Why don't we measure seismic signatures?

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ABSTRACT

There are three related problems with our approach to signature deconvolution. First, there is a confusion among geophysicists about the basis of the convolutional model itself, which leads to doubts about the value of measurements of the source signature. Secondly, it is not generally recognized that statistical methods of wavelet estimation are unreliable. Thirdly, many explorationists are unaware that it is practical in many cases to make meaningful measurements of the source signature.

The convolutional model of the reflection seismogram applies only for a point source, and is the convolution of the source signature with the impulse response of the earth, of Green's function, which contains all possible arrivals, including reflections, refractions, multiples and diffractions. Stabilized deconvolution of the data with a known band-limited signature is straightforward. The signature can be obtained by independent measurements, as described in the literature. The recovery of the elastic layer parameters from the band-limited impulse response of the earth, after removal of the source signature by deconvolution, is the problem of inversion, and is not discussed in this paper.

The theory of wave propagation does not support the commonly held view that a reflection seismogram can be regarded as a convolution of a wavelet with the series of normal-incidence primary reflection coefficients. This is true of both prestack and poststack data. Poststack seismic inversion schemes, based on this model, that use well logs to extract the wavelet for predicting lateral variations in lithology away from the wells, rely on the wavelet to be laterally invariant. Even if there is perfect shot-to-shot repeatability, this model must yield a different wavelet at every well, and therefore the extracted wavelet does vary laterally. These schemes are therefore self-contradictory and, in the worst cases, their results are likely to be worthless.

Published methods for determining the source signature from measurements for the land vibrator, marine seismic source arrays, and dynamite on land are summarized. None of these methods appears to be in use. A Vibroseis example is included to show that the signal transmitted into the ground by the vibrators does not closely resemble the predetermined sweep, as is normally assumed. The transmitted signal could be determined in processing from measurements of the vibrator behaviour that are made in production for vibrator control, if only these measurements were recorded. Normally they are not. Instead of using measurements to determine the signature, the exploration industry relies on wavelet estimation methods that depend on both a model and statistical assumptions that have no theoretical justification.

INTRODUCTION

There is an unfocused discussion among geophysicists about the value of measuring the source signature. On one hand, there are those who argue that the seismograms are so complicated that our mathematical descriptions explain only a small part of the data, and therefore a statistical approach to data processing is needed. On the other hand, there are those who argue that it is necessary to be as deterministic as possible; as a consequence of this second approach, it pays to measure the source signature.

In the first approach, led by Enders Robinson and Sven Treitel and based on the original stationary time series model of Wold (1938), the statistical properties of the data are exploited to remove the wavelet and attenuate near-surface reverberation (see especially Robinson, 1954; Treitel and Robinson, 1967; and Peacock and Treitel, 1969). Following this work, there has been a continuous stream of papers on
statistical approaches to deconvolution, including homomorphic deconvolution (Ulysh, 1971). Many of these famous papers are included in the two-volume book by Webster (1978). The original approach of Robinson and Tretelie is fully developed in their book (1980). Until reliable measurements of the source signature were possible, the statistical approach was, in fact, the only one available.

There is no doubt that the statistical approach to data processing has allowed the industry to make tremendous progress, especially offshore in the suppression of unwanted water-layer multiples. For the deterministic approach, the major triumph is probably the Vibroseis method. No serious geophysicist would dream of trying to apply the statistical approach for the determination of the Vibroseis wavelet to data that had not first been correlated with the sweep.

In this paper I discuss on the influence of the seismic source on the nature of the seismogram. I argue that the deterministic approach is the correct one, provided the seismogram satisfies the convolutional model. Since there is a great deal of confusion in the literature about the model, I give a brief outline of it below.

THE CONVOLUTIONAL MODEL.

In the convolutional model of the seismogram, the key concept is linearity. For perfectly elastic media, this is expressed by Hooke’s law relating stress and strain. For imperfectly elastic media, in which elastic energy is converted to heat by mechanisms known collectively as internal friction, a variety of different models have been proposed. Linear visco-elastic models, in which the stress depends linearly on both the strain and the rate of change of strain, include Voigt solids, Maxwell solids, and more general models (Kolsky, 1963). The propagation of waves in the medium is governed by the stress-strain relation, conservation of mass, and Newton’s laws of mechanics.

The effect of the source signature on the seismogram is independent of the layer parameters of the earth, as illustrated using the simplest case of lossless fluids. Figure 1 shows a seismic reflection experiment with a point source of volume injection in the top layer (it could of course be in any layer). Consider a point source at \((x, y, z)\) in layer 1, with propagation velocity \(c_1\). The wave equation for the pressure in this layer is then

\[
\nabla^2 p - \frac{1}{c_1^2} \frac{\partial^2 p}{\partial t^2} = -8(x-x_i)\delta(y-y_s)\delta(z-z_s)s_i(t),
\]

\[
\nabla^2 p - \frac{1}{c_n^2} \frac{\partial^2 p}{\partial t^2} = 0,
\]

where \(s_i(t)\) is the source time function and has units of pressure times distance (e.g., bar-m), and the asterisk denotes convolution. The temporal convolution originates right here, at the source. Note that the source is introduced spatially as a discontinuity in the wave field. This is an approximation which holds only for sources whose dimensions are very small compared with the smallest wavelengths they generate. In the \(n\)th layer there are no sources and the wave equation for the pressure is homogeneous:

\[
\nabla^2 p = 0.
\]

in which \(c_n\) is the velocity of propagation in layer \(n\). The wave fields in adjacent layers are connected by the boundary conditions: at the interface between any two layers, the stress and normal component of the displacement are continuous. There is no need for the boundaries to be horizontal, or even planar.

