SSI G.Gun Technical Brochure gives the measured peak pressure for both G. Guns and sleeve guns, all recorded under the same conditions. We have plotted these data in Fig. 5 against the cube-root of the gun volume. Measurements on other single airguns (Fig. 2, Vaage, Haugland and Utheim 1983) suggest that the amplitude of the peak pressure is linearly proportional to the cube-root of the volume, for a broad range of volumes. We have drawn a best-fitting straight line through the data and have used this line to determine the peak amplitude for a given gun volume. For each string a single hydrophone has been calibrated, averaging the calibration results of several single guns, using (10) for each result. The absolute hydrophone sensitivities thus computed are given in Table 2. Note that these sensitivities vary by a factor of about 4. This is typical for good quality hydrophones made to have the same sensitivity.

Definition of the relative motion of the bubbles to the hydrophones

We determined the relative motion from the same single-gun data set required to determine the sensitivities of the near-field hydrophone recording channels. Equations
were modified by multiplying both sides by \( C_i \) to yield

\[
C_i v_i(t) = C_i \sum_{k=1}^{n} \frac{1}{r_{ik}(t)} s_k \left( t - \frac{r_{ik}(0)}{c} \right) - \frac{1}{R_{ik}(t)} s_k \left( t - \frac{R_{ik}(0)}{c} \right), \quad i = 1, \ldots, m,
\]

(21)

in which the Doppler effects have been ignored, the \( C_i \) have already been determined and are given in Table 2, and

\[ m \geq n. \]

(22)

Using (21), rather than (9) enabled us to deal directly with the numbers on tape, rather than the voltages, which we do not know. Equation (22) expresses the existence of the spare hydrophones. Using only \( n \) hydrophones, we searched for a combination of velocities \( v_x \) and \( v_z \) that minimized the sum of the energies of the resulting notional source signals from the guns that did not fire. This is the same as finding a least-squares solution for \( v_x \) and \( v_z \). The distance between the gun strings was sufficiently large to neglect interaction between the strings. So, for the port inner string \( m \) was 7 and \( n \) was 6; for the middle string \( n \) and \( m \) were both 6; and for the port outer string \( m \) was 7 and \( n \) was 6. Figure 6 shows the energy plot for the determination of \( v_x \) and \( v_z \) for the port outer string, with a clear minimum. The horizontal and vertical velocities are \( 0.5 \pm 0.1 \) m/s and \( 1.5 \pm 0.1 \) m/s, respectively. It is possible to obtain more precise figures, but this is not worthwhile for two reasons. First, a change of less than \( 0.1 \) m/s does not give a significant change in the computation of the notional source signatures; second, the vertical velocity is not even constant, as explained above.

### Calculation of the notional source signatures

When several airguns are fired simultaneously, each one produces its own oscillating bubble, with its own radiating spherical wave. Each of these waves modifies the pressure acting externally on the other bubbles, causing the dynamics of oscillation to be different from that of the bubbles oscillating on their own. This mutual interaction depends on the gun geometry, the sizes of the guns in the array, and the relative times of firing of the guns within the array. If all these parameters are constant, the outgoing signal, or 'notional source signature' (Ziolkowski et al. 1982) of any gun is constant. If any of these parameters change, all the notional source signatures must change.
Figure 6. Search for relative horizontal and vertical velocities for the port outer string by minimization of the energy.

Figure 7. Part of a shot record, showing simultaneous acquisition and recording of near-field hydrophone source measurements on the data channels to the left, and seismic reflection data on the remaining channels to the right.
Figure 7 shows part of a typical shot record in which the near-field hydrophone signals are displayed on the first 36 channels on the left and are recorded through the same filters as the data channels, shown on the right. The time scale for all traces in this figure is the same. The long train of pressure oscillations due to the oscillations of the bubbles can clearly be seen on the near-field hydrophone signals.

In (4) the notional source signatures are the $s_k(t)$. Equations (4) were modified by multiplying both sides by $C_i$,  

$$C_i \hat{v}_i(t) = C_h \sum_{k=1}^{n} \left( \frac{1}{r_{ik}(t)} s_k \left( t - \frac{r_k(0)}{c} \right) - \frac{1}{R_{ik}(t)} s_k \left( t - \frac{R_k(0)}{c} \right) \right), \quad i = 1, \ldots, m,$$

(23)

enabling us again to deal directly with the numbers on tape. The resulting equations were then solved for the $s_k(t)$ using the method of successive approximations described by Parkes et al. (1984), in which the channel sensitivities $C_h$ and the time-dependent distances $r_{ik}(t)$ and $R_{ik}(t)$ are known and were determined from the calibration data as described above. Figure 8 shows the notional source signatures calculated for the port outer string for one shot.

