The signature of an air gun array: Computation from near-field measurements including interactions—Practical considerations

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Abstract

We have refined our system for calculating the signature of an interacting air gun array from near-field measurements of its pressure field. We use an iterative technique to calculate a notional array of noninteracting sources from the near-field hydrophone measurements. The notional signatures form the basis for calculating the array signature in any direction. The success of our iterative technique depends upon prudent positioning of the hydrophones, one close to each air gun.

In normal operation the forward motion of the hydrophones and upward motion of the air gun bubbles are important effects which must be included in the equations. A linear model for this motion is adequate and improves the method significantly. The vertically traveling "far-field" signature calculated by our extended method matches an equivalent "far-field" measurement very closely.

We present array signatures obtained in very bad weather conditions (force 8). In this extreme test the signatures are very stable from shot to shot. Therefore it is not necessary to calculate the array signature every shot; however, continuous recording of near-fields should still be carried out as a check on signature stability.

Introduction

It is becoming well known that the superposition of signatures of air guns fired in isolation provides a very inadequate description of the combined signature of the same air guns fired simultaneously in an array. This is because the dynamics of each air gun bubble are changed by the time-varying pressure field created by the other bubbles of the array. In a previous paper (Ziolkowski et al, 1982) we presented a theory of the interactions between the bubbles which shows that an interacting array is equivalent to a "notional" array of noninteracting oscillating bubbles. If there are n guns in the array, then n independent measurements of the pressure field of the full array may be used to determine the n notional source signatures. The signature of the array at any point in the water may then be calculated by superposition of these notional sources, scaled and delayed relative to each other according to distance and direction. The method is confirmed by experiment.

In this paper we present further details of the method and discuss practical considerations relevant to the operation of the system in production. In particular, we describe a numerical method for deriving the notional sources from the pressure field measurements. We discuss the complications introduced by the relative motion of the hydrophones and bubbles, and show how the equations may be solved to take this motion into account. Finally, we present measurements of array stability and discuss its importance.

Calculation of the Notional Sources

The signatures presented throughout this paper (with the exception of Figure 5) have been derived from near-field hydrophone measurements made on an air gun subarray of 910 inches³ (Parkes et al, 1982). The subarray geometry and air gun sizes are shown in Figure 1. Each gun of the subarray is equipped with a permanent near-field hydrophone at a distance of 1 m. We discuss the siting of the hydrophones in more detail in a later section.

In this section, we restrict ourselves to a frame of reference in which the hydrophones and air gun bubbles are stationary. If there are n bubbles and we place a hydrophone 1 m away from each bubble, then the voltage output \( h_j(t) \) of each of the hydrophones is as follows:

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Fig. 1. Schematic of the subarray geometry showing the relative positions of air guns and hydrophones.

\[
\frac{h_j(t)}{s_j} = \sum_{i=1}^{n} \frac{1}{r_{ij}} \cdot p_i(t - \frac{r_{ij} - 1}{c}) + \sum_{i=1}^{n} \frac{R}{(r_{j})_{ij}} \cdot p_i(t - \frac{(r_{j})_{ij} - 1}{c}),
\]

where \( p_i(t) \) is the \( i \)th notional source pressure signature at a range of 1 m and \( r_{ij} \) is the distance from the \( i \)th bubble to the \( j \)th hydrophone; \( c \) is the speed of sound in water. The virtual images or ghosts of the notional sources are accounted for by the second summation, in which the distance \( (r_{j})_{ij} \) is from the virtual image of the \( i \)th bubble to the \( j \)th hydrophone. \( R \) is the reflection coefficient at the sea surface (normally \(-1\)) and \( s_j \) is the sensitivity of the \( j \)th hydrophone (volts/bar).

If we make \( n \) measurements, \( h_j(t) \), then we may solve for the \( n \) notional sources \( p_i(t) \). For independence and easy solution of these equations, the \( n \) hydrophone measurements must be at independent positions in the wave field. This condition is well met by the hydrophone geometry of Figure 1. We may rewrite equation (1) as

\[
p_i(t) = \frac{h_j(t)}{s_j} - \sum_{i=1}^{n} \frac{1}{r_{ij}} \cdot p_i(t - \frac{r_{ij} - 1}{c}) - \sum_{i=1}^{n} \frac{R}{(r_{j})_{ij}} \cdot p_i(t - \frac{(r_{j})_{ij} - 1}{c}),
\]

in which we have used the fact that \( r_{ij} = 1 \) m for \( i = j \).