Let the response of the earth at the receiver, with coordinates \((x_r, y_r, z_r)\), be the pressure \(p_{r}(t)\), where subscripts \(s\) and \(r\) indicate the dependence on the source and receiver positions, respectively. The effect of the source time function is a convolution, and the response at the receiver may be written as

\[
p_{r}(t) = s_i(t) * g_{r}(t),
\]

where \(g_{r}(t)\) is the response at the same receiver position to an impulse in units of pressure times distance at the same source position. This is the convolutional model of the seismogram.

If the earth had been modeled as linear elastic layers, or linear viscoelastic layers, rather than fluid layers, the argument would be exactly the same. The equivalent wave equations would be different, of course, but the source term would appear only in the right-hand side of the inhomogeneous equation corresponding to equation (1). Because the stress-strain relation is linear, the response is equal to the convolution of the time function \(s_i(t)\) of the point source with the impulse response, or Green’s function, \(g_{r}(t)\) of the earth. The properties of the medium and the source-receiver geometry determine this impulse response.

If the stress depends on the rate of change of strain, for example in a Maxwell solid or a Voigt solid, there is attenuation, which is likely to be different in every layer.

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**Fig. 1.** A monopole source and point receiver: the basis for the convolutional model of the seismogram.
Attenuating and elastic media have different impulse responses \( g_p(t) \). The source time function is independent of this—it is the signature at the source.

**THE CHANGING WAVELET MODEL OF THE SEISMOGRAM**

There is another description of the reflection seismogram (e.g., Ricker, 1940; Dobrin and Savit, 1988), in which the seismic wavelet has a character determined both by the source and by transmission effects. In this model, which I call "the changing wavelet model," the source signature is modified by its propagation through the earth. The modifications are caused by elastic transmission effects such as peg-leg multiples, that add a tail to the wavelet, and by anelastic absorption that may be expressed by a linear relation between the stress and the rate of change of strain, and which attenuates the high-frequency energy, causes dispersion, and also lengthens the wavelet. On its way back from a reflecting interface, the wavelet is further modified by transmission through the overburden to the receivers. From this model of the seismogram, many geophysicists regard the measurement of the source signature as irrelevant. Conventional statistical methods of wavelet estimation therefore usually take a window of data, including the target zone, from which to make an estimate.

The shape of the returning wavelet is determined not only by the source signature, but also by its travel path. Primary reflections and multiple reflections that arrive at the same time have traveled different paths and therefore have different wavelets. In fact, in this model there is a different wavelet for every different arrival. The seismogram in this model is the sum of these different arriving wavelets; it cannot be written as the convolution of some fixed wavelet with some other physically meaningful time function, even in some window of the data.

Exploration geophysicists must tie the seismic reflection data to well data in order to interpret the horizons. This is a chronic source of difficulty, where deconvolution clearly has a role to play. In their textbook, Dobrin and Savit (1988) are especially confusing about this problem. They explain (p. 296) that "the reflectivity time series" is the series of normal incidence reflection coefficients calculated from the density and velocity logs run in the well. On p. 297 they say that the seismogram can be described as a convolution of "the wavelet" with "the reflectivity." However, on p. 298 they say: "The relationship between the seismic trace and the sonic log is complex and not open for simple or naive interpretation. The band-limited nature of the seismic data, the thin-bed interference or tuning effect, and other factors of noise, multiples, etc., combine to obscure the desired, simple relationship among amplitude, reflection coefficient, and velocity (or acoustic impedance contrast)."

Clearly, there ought to be a tie between the seismic data and the well data, but it cannot be achieved by simple deconvolution because the seismic data cannot be described as a convolution of some wavelet with the sequence of primary reflection coefficients. This is clear from the literature on wave propagation in layered media (e.g., Ewing et al., 1957; Wuenschel, 1960; Goupillaud, 1961; Kennett, 1983).

**EVALUATION OF STATISTICAL DECONVOLUTION METHODS**

Nondeterministic, or statistical wavelet estimation methods, estimate "the wavelet" from the seismic data. They assume the data in a given window can be described as the convolution of "a wavelet," say \( w(t) \), with some other function, say \( r(t) \), which is very often called "the reflectivity series:"

\[
p(t) = w(t) \ast r(t)
\]

In normal everyday geophysics, the term "reflectivity series" refers to the series of normal-incidence primary reflection coefficients calculated from velocity and density logs measured in a well (e.g., Dobrin and Savit, p. 296). Whatever the function \( r(t) \) may be, it is certainly not this reflectivity series. Many geophysicists are simply unaware of this. This applies equally to prestack and to poststack data. The poststack situation is treated in more detail later.

The physical interpretation of "the wavelet" \( w(t) \) is equally mysterious, since none of these methods claim to extract the source time function \( s(t) \). If they did, they would be open to refutation by measurement of this function. The wavelet that is estimated and removed by the statistical deconvolution methods depends on the assumptions of the estimation method: each method has different assumptions and estimates a different wavelet from the same data. There is no clearly defined test for evaluating these methods, so no one can say that any particular method is wrong. This brings these methods and the theories on which they are based into the realm of metaphysics, rather than science.