**Calculation of far-field signatures including source ghosts**

Once the notional source signatures are known, the pressure can be calculated anywhere in the water according to (2). The point $(x, y, z)$ can be chosen to be in the far-field, 1000 m vertically below the source. There will be significant errors if the calculation is performed in the discrete time domain, since the time delays from each source to the point $(x, y, z)$ are unlikely to be integer numbers of samples. This problem can be overcome if the calculation is carried out in the frequency domain using the following formulation:

$$P_d(x, y, z, \omega) = \sum_{k=1}^{n} \left[ S_{ik}(\omega) \exp \left( i \omega \frac{r_k(0)}{c} \right) - S_{Rk}(\omega) \exp \left( i \omega \frac{R_k(0)}{c} \right) \right],$$

(24)

in which

$$S_{ik}(\omega) = \int_{-\infty}^{\infty} \frac{s_k(t)}{r_k(t)} \exp \{ i \omega t \} \, dt$$

(25)

and

$$S_{Rk}(\omega) = \int_{-\infty}^{\infty} \frac{s_k(t)}{R_k(t)} \exp \{ i \omega t \} \, dt.$$  

(26)

The phase shifts corresponding to the time delays can be computed exactly. This calculation can be made for any point in the water. As an example Fig. 9 shows far-field signatures for the vertical and for 20°, 40° and 60° to the vertical, measuring the angle away from the vessel, i.e. towards the cable.
Figure 8. The notional source signatures calculated for the port outer sub-array for one shot.
Figure 9. The far-field signature of the full airgun array (treating the starboard array as identical with the port array) at different angles in the vertical plane containing the ship’s track, with the angle increasing away from the vessel for 0°, 20°, 40°, 60°.
Test of calculation method against spare near-field measurements

As mentioned above, once the notional source signatures have been calculated, the pressure can be calculated at any point in the water, for instance at any of the spare hydrophones. There were two spare hydrophones: on the port outer string and on the port inner string (see Fig. 3), and the sensitivities of both hydrophone channels are known from the calibration data (Table 2). The recorded signals can be divided by the absolute channel sensitivities to determine the pressure signals at the hydrophones. The pressure signal at each spare hydrophone can also be calculated according to (8), but in the frequency domain, along the lines expressed by (24), (25) and (26). The comparison of the predicted and measured pressure signals at the two spare hydrophones is shown in Figs 10 and 11. The rms errors for the spare hydrophones on the port outer and port inner strings were 2.8% and 5.3%, respectively.

There are several sources of error. First, there is an error of a few cm in the determination of the position of the gun ports on deck. Second, the gun–hydrophone geometry in the water is not likely to be exactly the same as it is on deck and this change may be a few cm. Third, both these sources of error feed into our estimate of the hydrophone sensitivities, creating additional errors in the calculation. Fourth, there is a very small interaction among the strings of guns, which we neglected in our calculation because the strings were observed from the back deck to be at least 15 m apart. To have included the small interactions among strings would have required a knowledge of the inter-string geometry to far greater precision than was available on this occasion.

It is our opinion that the rms errors of a few percent that we have observed here are acceptable but they could be improved (a) by having properly-calibrated hydrophones

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and (b) by restricting the movement of the guns, so that the gun–hydrophone geometry as measured before the guns enter the water is as near as possible to the true geometry in the water.

This is a rigorous test of the theory of Ziolkowski et al. (1982) and Parkes et al. (1984), provided the spare hydrophones are placed in positions where they can be considered to give independent information. That is, the $m$ hydrophones are all independent if any $n$ of them can be used to find the notional source signatures. This test can be done as a matter of course on every shot record.

**Conclusions**

For the first time a commercial marine seismic survey has been completed with the source wavefield measured for every shot and with a continuous check on both the quality of the measurements and the reliability of the computed wavefield.

The wavefield of the airgun array was measured using an array of near-field hydrophones on the Seismograph Service Limited (SSL) Seisventurer in October 1992. The method of Ziolkowski et al. (1982) and Parkes et al. (1984) was used to determine the wavefield of the airgun array and requires as many near-field hydrophones as there are source elements. Two spare hydrophones were added to provide checks on the quality of the measurements and on the calculation of the wavefield.

The positioning of the hydrophones is important. They should be as close to the guns as possible, but not so close that the afterflow term in the pressure field becomes significant. They should also be as far from the sea-floor as possible to minimize the interference from sea-bottom reflections. In practice these two requirements force the location of the hydrophones to be above the airguns and no closer than about 1 m. If

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**Figure 11.** Comparison of measured and predicted pressures at the spare hydrophone on the port inner string: measured ($\cdot$); predicted($\cdot\cdot$). The rms error is 5.3%.

there are $n$ source elements and a total of $m$ hydrophones ($m \geq n$), the $m$ hydrophones are all independent if any $n$ of them can be used to determine the notional source signatures by inversion. This configuration was observed in this experiment.

