Geophysical Prospecting, 1980, 28, 902–918.

SOURCE ARRAY SCALING FOR WAVELET DECONVOLUTION*

A.M. ZIOLKOWSKI**

ABSTRACT

ZIOLKOWSKI, A.M. 1980, Source Array Scaling for Wavelet Deconvolution, Geophysical Prospecting 28, 902–918.

A seismic source array is normally composed of elements spaced at distances less than a wavelength while the overall dimensions of the array are normally of the order of a wavelength. Consequently, unpredictable interaction effects occur between element and the shape of the far field wavelet, which is azimuth-dependent, can only be determined by measurements in the far field. Since such measurements are very often impossible to make, the shape of the wavelet—particularly its phase spectrum—is unknown.

A theoretical design method for overcoming this problem is presented using two scaled arrays. The far field source wavelets from the source arrays have the same azimuth dependence at scaled frequencies, and the far field wavelets along any azimuth are related by a simple scaling law. Two independent seismograms are generated by the two scaled arrays for each pair of source-receiver locations, the source wavelets being related by the scaling law.

The technique thus permits the far field waveform of an array to be determined in situations where it is impossible to measure it. Furthermore it permits the array design criteria to be changed: instead of sacrificing useful signal energy for the sake of the phase spectrum, the array may be designed to produce a wavelet with desired amplitude characteristics, without much regard for phase.

1. INTRODUCTION

Distributed arrays of point sources are used for increasing the power of the source, for shaping the far field wavelet, and for improving the directivity of the radiation. If the distance between individual point sources within such an array is less than about a wavelength, the interaction effects between these individual sources are significant. For most point sources, these interaction effects are not well understood, and the far field wavelet of such an array of point sources cannot be calculated from a knowledge of the individual far

* Received September 1979, revised version May 1980.

** Consultant to The British National Oil Corporation, 38 Hans Crescent, London SW1 0ND, UK.

field source wavelets. It must be measured in the far field. Since this measurement is very often awkward or impossible to make, the far field wavelet of such an array is very often unknown.

In reflection seismology initial data processing aims to recover the impulse response of the earth from the recorded seismogram. It is essential to know the shape of the wavelet to perform this recovery. If the wavelet cannot be measured, some of its properties must be assumed. The most difficult property to determine is the phase spectrum. In the absence of any information it is *convenient* to assume that the wavelet is minimum phase. But if this assumption does not hold, and if no far field measurement of the wavelet is available, the initial deconvolution is likely to be unsuccessful—unless some other assumptions are found to give more pleasing results.

Errors in the knowledge of the phase spectrum of the far field wavelet can have a profoundly detrimental effect on the quality of the final section, and it has therefore become extremely important to minimize them. Very often it is an acknowledgment of his ignorance of the phase spectrum which provides the designer of an array with his overriding constraint. In the marine environment, for example, it has become essential to remove "bubble pulses" to obtain adequate resolution.

It is the aim of this paper to offer a design method for two-dimensional arrays which permits recovery of the far field wavelet in any azimuth, whatever the phase spectrum of the wavelet, whatever the degree of interaction between elements of the array, and even in situations where it would be impossible to measure it. This would free the designer from one of his most severe constraints and thus allow him to design the array to satisfy his other criteria. For example, it would then be possible simply to design the source array to generate maximum power over a given frequency bandwidth—irrespective of the duration of the signal and its phase spectrum.

Before considering the design method, it is worthwhile considering the example of air gun arrays in which all the general points outlined above are illustrated.

2. The Example of Air Gun Arrays

Although the air gun has many practical advantages (Giles 1968) its signature suffers from a special combination of disadvantages. It lacks power, it has a multi-peaked spectrum, and, in the time domain, it is inconveniently long and oscillatory; moreover, it is not minimum-phase (Berkhout 1970, Ziolkowski 1971). These disadvantages can be overcome quite easily if tackled individually. The problem is to design an array which will overcome them all simultaneously.

If, for example, the only problem with the air gun were the phase spectrum of its signature, this could easily be solved by measuring the waveform in the "near field" and using this measurement to derive a deconvolution filter—as in Maxipulse processing. This would work because an air gun bubble, like Maxipulse, Vaporchoc or sparker, is a point source at wavelengths of seismic interest and therefore its wavefield is spherically symmetric. Furthermore, the amplitude of the pressure wave (but not the particle velocity wave) decreases linearly with increasing distance from the source, even at distances small compared with a wavelength (Safar 1977). Thus the region in which the pressure amplitude is a linear function of distance from the source extends from infinity to a very small distance from the source (see Appendix B) (there is of course a small nonlinear region between the bubble itself and this linear region (Ziolkowski 1970, 1977a, b).