It has become customary to let the interpreter decide what the correct wavelet is, rather than using a well-defined test. I quote two examples of this custom, but many more can be found in the geophysical literature. In their paper on deconvolution by autoepstral windowing, Schuster and Wagner (1985) include the following statement in their conclusions: "A difficulty in applying seismic theory to field data stems from not knowing the true answer. Therefore, the validity of the final output is usually subject to the interpreter's personal judgement" (my emphasis). This view is echoed by Brown et al. (1988), who include the following statement in the opening paragraph of their paper on wavelet estimation: "Although the wavelet estimation problem can become buried in mathematics, the interpreter is best suited to judge the quality of the estimated wavelet and the applicability of the wavelet to the final interpretation" (my emphasis).

The truth is independent of the interpreter's personal judgment. However, there is a valid point that these statements are driving at: the seismic data are supposed to tie to the well data. If the seismic reflection data do not tie with synthetic seismograms calculated from the well logs when the estimated wavelet \( w(t) \) is removed from the data, the interpreter can blame the deconvolution. This would be reasonable if the seismogram were correctly expressed by equation (4). The conditions under which this model would be valid are as follows: (1) the earth must consist of plane horizontal layers; (2) internal multiples must be negligibly small; this is true if the primary reflection coefficients are infinitesimal; (3) there are no reflections from the free
Measuring Seismic Signatures

The stationarity assumption states that the statistical properties of \( p(t) \), estimated over an ensemble of time windows each of adequate length, are independent of the time origin of the windows. Seismic reflection data are clearly not stationary; they do not exist before the shot is fired and are effectively zero a few seconds later. The application of some time-dependent gain function attempts to alter this. Unless the gain correction is exponential, it ensures that the convolutional model is violated by distorting the wavelet unevenly down the trace (Ziolkowski, 1984).

The stationarity assumption is critical in conventional predictive deconvolution (Peacock and Treitel, 1969). Stationarity is assumed in order to estimate the autocorrelation of the “predictable part” of the seismogram—the multiples—from the data. This can then be deconvolved from the data to recover the “unpredictable part”—the primary reflections or, “reflectivity”, \( r(t) \). However, since the data are not stationary, the so-called “unpredictable part” is not unpredictable.

The autocorrelation is computed from a window of data that includes the zone of interest. Peaks and notches in the autocorrelation function are caused not only by multiples, but also by the correlation of primary reflections with each other. This occurs especially in sequences of cyclic deposition and for sequences where there are a few large reflectors. In predictive deconvolution such correlations are interpreted as multiples. The predictive deconvolution operator works on the early part of the data to “predict” and subtract correlated information which arrives later. If it removes multiples and uncovers primary reflections, we are happy. If it removes later primary reflections, of adds events which come to be interpreted falsely as later primaries, we are not at all happy. We could compute the autocorrelation function using a shallower window that does not include the zone of interest, but there are two problems with this. First, since there are no data until the shot instant, the windows cannot be made long enough without including the zone of interest. Second, if the data are not stationary, the autocorrelation, computed over any window, must contain undesired correlations between primary reflections as well as the desired correlations caused by multiples.

The stationarity assumption says that the problem of correlation between primary reflections does not arise. We may think of the seismogram as a small piece of an infinitely long stationary time series, whose autocorrelation is more or less the same wherever we choose to put the window over which it is estimated, provided the window is long enough. It does not matter to the estimate where in the sequence the window is located; it can just as well include the zone of interest as not. Thus, provided the data are stationary, we are perfectly entitled to use windows that include the zone of interest.

Unfortunately, assuming the data are stationary does not make the data stationary. This is an assumption that is testable, but the tests are rarely made. If the data are stationary and reasonably broad band, the phase spectrum should be a random function of frequency, and the phase distribution should be uniform in the range \(-\pi\) to \(+\pi\) (e.g., Rice, 1954); this test is well known. The problem is to find a figure of merit to determine how much deviation from this uniform distribution causes the stationarity assumption to
break down (Treitel, 1989). Without this figure of merit it is impossible to say whether the stationarity assumption is good or bad. The validity of the assumption probably varies from area to area. There is every reason to expect seismic data to be nonstationary and for accidental (unwanted) correlations between primary reflections to be the rule, rather than the exception.

The assumption of whiteness is a second critical assumption in most waveform estimation methods, for the amplitude spectrum of the reflection response is normally not known. The assumption is critical in both conventional least-squares spiking deconvolution and in minimum-entropy deconvolution. In comparing least-squares with minimum-entropy deconvolution, Wiggins (1985) stresses the importance of the whiteness assumption. His clearest statement is in the opening sentences: "I examine here the blind deconvolution problem where an observed time series consists of the convolution of an unknown wavelet with a white, stationary, random sequence. My goal is to remove the effects of the wavelet so the underlying random sequence can be examined free from the smoothing effects of the wavelet" (my emphasis).

In the frequency domain equation (4) becomes

$$P(\omega) = W(\omega) R(\omega),$$

in which the convolution becomes a multiplication. The whiteness assumption says that the amplitude spectrum of the response \( R(t) \) is a constant, say, \( \sigma \):

$$|P(\omega)| = \sigma |W(\omega)|,$$

from which it is clear that the amplitude spectrum of the wavelet is simply a scaled version of the amplitude spectrum of the data. This assumption is made purely for mathematical convenience. Neither equation (4) nor equation (5) can be solved. The whiteness assumption says that the response \( R(t) \) has no influence on the amplitude spectrum of the seismogram. This assumption is not supported by any theory.