We have shown how the method of computing the wavefield can be checked using one shot record in which the rms errors at the spare hydrophones were 5.3% and 2.8%. Such errors could be reduced in practice (a) by having properly-calibrated hydrophones and (b) by restricting the movement of the guns, so that the gun–hydrophone geometry as measured before the guns enter the water is as near as possible to the true geometry in the water.

We recommend that this check be implemented as a normal quality control procedure whenever it is desired to measure the source wavefield. For example, within Geco-Prakla’s Trisor system, this could be implemented by adding an extra hydrophone on each gun string.

**Acknowledgements**

We gratefully acknowledge the cooperation of Seismograph Service Limited for obtaining the data used in this paper, and for allowing us to use it for research purposes. In particular, we thank Ian Cheshire, Director; Rex Lugg, Manager of the Marine Division; Paul Tollefson, for obtaining permission to carry out the survey from all the oil companies in the Inner Moray Firth; John Connor, Party Chief; Lloyd Peardon, Manager of Research; and Robert Laws of Research, who also came on board and did some of the initial work in the determination of the hydrophone sensitivities. We are also very grateful to the Master and crew of the seismic vessel *Seisventurer*. The data are now the property of Geco-Prakla and are the subject of a joint venture between Geco-Prakla and the University of Edinburgh.

A.Z. thanks the Petroleum Science and Technology Institute (PSTI) for funding the chair of Petroleum Geoscience at Edinburgh. The work reported here was based on a study funded by PSTI and Total Oil Marine entitled “Does it make sense to measure the source signature continuously in marine seismic exploration?”, which was completed in February 1994. R.J. is now funded under NERC Project No. GR3/9064.

**Appendix**

**The pressure near an airgun bubble**

This appendix demonstrates that an oscillating bubble creates a pressure field that propagates outwards with the velocity of sound $c$, and decays in amplitude linearly with the distance $r$ from the centre of the bubble, for distances $r$ greater than about 1m. At distances less than 1m from the centre of the bubble there is a non-linear contribution to the pressure field which cannot be neglected.
Basic Equations

Consider a typical airgun bubble in water, oscillating radially with a diameter varying from a few centimetres to more than 1m, with pressure radiation in the frequency range 5–100 Hz. Neglecting gravity and the effect of the surface of the water, the pressure $p$ and particle velocity $v$ in the water are determined by the radial distance $r$ from the centre of the bubble and time $t$. The equation of motion of the water is

$$\frac{\partial p(r,t)}{\partial r} = -\rho(r,t) \frac{Dv(r,t)}{Dt}, \quad (A1)$$

in which $\rho$ is the density of the water and the total Lagrangian time derivative is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v(r,t) \frac{\partial}{\partial r}. \quad (A2)$$

The first term on the right-hand side of (A2) is the partial derivative with respect to time, and the second, non-linear, term arises from following the particle. Substituting (A2) in (A1) and dividing by $\rho$ yields

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = -\frac{\partial v}{\partial t} - v \frac{\partial v}{\partial r}. \quad (A3)$$

The pressure and density in the water are related via the speed of sound, according to the definition

$$c^2 = \frac{dp}{d\rho}, \quad (A4)$$

from which it may be deduced that moderate changes in pressure correspond to very small changes in density. Since the flow is radial, and therefore irrotational, the particle velocity can be described as the gradient of the particle velocity potential $\phi$:

$$v(r,t) = -\frac{\partial \phi(r,t)}{\partial r}. \quad (A5)$$

Substituting for $p$ from (A4) in (A3) and using (A5) in (A3) yields

$$\frac{c^2}{\rho} \frac{\partial \rho}{\partial r} = \frac{\partial^2 \phi}{\partial r^2} - v \frac{\partial v}{\partial r}, \quad (A6)$$

which may be integrated with respect to $r$ to give

$$c^2 \log \left( \frac{\rho}{\rho_u} \right) = \frac{\partial \phi}{\partial t} - \frac{v^2}{2}. \quad (A7)$$

in which $\rho_u$ is the undisturbed density of the water, and the particle velocity potential and its derivatives are all assumed to be zero at infinite distance from the bubble. Equation (A7) is Bernoulli’s equation, in which the second term on the right-hand side is non-linear and is significant very close to the bubble, but becomes negligible, as will be shown, at distances greater than about 1 m from the bubble. This non-linear term arises directly from the second non-linear term in the total time derivative (A2).
First-order relationship for the particle velocity potential

