We distinguish two approaches to geophysical inverse problems. One is the opposite, or inverse, of the forward problem. Signature deconvolution and migration are examples of this kind of inversion. The data are inverted to arrive at the earth model. The other approach is iterative forward modelling, in which we try to match the measured data with synthetic data created with a model and the given theory. In recent years Bayes's rule has been a popular way to measure the quality of the result obtained by iterative forward modelling. We focus on the philosophical arguments for the two approaches and on the difficulties in applying them to geophysical inverse problems.

The differences between these two approaches are related to the problem of induction. In 1739, the philosopher David Hume (see, e.g., 1984) split the problem of induction into two: a logical problem and a psychological problem. The logical problem of induction is expressed by the question: Can we prove that a universal theory is true? Hume proved that the answer is no. The instances of our experience are always an infinitesimally small fraction of the total number of possible instances. For example, all our experiences of gravitational attraction at non-relativistic velocities corroborate Newton's law of gravitation. We are confident that Newton's law applies throughout the universe. But the law has not been proved. In fact, compared with the total amount of evidence we should need to prove Newton's law throughout the whole universe, our experience is infinitesimally. The probability (in the sense of the calculus of probability) of Newton's law of gravitation being true is then infinitesimally small.

Although Hume solved the logical problem of induction, there still remained the so-called psychological problem of induction: Why do people have confidence in their theories? Why do people believe that their past experience has relevance to the future? For example, why do we believe the sun will rise tomorrow? In the first half of this century, Popper (1934, 1959, 1969, 1986) solved this problem. Although Hume is quite right that we can never prove a universal theory to be true, we are able to prove that a theory is false. This can be established via some test against reality. If we test a theory, exposing it to risks designed to question the axioms on which it rests, there is the possibility that the theory will fail. If it does fail, we have to find new axioms, or laws. If it survives the tests, and continues to survive new tests, our confidence in it is increased. However, since the evidence from these tests is infinitesimal compared with the amount of evidence we should need to prove the theory, the absolute probability of the theory being true is still always zero. Not all the theories we use can be put at risk in the way suggested by Popper. Popper uses this distinction as the demarcation between scientific and non-scientific theories. According to Popper, theories are scientific if they are framed in such a way that they may be put at risk. Theories are not scientific if they cannot be put at risk.

How do we decide whether a given test has refuted a (scientific) theory? This is not always clear-cut. If there is only one well-established and accepted theory, it is very difficult, psychologically, to accept the evidence from a given test that the theory has been refuted. As Lakatos (1970) argued, in this situation ad hoc explanations are then always proposed to account for the conflict of the evidence with the theory. He argued that you really need to have competing theories. If a rival theory is able to explain the
new evidence as well as everything that was explained by the first theory, it is more powerful and will displace the first theory.

There are geophysicists who take a probabilistic view of the world and express this in mathematical terms using Bayes's rule (See, for example, Howson and Urbach, 1989). These geophysicists do not put the theory at risk, but they do question the model which, together with the theory, yields synthetic data which are a match to the measured data. They ask: What is the probability of the model being true, given the data? Bayes's rule allows them to answer the question as follows:

$$P(\text{model} \mid \text{data}) = \frac{P(\text{data} \mid \text{model}) P(\text{model})}{P(\text{data})}$$

in which $P$ denotes the probability. The a priori information is given by $P(\text{model})$, and we "learn" from the data via the likelihood $P(\text{data} \mid \text{model})$. The probability of the data $P(\text{data})$ is usually considered to be a constant in order to make $P(\text{model} \mid \text{data})$ also a probability function.

In our opinion the main objection to the Bayesian approach as a philosophy, is the use of probabilities, in the sense of the calculus of probability. The presence of $P(\text{model})$, the a priori distribution, poses the main difficulty. We do not know whether the chosen parameterisation of the model is true. There are, in principle, an infinite number of ways in which we can parameterise the model. The probability of a given parameterisation being true is therefore zero. But we can use Bayes's rule only if the probability of the parameterisation is finite. In fact, Bayes's rule applied in the sense in which it is used here only defines a given optimisation problem. Of course, the optimisation problem is very interesting and geophysicists need methods to solve this problem, but it is not the same as determining the probability that the solution to the geophysical inverse problem has been found.

So, what can we do then as geophysicists? We should certainly question our parameterisations. How many significant parameters do we have? How many significant data do we have? How do we define significant data? In order to solve geophysical inverse problems with scientific theories, we believe we should have at least three requirements. First, we should put our theories at risk and test every step in a given theory. Secondly, we should always have fewer parameters than significant data. In this way we have an overdetermined system and can quantify our errors. Finally, we should always pose the question: What would we regard as a refutation of our solution? We illustrate these points using examples taken from the field of geophysics.

References