It has been known for years that the normal-incidence reflection response of a horizontally layered earth lacks low frequencies. This was first noticed by O’Doherty and Anstey (1971). A thin layer has a low-cut reflection response, with the amplitude of the response proportional to the frequency (Widess, 1973; Kooihoed and de Voogd, 1980). Any layer behaves like a thin layer if the frequency is low enough, so this lack of low-frequency energy is inevitable (Ziolkowski and Fokkema, 1986).

It is sometimes assumed (e.g., Yilmaz, 1987, p. 85, assumption 5) that the randomness of the reflection coefficients has something to do with the assumed whiteness of the reflection response. This is false on two grounds: from signal theory and from wave propagation theory. Arguing from signal theory, we regard a signal as white if its amplitude spectrum is a constant, independent of frequency; for example: a delta function, or white random noise. The difference between the delta function and the random noise is in the phase spectrum. The phase spectrum of random noise is random: it is independent of the frequency. The phase spectrum of the delta function is not random: it is a linear function of frequency. So randomness and whiteness are independent signal properties; whiteness refers to the amplitude spectrum and randomness to the phase spectrum. Therefore whiteness cannot be a consequence of randomness, and the randomness of the reflection coefficient series is irrelevant to the whiteness of the reflection response.

Arguing from wave propagation theory, we know that the reflection response of a layered earth depends upon a number of factors, including the reflection coefficients. Fokkema and Ziolkowski (1987) showed theoretically and with examples that the randomness of the reflection coefficient series is irrelevant to the whiteness of the reflection response. If there is no free surface and the layered earth is considered as sandwiched between two half-spaces, the amplitude spectrum of the response depends only upon the angle of incidence of the wavefront on the deepest interface. If this angle exceeds the critical angle, the wave transmitted into the lower half-space is inhomogeneous, or evanescent. It does not contain any propagating energy. The interface behaves as a perfect reflector and, if the layers are elastic, all the incident energy is reflected and returns eventually to the upper half-space. The reflection response is then all-pass, or white. If the angle of incidence on the lowest interface is less than the critical angle, energy propagates into the lower half-space, and the spectrum of this transmitted energy depends upon the layer parameters and the angle of incidence. It follows that the reflection response is not then all-pass and therefore not white. If the free surface is included in the analysis, it introduces peaks and notches in the spectrum of the plane-wave response, both for precritical and postcritical incidence on the lower half-space (Fokkema and Ziolkowski, 1987).

In conventional seismic data processing all shallow refractions and postcritical reflections are removed by the muting process at an early stage; deeper refractions and postcritical reflections are not observed because of the limited offset range. So, after muting, only reflections at precritical incidence remain. The spectra of the resulting data are therefore nonwhite, containing many peaks and notches caused by the interference effects in the earth, including the free surface, and exhibiting the well-known low-cut filtering effect. The reflection response of every trace in every near normal-incidence shot gather is nonwhite, before or after muting. Averaging the spectra does not make the reflection response white: it only smooths the spectrum. Any estimate of the amplitude spectrum of the source signature, or wavelet, from averaged spectra of shot gathers depends, finally, on the whiteness assumption, which is wrong.

The third problem—finding the phase of the wavelet given its amplitude spectrum—is well-known to be insoluble. The conventional minimum-phase assumption is invalid for every known seismic source, with the exception of dynamite on land, for which it may be approximately correct (Schneider, 1982). The phase spectrum of the wavelet is not related to the amplitude spectrum, and, as we have seen, the amplitude spectrum of the wavelet cannot be obtained from the precritical seismic reflection data. It follows that neither the amplitude spectrum nor the phase spectrum of the wavelet can be obtained from the precritical seismic data.

Following the work of Oppenheim (1965) and Schafer (1969), there was considerable interest in homomorphic deconvolution. Early geophysical papers on this are by Ulrych (1971) and Stoffa et al. (1974). The idea behind
method is essentially to take the logarithm of equation (6): the multiplication becomes an addition. The log-frequency data are then Fourier transformed to a domain known as the complex cepstrum. It is assumed that the wavelet is associated with the slowly varying part of the spectrum, which corresponds to the low cepstrum numbers, while the earth response is associated with the rapidly varying part of the spectrum, which corresponds to the high cepstrum numbers. This assumption has much in common with the stationarity assumption in predictive deconvolution. Separation of the wavelet from the earth response is achieved by low-pass filtering to extract the wavelet (Ulrych), or by high-pass filtering to extract the earth response (Stoffa et al.). After inverse Fourier transformation, the return log frequency to frequency requires the minimum-phase assumption. The phase may then be unwrapped (Stoffa et al., 1974). These transformations do not make the fundamental problem of deconvolution go away, of course: the law of conservation of misery is obeyed. In practice, the wavelet and the earth response overlap in the complex cepstrum domain, so neither the wavelet nor the response can be extracted correctly from the data.

In conclusion, the estimate of "the wavelet" by statistical deconvolution methods relies on a model, equation (4), that has no theoretical support, and on a combination of assumptions about the whiteness of the response of the earth, about the statistical properties of the data, and about the phase of the wavelet, that are mathematically unavoidable but have no theoretical justification.

THE USE OF WELL LOGS TO ESTIMATE THE WAVELET

It is not uncommon to use well logs to estimate the wavelet in the seismic data. In this section I briefly summarize and then question the theory. The theoretical support for this practice is weak. Nevertheless, it is worth discussing because it is common and has not been given a proper airing in the literature.