If \( r_{ij} (i = j) \) is small compared with \( r_{ij} (i \neq j) \), then the hydrophone measurements \( h_j(t) / s_j, i = j \), are a good first approxi-

Fig. 2. (a) Shows a hydrophone measurement at the 75 inches\(^3\) gun position, with the full subarray firing. (b) is the calculated notional source; notice the first approximation similarity between (a) and (b). (c) is the simulated hydrophone measurement calculated from the 7 derived notional sources.
signature of an air gun array

Fig. 3. Data for a shot in which only gun 1 was fired. (a) shows the 7 hydrophone measurements. Hydrophone 1 is the top signature; hydrophone 7 is the bottom signature. (b) shows the derived notional sources assuming no relative motion. (c) shows notional sources derived including relative motion terms.

We wish to refer the notional signatures we derive from equations (2) to some convenient fixed distance. The usual standard is 1 m; the signatures are then expressed in units of bars at 1 m, or bar-meters. Having parameterized distance, we may then calculate the signature at any other distance. The pressure field around a source drops off as 1/r. As discussed above, the bubble-to-hydrophone distance is a function of time r(t) (a different function for each hydrophone/bubble pair). Therefore our hydrophone measurements will contain distance and hence amplitude dependences of the form r(t)/r(0), where r(0) is the distance at time zero. If r is large (i.e., the far field), then in the time interval of interest (∼ 1/2 sec), r(t)/r(0) will not differ significantly from unity. However, in the near-field (r ∼ 1 m) the dependence can be large and must be included in our equations. Therefore relative motion introduces significant amplitude variations in the near-field. Doppler shifts may be neglected since the speeds under consideration are small compared with the speed of sound in water, and the duration of the signal is short. The net change in traveltime between any bubble and any hydrophone may therefore be neglected.

If we assume that the drag on the air gun bubbles causes them to stop with respect to the water as soon as they are formed, and that thereafter their only motion is upward at constant velocity, then the terms r_ij of equations (1) and (2) can be obtained from

\[ r_{ij}(t) = r_{ij}(0) + v_{ij}t, \]

where the v and r terms are vectors, and v_ij is the relative velocity between hydrophone j and bubble i.

A good test of a linear velocity model [equations (2) and (3)]

RELATIVE MOTION EFFECTS

In reality, the bubbles and hydrophones are not stationary with respect to each other. The buoyant bubbles rise through the water, and the hydrophones, which are attached to the subarray, are towed forward at the normal production speed of about 5 knots. (Note this is the speed over the sea floor; depending upon the currents, the true water speed may be different.)
is provided by an experiment in which all guns but one in a
seven-gun subarray are turned off. The single gun is then fired
and the signal at the seven near-field hydrophones recorded.
The solution of equations (2) and (3) should produce a full
notional source at the position of the gun, and zeros for the
other six notional sources. This experiment was carried out
with the boat (and hence hydrophones) moving at 5 knots.
Figure 3a shows the seven hydrophone recordings. Gun 1 was
the only active gun. Figure 3b shows notional sources calcu-
lated neglecting relative motion effects. The residuals at posi-
tions 2 to 6 are well below the hydrophone measurements;
however, there are noticeable deviations, particularly at posi-
tion 2. Figure 3c shows the solution including a hydrophone
forward velocity of 1.8 m/sect and a bubble rise velocity of 1.0
m/sect (these were best-fit values). At position 2, in particular,
the residual is now much smaller. However, more importantly,
the signature at position 1 is quite different from that in Figures
3a and 3b. The negative excursions decay in size, exactly as we
would expect for a damped oscillator such as an air gun bubble.
The data for position 2 are shown on an expanded scale in
Figure 4. On top is the hydrophone measurement, at center is
the calculation neglecting relative motion, and below is the
calculation including motion. Clearly, the linear velocity model
of equation (3) is very effective over most of the signature.
However there are residual errors, particularly at the start of
the signature. Among the causes of these residuals are the small
errors in our knowledge of the hydrophone sensitivities, and the
absence of bubble deceleration terms in our model.
The iterative method of solving the system of equations (for
full array firings) still works well when velocity terms are in-
cluded, but the computation time increases. Furthermore, if the
chosen velocity model is very inaccurate, the equations become
insoluble. In Figure 5 we illustrate the precision now achieved
by the method. The signatures shown are for a larger 1390
inches³ subarray, with gun depths of 5 m. The top signature is a
far-field hydrophone recording made approximately 100 m ver-
tically below the subarray. Below is shown the equivalent sig-
nature calculated by our method including relative motion
effects. Clearly, the detailed agreement between the two signa-
tures is extremely good.

**SITING THE HYDROPHONES**

We have already pointed out above that, if the hydrophones
are placed close to the guns, then the hydrophone measure-
ments provide a good first approximation to the notional
sources; and equations (2) are independent and may be solved
iteratively. However, large relative motion effects in the
measurements are a direct consequence of this proximity. In the
subarray of Figure 1 there is a hydrophone approximately
0.7 m above and 0.7 m behind each gun (the largest gun is at the
front of the array). When the hydrophone moves to a position
directly above the bubble, the initial hydrophone/bubble sepa-
ration of 1 m decreases to 0.7 m. As the hydrophone moves
farther forward, the separation increases again through 1 m and
beyond. If the hydrophones were in front of the guns, the initial
separation would increase continuously from the start so the
second-order terms in equations (2) would increase more
quickly as hydrophone \( i \) moves towards gun \( (i - 1) \), and so
forth. On the other hand, the hydrophones cannot be put very
much closer to the guns than 1 m, for they will become en-
veloped by the oscillating bubbles. In conclusion, therefore, the

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**Fig. 4.** Shows position 2 of Figure 3 on an expanded scale.