It is necessary now to find an expression for the particle velocity potential, using the equation of continuity,

$$\frac{1}{\rho} \frac{D\rho}{Dt} + \frac{2v}{r} + \frac{\partial v}{\partial r} = 0. \quad (A8)$$

From (A7) and (A8), a first-order expression for the particle velocity potential may be found by linearizing the equations. Linearization is approximation of the total time derivative in (A2) by the partial time derivative:

$$\frac{D}{Dt} \approx \frac{\partial}{\partial t}, \quad (A9)$$

and applies where the particle velocity is very small compared with the speed of sound. The non-linear term in (A7) then vanishes and the linearized version of (A7) is

$$c^2 \log_e \left( \frac{\rho}{\rho_0} \right) = \frac{\partial \phi}{\partial t}, \quad (A10)$$

while the linearized version of (A8) is

$$\frac{1}{\rho} \frac{\partial \rho}{\partial t} + \frac{2v}{r} + \frac{\partial v}{\partial r} = 0. \quad (A11)$$

Differentiating (A10) with respect to time and subtracting the result from (A11) yields

$$\frac{2v}{r} + \frac{\partial v}{\partial r} + \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 0. \quad (A12)$$

Substituting for \(v\) from (A5) in (A12) yields

$$\frac{2}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 0, \quad (A13)$$

which may be written as

$$\frac{\partial^2 (r\phi)}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 (r\phi)}{\partial t^2} = 0. \quad (A14)$$

Equation (A14) is the linear acoustic wave equation in spherical coordinates and has the well-known solution

$$\phi = \frac{1}{r} f \left( t - \frac{r}{c} \right), \quad (A15)$$

in which \(f(t)\) is known as the wave function. This may now be used to obtain expressions for the particle velocity and pressure.

Expressions for pressure and particle velocity in terms of the wave function

First, return to (A3), where, with very little error, we now regard the density as...
constant. Using (A5) in (A3) as before and integrating, yields

\[
\frac{p - p_w}{\rho} = \frac{\partial \phi}{\partial t} - \frac{v^2}{2}. \tag{A16}
\]

Using (A15) in (A16) gives

\[
\frac{p - p_w}{\rho} = \frac{1}{r} f'(t - \frac{r}{c}) - \frac{v^2}{2}, \tag{A17}
\]

where the prime indicates differentiation with respect to the argument. The particle velocity can be obtained from (A5), given (A15), and is

\[
v = \frac{1}{r} f(t - \frac{r}{c}) + \frac{1}{rc} f'(t - \frac{r}{c}). \tag{A18}
\]

The range of significance of the non-linear \(v^2/2\) term in the radiated pressure

The problem is now to determine the range of significance of the \(v^2/2\) term in (A17), and it may be approached by considering a single frequency. Let

\[
f(t) = \exp\{i\omega t\}. \tag{A19}
\]

Then (A17) becomes

\[
\frac{p - p_w}{\rho} = \frac{i\omega}{r} \exp\{i\omega (t - \frac{r}{c})\} - \frac{v^2}{2}, \tag{A20}
\]

while (A18) becomes

\[
v = \frac{1}{r} \exp\{i\omega (t - \frac{r}{c})\} + \frac{i\omega}{rc} \exp\{i\omega (t - \frac{r}{c})\}. \tag{A21}
\]

Substituting for \(v\) from (A21) into (A20) gives

\[
\frac{p - p_w}{\rho} = \frac{i\omega}{r} \exp\{i\omega (t - \frac{r}{c})\}\left[1 - A(\omega, r)\right], \tag{A22}
\]

in which,

\[
A(\omega, r) = \exp\{i\omega (t - \frac{r}{c})\}\left[\frac{1}{r^3 \omega} + \frac{2}{r^2 c} + \frac{i\omega}{rc^2}\right]. \tag{A23}
\]

is the influence of the non-linear term. For \(c = 1500\ \text{ms}^{-1}\) and frequencies 5–100 Hz, it is clear that the first term in the square brackets is the largest at small \(r\) and low frequencies. For \(r = 1\ \text{m}\), this term has a magnitude of about 0.03 at 5 Hz, decreasing approximately linearly with frequency and decreasing inversely as the cube of the distance. It follows that at distances greater than about 1 m the non-linear term in the pressure field can be neglected, and the pressure field can be written from (A17) as

\[
\frac{p(r, t) - p_w}{\rho} = \frac{1}{r} f'(t - \frac{r}{c}), \ r \geq 1 \ \text{m}. \tag{A24}
\]

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For ranges significantly less than 1 m this approximation is not valid. For example, for \( r = 0.3 \) m, the magnitude of \( A(\omega, r) \) is approximately 1 at 5 Hz, while for \( r = 0.2 \) m, the magnitude of \( A(\omega, r) \) is approximately 4 at 5 Hz.

References