The method is essentially as follows. The data are processed in the normal way to stack. It is then assumed that the stacked seismic data can be regarded as the convolution of the reflection coefficient series with some wavelet. The reflection coefficient series is calculated as the reflection response of an impulsive plane wave normally incident on a horizontally layered earth, whose layer parameters are derived from the densities and compressional-wave velocities of the well log. A wavelet is found that, when convolved with this series, yields the stacked trace for the common-midpoint (CMP) immediately over the well, as shown in Figure 2. White (1980) discusses how to assess the statistical (though not the physical) reliability of the wavelet determined in this way.

Compare Figure 3a with Figure 3b. Figure 3a shows a point source and a line of receivers over a plane, horizontally layered earth. This model is the basis for the processes of normal moveout (NMO) and stack on common-midpoint (CMP) data (Mayne, 1956, 1962). It has worked well for nearly thirty years and is clearly an excellent approximation. The CMP data are massaged by a series of prestack processes including muting, spherical divergence correction, a (usually) different prediction-error filter on every trace, dip moveout (DMO) and NMO correction, and then stacked. Some of these processes, especially NMO, are nonlinear. The linearity that is essential for the validity of the convolutional model is destroyed. The original multichannel data contain spherical divergence effects, $P$-$S$ conversions, interbed multiples, source and receiver ghosts, and every reflection is angular-dependent, including the free-surface reflections. The NMO stretch distorts the wavelet unevenly down every trace and increasingly with the offset of the trace (Dunkin and Levin, 1973). The resulting stacked trace, whatever it is, is not the convolution of the original source signature (or any other wavelet derived from it by a physical process) with the primary reflectivity series.

Figure 3b shows a plane wave normally incident at the surface of a horizontally layered half-space. Conventional processing to stack certainly does not convert data obtained with the configuration of Figure 3a to those of Figure 3b. It

![Fig. 2. The reflectivity series r(t), calculated from the sonic and density logs, is the input to a linear filter w(t) whose output is the stacked seismic trace x(t) from a CMP gather at or near the well position. The input and output are known, so the filter w(t), regarded as the seismic wavelet, can be obtained by deconvolution.](image)

![Fig. 3. (a) A monopole source and a series of point receivers embedded in the uppermost layer of an elastic stratified earth. The uppermost layer is bounded at the top by a free surface. (b) A plane wave in a half-space normally incident on an elastic stratified earth with the same parameters as in (a). The upper half-space in model (b) has the same parameters as the uppermost layer in model (a).](image)
would be possible to achieve this with the following processing sequence: (1) decompose the raw CMP gather into plane waves; that is, transform the data to the intercept time-slowness (\(r-p\)) domain (e.g., Brysk and McCowan, 1986); (2) remove all plane-wave components except the one at normal incidence (\(p = 0\)); (3) remove the effect of the free surface. The result is the plane-wave response at normal incidence, including all internal multiples. This processing sequence is totally different from conventional processing and has never been done, partly because step (3) cannot yet be done on real data. A scheme for performing this step was outlined by Berkowitz (1982). In order to do (3), it is necessary to know both the source signature, as defined in equation (1), and the recording system sensitivity (not normally available). Although it is in principle possible to calibrate the recording system such that the numbers recorded on tape can be converted to true pressure at the hydrophone or true particle velocity at the geophone; in practice this calibration is never done and the calibration factor is unknown.

The filter shown in Figure 2 is not "the seismic wavelet;" it is simply a filter that converts one wiggle trace to another. Since this "wavelet" is not the uncontaminated source time function, it contains components of the earth response. Indeed, it is supposed to contain all the components of the earth response that we want to remove, including free-surface effects and internal multiples. Even if there were perfect shot-to-shot repeatability through all the surveys shot in a given prospect, this method of wavelet estimation would give a different "wavelet" at every well—assuming the wells were all different. Indeed, in the case of perfect shot-to-shot repeatability, lateral variations in geology are the only cause of variations in the extracted wavelet. The physical meaning of such a wavelet is therefore obscure. Such wavelet estimation techniques are used in poststack inversion and lateral prediction schemes. In the lateral prediction part of the scheme the basic assumption is that the "wavelet," extracted as shown in Figure 2, does not vary laterally, while the geology does. This assumption cannot be defended if the extracted wavelet varies significantly from well to well. It is therefore impossible to distinguish which lateral variations in the seismic data are to be assigned to variations in the normal-incidence primary reflection coefficient series of an equivalent plane horizontally layered earth, and which should be assigned to variations in "the wavelet." Any poststack inversion and lateral prediction scheme based on this method of wavelet extraction is therefore self-contradictory. In the worst cases, the results of such a scheme are likely to be worthless.

It is possible to use the well-log data to calculate an offset-dependent synthetic seismogram from a point source, putting in all the \(P-S\) conversions, interbed multiples, source and receiver ghosts, etc. The brute-force way to do this is by finite-difference methods, but memory limitations even in current computers are a problem for a model with a large number of layers and a point-source configuration. A more elegant way for plane horizontal layers is to use the reflectivity method (e.g., Kennett, 1983). The resulting offset-dependent data could be compared with the CMP data and a multichannel estimate of the wavelet obtained. Anyone who has ever tried to do this knows how difficult it is to get the synthetic seismogram to look like the seismic data. This is not because of attenuation: if a sufficient number of layers is used, it is quite possible to match the bandwidth of the seismic data without introducing frequency-dependent attenuation. The problem is that there are usually reflections present in the data and not present in the synthetic, and vice versa. Questions always arise about the validity of the well logs and whether they have been edited correctly. At any rate, the proper calculation of an offset-dependent synthetic seismogram for a point source embedded in a horizontally layered earth is by no means a routine procedure even though the theory for it is well known. In conclusion, the proper use of well logs to determine the wavelet in seismic data is far from trivial and has not yet become a routinely viable method.