**Fig. 5.** Measured far-field signature compared with a signature
calculated by the method including relative motion effects,
documented in this paper. (The boat speed was about 2½ knots.)
The dynamics, towing method, and gun suspension system of the subarrays* used to develop our method ensure a high degree of array stability (Parkes et al, 1982). To illustrate this stability, hydrophone measurements obtained while shooting a test line in the North Sea have been analyzed. Gun depths of 5 m were used, and it should be stressed that the weather conditions at the time were extremely bad (force 8 or 9). Figure 6 shows a sequence of signatures from about 50 shots. Each signature has been obtained by directly summing the 7 near-field hydrophone measurements of a single subarray. They are not therefore the true array signature, in that the ghosts and second-order terms of equations (2) have not been removed. However they are closely related to the true signatures and serve to illustrate signature and hence array stability, even in the extreme conditions of the test.

To illustrate this stability further, Figure 7 shows a similar sequence of signatures. A spiking filter was derived from the average of the shown signatures. This single filter was then applied to the signatures, with the results shown in the figure.

*This array system was developed by Seismic Profilers A/S of Norway.

**FIG. 6. A sequence of subarray signatures recorded in extreme weather conditions (force 8 or 9).**
Fig. 7. A further sequence of signatures which have been spiked using a single spiking filter calculated from the average of the signatures shown.

Fig. 8. Crosscorrelation coefficients between signatures for individual shots and an average signature over the line. A high degree of signature stability over the 10 km is evident, despite the extreme weather conditions.

Fig. 9. Expected drop in correlation when individual guns fail to fire. Also shown is the expected level of variability when all guns are firing.
The effectiveness of the single spiking filter, and hence similarity of the signatures, is striking.

A weighted average signature was computed for the full test line of 10 km. The signature for each shot was then correlated with this average. Identical signatures would produce a cross-correlation coefficient of 1. In Figure 8 the crosscorrelation coefficients (for zero lag) are plotted for the full line. The coefficients are all close to unity. The hiccup at 5.2 km was caused by a faulty data recording, in which the first 100 msec of the apparently normal signatures was absent from the recording.

To add perspective to Figure 8, Figure 9 shows the expected drop in correlation coefficient when a gun fails to fire. The data have been derived from trials in which the shown guns were purposely dropped out. The hatched region shows the expected limit of variability when all the guns continue to fire. It is clear that a drop-out of even the smallest gun (gun 7) can be positively detected in a single shot. If the gun stays out, then the crosscorrelation graph will remain at the lower level.

It is clear from the data discussed above that the radiation of the array system we have described is extremely stable, so we need compute the wave field only occasionally. However, we still require continuous near-field recordings as a check on stability and to provide the necessary data for recomputing the wave field if and when significant changes do occur.

**CONCLUSIONS**

We have presented further details of a method for calculating the wave field of an array of marine seismic sources, including interaction effects. The theory was presented in a previous paper (Ziolkowski et al, 1982); here we have described an iterative method for solving the equations. Ease of solution relies upon prudent positioning of the near-field hydrophones close to the air guns. However, because the hydrophones and air gun bubbles are in motion with respect to each other, this proximity introduces significant amplitude variation effects in the measurements. A model for these effects must be included in the equations. We have shown a simple linear velocity model to be effective. It would be an easy matter to include acceleration terms, if even higher accuracy were required.

The technique provides an accurate, deterministic method for calculating the wave field for any interacting array of marine sources, on a shot-by-shot basis. The method is not specific to air guns and could be applied to any source system (for example, water guns). Any shot-to-shot variability, such as timing errors, gun misfires, or drop outs, is automatically included by the technique. However, we have shown that the accompanying array system used to develop the method is extremely stable, even in weather conditions which would normally prohibit production acquisition. For such a system the calculation of the array signature and its directional dependence need only be carried out occasionally. As a quality control check, continuous recording of the near-field signatures is important and, fortunately, relatively inexpensive. If variability is detected beyond some unacceptable limit, according to some criterion, then a new wave field must be computed. The system described in this paper is now in operational use, opening the door to advanced designature analyses perhaps including directional dependence, and certainly extending the effective bandwidth of the marine seismic method considerably.

The technique described here and in Ziolkowski et al (1982) is the subject of a patent application.

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**REFERENCES**