**DETERMINATION OF THE SIGNATURE FROM MEASUREMENTS**

It is possible to measure the source signature in production, although this is not always easy. This needs to be stressed, since many geophysicists are unaware of it. I briefly consider the land vibrator, marine seismic sources, and dynamite.

The Vibroseis source problem was solved in principle by Sallais and Weber (1982), with further supporting evidence by Sallais (1984). In the Vibroseis method, the far-field wavelet (at least a wavelength from the source), in units of particle velocity, has the same time function as the time derivative of the force of the plate on the ground (Miller and Pursey, 1954). Assuming the baseplate is rigid, this ground force can be determined as a weighted sum of the vertical accelerations of the reaction mass and baseplate (Sallais and Weber, 1982). It is now common practice to measure both these accelerations and use their weighted sum as the feedback signal in the control system of the vibrator, as indicated in Figure 4. The control system adjusts the flow of hydraulic fluid in the vibrator to ensure that the ground force signal is in phase with the predetermined reference sweep. More sophisticated systems also ensure that the amplitude of

![Fig. 4. "Ground force" servo control. Assumining the baseplate is rigid, the ground force is equal to a weighted sum of the reaction mass and baseplate accelerations (Sallais and Weber, 1982). The servo control system aims to make this weighted sum signal in-phase with the predetermined sweep.](image-url)
the ground-force signal is proportional to the amplitude of the sweep, with a time-invariant constant of proportionality. This is called ground-force amplitude control.

In the far field, and considering only compressional waves, the reflection seismogram may be expressed as

$$v(t) = \frac{df(t)}{dt} * g(t),$$

where \(v(t)\) is the velocity measured at the geophone, \(f(t)\) is the force of the baseplate on the ground, or ground force, \(g(t)\) is the earth impulse response and the asterisk denotes convolution. In the frequency domain this may be expressed as

$$V(\omega) = -i\omega F(\omega)G(\omega).$$

In normal Vibroseis processing, the received data are cross-correlated with the sweep; this is equivalent to multiplication by the complex conjugate of the Fourier transform of the sweep:

$$V(\omega) = \{-i\omega F(\omega)S_w(\omega)\}G(\omega),$$

where \(S_w(\omega)\) is the Fourier transform of the sweep and the bar denotes complex conjugate. The Fourier transform of the resulting wavelet is the term in braces. In the time domain, this wavelet is the convolution of the time derivative of the ground force with the time reverse of the sweep. This wavelet is clearly not equal to the autocorrelation of the sweep, as is usually assumed in the processing of Vibroseis data (e.g., Ristow and Jurczyk, 1975; Lines and Clayton, 1977; Bickel, 1982; Brötz et al., 1987). To determine the wavelet, it is necessary to know the ground force. The feedback signal is not the same as the sweep, because the control system is always reacting to events. The feedback control system is causal; it tries to compensate for nonlinearities in the system, but does not remove them entirely.

For example, harmonic distortion introduced by the servo valve is transmitted to the baseplate by the hydraulic drive system. The transmitted signal contains this harmonic distortion, while the pilot sweep does not. So, the only additional thing that needs to be done to enable the wavelet to be recovered is to record the baseplate and reaction mass accelerations that have already been measured. This is possible but is not normally done.

The analysis of Sallas and Weber ignores the bending of the baseplate. This was shown to be important, especially at higher frequencies—typically above about 80 Hz—by Baeten (1989), who also developed a theory to allow the bending forces to be taken into account using straightforward measurements on the vibrator.

As has been known for many years (e.g., Farron et al., 1970), for the marine monopole source only a single hydrophone in a known position relative to the source and the free surface is required to determine the whole wave field. The well-known solution of equation (1) for the pressure at a radius \(r\) from the source in an infinite homogeneous medium is

$$p(r, t) = \frac{1}{4\pi r} s \left( t - \frac{r}{c} \right),$$

where the subscripts have been dropped. Now consider the monopole source a distance \(D\) below the free surface, and a hydrophone a distance \(r\) from the source and a distance \(R\) from its virtual image, as shown in Figure 5. The hydrophone measures a direct wave and a wave reflected from the sea surface:

$$p_A(t) = \frac{1}{4\pi R} s \left( \frac{r}{c} - \frac{R}{c} \right) - \frac{1}{4\pi R} s \left( \frac{r}{c} - \frac{R}{c} \right).$$

This equation may be written as

$$p_A(t) = s(t) * \left\{ \frac{1}{4\pi R} s \left( \frac{r}{c} - \frac{R}{c} \right) - \frac{1}{4\pi R} s \left( \frac{r}{c} - \frac{R}{c} \right) \right\},$$

where \(s(t)\) now includes the interaction of the source with the free surface.

The term in the braces is the Green's function for this problem and may be compared with equation (3). In normal exploration problems the source depth \(D\) is a few meters and the sea water is usually much deeper. Scattering from the sea floor and deeper interfaces may be neglected, due to spherical divergence, if \(r\) is small, say 1 m. The Green's function is known if \(r, R,\) and \(c\) are all known. Then \(s(t)\) can be obtained by deconvolution, most easily done by division in the frequency domain:

$$S(w) = \frac{4\pi P_A(w)}{1 - \exp (iwR/c) - \frac{1}{R} \exp (iwR/c)}.$$

This division is stable since \(R > r\) (see Figure 5).

For arrays of interacting marine seismic sources—for example, air-gun arrays and water-gun arrays—the problem

![Fig. 5. A point source below the sea surface. The hydrophone measures a direct wave and a reflected wave.](image-url)
was solved by Ziolkowski et al. (1982). The solution is essentially a generalization of the method for the single monopole source, as illustrated in Figure 6. \( s_j(t) \) is the source function of the \( j \)th monopole, including all interaction effects, \( r_j \) is the distance from the source to the \( j \)th hydrophone, \( R_j \) is the distance from its virtual image to the hydrophone. The pressure measured at the hydrophone is then

\[
p_j(t) = \frac{1}{4\pi} \sum_{i=1}^{n} \frac{1}{R_{ij}} s_i \left( t - \frac{r_{ij}}{c} \right) - \frac{1}{4\pi} \sum_{i=1}^{n} \frac{1}{R_{ij}} s_i \left( t - \frac{R_{ij}}{c} \right).
\]

This equation can be seen as a superposition of \( n \) weighted and delayed unknown source functions \( s_i(t) \). Ziolkowski et al. (1982) showed on real data that these source functions can be found if \( n \) independent hydrophones are used. Clearly, once the source functions are known, the wave field can be calculated everywhere in the water. Parkes et al. (1984) tested the theory in a deep-water experiment. They computed the far-field signature from the source functions recovered from the near-field measurements for a given shot, and compared the result with a far-field measurement made for the same shot. The agreement between the measurements and the calculation was excellent, so the theory was not refuted. The method was tested again in the Delft air gun experiment (Ziolkowski, 1987), in which it was shown that the source signature could be recovered from near-field measurements even when the synchronization of the air guns was in error by as much as 100 ms. This method is the only one available for determining the source signature in production on the continental shelf. The invention is protected by a GECO patent (Ziolkowski et al., 1981; Ziolkowski et al., 1984). It is not used.

The problem of the dynamite signature has not been solved. It is impossible to measure it directly, without contamination from scatterers. Two solutions have been proposed, but have not yet been tested. First, Ziolkowski et al. (1980) proposed using two shots of different charge size at each shotpoint. The impulse response is the same for each shot, but the source signatures are different. The seismograms are then

\[
v_1(t) = s_1(t) * g(t),
\]

\[
v_2(t) = s_2(t) * g(t),
\]

in which \( s_1(t) \) and \( s_2(t) \) are the unknown signatures and \( g(t) \) is the unknown impulse response. The source signatures are related by a scaling law proposed by Ziolkowski and Lerwill (1979):

\[
s_2(t) = \alpha s_1(f/\alpha),
\]

in which \( \alpha \) is the cube root of the ratio of the energy of the second shot to the energy of the first shot. This law assumes that the fraction of energy radiated by the source depends only on the properties of the medium and the type of explosive, but not on the mass of the explosive. Equations (15), (16), and (17) are three equations containing three unknowns. Ziolkowski et al. (1980) showed how they may be solved in the presence of noise. The method is open to refutation by a third shot, as described by Ziolkowski (1982). Experimental data for this test were obtained near Tubbergen in the Netherlands in April 1990, but have not yet been analyzed.

A second solution for the dynamite problem was proposed by Fokkema and Ziolkowski (1987) and Baeten et al. (1990). The same experimental data set will be used to test this second method. The proposed method relies on the whiteness of the reflection response for reflection data that have horizontal slownesses greater than the reciprocal of the compressional-wave velocity of the deepest layer. To isolate these postcritical reflection data from the precritical data requires a transformation of the CMP data to the intercept-time slowness domain (Brysk and McCowan, 1986). In principle the presence of the free surface allows the amplitude and phase spectra of the source signature to be obtained from two postcritical traces.

Meaningful measurements of the source wave field can now be made, but they are not. The obstacles to these measurements are clearly not technical.

**A FIELD EXAMPLE**

I include one example taken from Baeten and Ziolkowski (1990) using data from the Tubbergen experiment. In Vibroseis processing, it is normally assumed that the signal put into the ground is equal to the sweep, regardless of the signal used for feedback and control (e.g., Ristow and Jurczyk, 1975; Lines and Clayton, 1977; Bickel, 1982; Brötz et al., 1987). It was argued above that the feedback system cannot cope with the harmonic distortion introduced by the drive.
system of the hydraulic vibrator, so the feedback signal and the pilot sweep are not the same.

Figure 7, which shows the first second of a 10 s, 10–130 Hz linear upsweep with a cosine taper of 200 ms applied at both ends of the sweep, illustrates the difference. The first trace is the reaction mass acceleration; the second trace is the baseplate acceleration. Both traces, especially the latter, clearly exhibit harmonic distortion. The third trace is the predetermined sweep; this is the signal from the recording truck, recorded on an auxiliary channel. The fourth signal is the pilot sweep from the vibrator electronics; this should be identical with the predetermined sweep from the recording truck and it is nearly identical. The feedback system adjusts the force in the hydraulic drive system to ensure that the feedback signal has the same amplitude and phase as this pilot sweep. The fifth signal is the feedback signal, which is the weighted sum of the baseplate and reaction mass accelerations (Sallas and Weber, 1982; Sallas, 1984). This is clearly not the same as the pilot sweep. The sixth signal is the best estimate of the true ground force, based on the baseplate and reaction mass accelerations and including the flexing of the plate. This is called the “flexural rigidity signal” (Baeten, 1989; Baeten and Ziolkowski, 1990). The errors in both amplitude and phase incurred by not recording the measured baseplate and reaction mass accelerations, and therefore being forced to assume that the ground force is equal to the sweep, are shown in Figure 8 for the whole sweep.

Figure 8a shows the normalized amplitude spectra of the pilot sweep and the weighted sum, or feedback signal. They are far from identical. Figure 8b shows the normalized amplitude spectra of the pilot sweep and the flexural rigidity signal. Comparing Figures 8a with 8b, it can be seen that the energy of the true ground-force signal decays very significantly at high frequencies, due to the work done in bending the baseplate. Finally, Figure 8c shows the phase difference between the weighted sum signal and the pilot sweep, and the phase difference between the flexural rigidity signal and the pilot sweep. Above about 10 Hz the weighted sum signal shows a small average phase error—although there are large positive and negative errors at high frequencies. The true ground-force signal is in error by about 30–50 degrees in the central part of the spectrum.

These errors are large. Further errors are introduced by ignoring the time-derivative factor in equations (7), (8), and (9).

CONCLUSIONS

The changing wavelet model of the seismogram, in which the wavelet changes as it propagates, does not help formulate the deconvolution problem. The “wavelet” that is reflected from the target—that is, the original source signature modified by all the transmission properties along the travel path including irreversible frequency-dependent losses—cannot be incorporated in any convolutional model of the seismogram. The seismogram certainly cannot be described as a convolution of this wavelet with the normal-incidence series of reflection coefficients, or “reflectivity.” Therefore, this reflectivity cannot be recovered from the seismogram by deconvolution.

The convolutional model of the seismogram is derived using a point source, a point receiver, a linear relation between stress and strain in the rocks, and Newton’s laws of mechanics. The earth need not be composed of plate horizontal layers. The seismogram is simply the convolution of the source time function with the impulse response of the earth, or Green’s function (plus uncorrelated noise). The impulse response is what would be seen at the receiver if the source time function were an impulse, and if there were no noise; it includes all the propagation effects including attenuation. The source time function should be determined from independent measurements at the source. Methods for Vibroseis and marine seismic sources have been published, but are apparently not being used. Proposals for determining the dynamic wavelet have been published, but have not yet been tested.

The exploration industry is still relying on a model of the seismogram that cannot be supported by wave propagation theory and statistical wavelet estimation methods based on one or more of the following three assumptions: (1) the seismic data are stationary; (2) the earth reflection response at near-normal incidence is white; and (3) the seismic wavelet is minimum-phase. The first two assumptions are false, and the third assumption is false for all sources except possibly dynamite on land. Since there is no way to determine scientifically which of these methods is the most accurate, each interpreter uses his judgment to decide this matter on a case-by-case basis.

Another wavelet estimation method, using stacked seismic data and well logs, is shown to yield a wavelet that has no physical meaning: the wavelet so estimated must contain components of the earth reflection response. If there is good shot-to-shot repeatability, but lateral variations in the geology, the extracted wavelet must vary from well to well. However, any poststack inversion and lateral prediction scheme that depends on this method of wavelet estimation requires that the wavelet does not vary laterally, and is therefore self-contradictory. In the worst cases, the results of such a scheme are likely to be worthless.

![Fig. 7. (From Baeten and Ziolkowski, 1990) First second of vibrator signals from Tubbergen vibrator experiment: trace 1, reaction mass acceleration; trace 2, baseplate acceleration; trace 3, predetermined sweep from recording truck; trace 4, pilot sweep from the vibrator electronics; trace 5, weighted sum signal; trace 6, flexural rigidity signal.](image-url)
Fig. 8. (From Becton and Ziolkowski, 1990) Amplitude and phase spectra of the weighted sum signal and the flexural rigidity signal. (a) Normalized amplitude spectra of the predetermined sweep (solid line) and the weighted sum signal (dotted line), (b) normalized amplitude spectra of the predetermined sweep (solid line) and the flexural rigidity signal (dotted line), and (c) phase differences of the weighted sum signal with the predetermined sweep (solid line) and of the flexural rigidity signal with the predetermined sweep (dotted line).
Measuring Seismic Signatures

It is more scientific to determine the source signature by measurements, deconvolve the data for this signature, and then invert the resulting band-limited Green's functions using an inversion method based on sound-wave propagation theory. The Vibrosis field example shows that errors incurred by not determining the signature from measurements can be large. There is every reason to expect this to be the rule rather than the exception.

ACKNOWLEDGMENTS

This paper has taken a long time to get into half-decent shape. The views in it are my own, but I have had a lot of help in trying to express them. Unfortunately, the paper has become longer with each revision. I thank Sven Treitel, Tony Gangi, Roger Bilham, Roger Turpening, Jacob Fokkema, and especially Guido Baeten for helpful conversations, and Pat Lindsay, John Sallas, Norman Neidell (who also suggested the present title for the paper), Larry Lines, an anonymous reviewer, and the Associate Editor for their very helpful comments on earlier drafts. I am very grateful to the European Commission for supporting this work, and the Tubbergen experiment of April 1990 under project no. TH.01/121/88. I thank Elsevier for giving permission to use Figures 7 and 8.

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