However, this "Maxipulse approach" is seldom, if ever, adopted for single air guns, because the air gun waveform suffers from two other deficiencies: lack of power and a multi-peaked spectrum. These are remedied simply by using more guns and by using guns of appropriately different sizes. Most designers of air gun arrays find it essential to put certain guns sufficiently far apart so that they do not appear to interact (Kramer, Peterson and Walter 1968, Giles 1968, Giles and Johnson 1973, Nooteboom 1978). The required distance between guns is of the order of several meters. Nooteboom (1978) gives an experimental formula for this distance

$$D = \frac{5.1(P_1 V_1)^{1/3}}{P_s^{1/3}}$$

where D is the distance between guns, P_1 is the air pressure, P_S is the static water pressure and V_1 is the volume of the bigger gun.

When more than a few guns are put together in this way the dimensions of the source are no longer small compared with a wavelength and the array ceases to behave as a point source (see Appendix A). The "Maxipulse approach" to the solution of the problem must then fail.

A modern array of twenty or more guns (Nooteboom 1978) is certainly not a point source, and the acoustic radiation it generates is not spherically symmetric. Furthermore, the distance to the "far field" region—in which the *shape* of the waveform depends only on azimuth, and its amplitude is inversely proportional to the range—is normally of the order of 100 m from the center of the array. In fact, Nooteboom (1978) found that, at a range of 40 m, the measured signal was "not suited to derive deconvolution filters to correct for signal shape". In other words, 40 m was not far enough away to be in the far field. This is confirmed by Hubbard (1978) who met with more success with arrays of the same sort of size by using a detector hydrophone at a range of 90 m.

For arrays of such size, the vertically-travelling far field signal can be measured *only* if the hydrophone is in the far field vertically below the array *and* if the water is deep enough to cause the reflected signal from the sea-floor to arrive too late to interfere with the direct measurement. In shallow water surveys, it is obviously impossible to meet these requirements, and so the far field signature cannot be measured. Yet there is normally sufficient variation of the signal along a line for continuous measurement to be extremely desirable (Nooteboom 1978, Hubbard 1978).

The distances between the guns are normally large enough to allow the guns to behave reasonably independently, but, unfortunately, they are small enough that there is sufficient interaction between them for the far field signature to differ significantly from what it would be if the individual air gun signatures were combined by simple superposition. For the principle of superposition to hold the bubbles would have to be truly independent, in which case the gun spacing would have to be of the order of a wavelength. Since there is no adequate theory to describe the distorting effect of the interactions on the shape of the far field waveform, this waveform is not known. This point is considered further in section 5 of this paper.

So, while the introduction of arrays of air guns has solved the problems of the low power and poor amplitude spectrum of the signal from a single gun, it has reintroduced the problem of determining the phase spectrum of the far field wavelet. And this is *the* problem to which designers of air gun arrays have addressed themselves.

The way this is tackled conventionally is simply to suppress the "bubble pulses" in the tail of the wavelet. The idea is that the far field wavelet thus becomes so short that either it needs no further compression, or else it can be adequately compressed using the assumption that it is minimum-phase. Larner, Hale, Misener and Hewlett have shown at the tenth offshore conference 1979 that if the primary-to-bubble ratio (a measure of shortness) is at least 10, the minimum-phase assumption is an adequate approximation for the derivation of an inverse filter. In the presence of noise the minimum-phase assumption certainly works (according to Larner et al.) a lot better on the signal from a "tuned" air gun array than on the signal from a single gun.

"Tuning" simply becomes the process of reducing the tail-end energy of the vertically-travelling far field wavelet to the point where the minimum-phase assumption will do. Since the individual air guns in the array are placed such that their bubbles oscillate almost independently of each other, there is no attenuation of the bubble oscillations *at the source*. The attenuation of the "bubble pulses" in the vertical direction is achieved by destructive interference (Safar 1976). It follows, by conservation of energy, that this acoustic energy is being radiated in directions other than the vertical and is therefore an uncontrolled source of noise. A more elegant approach to the problem of tuning is given by Safar (1976), who exploits the bubble interaction that occurs when guns of the same size are close together in order to increase the damping of the bubble oscillations. The energy which would otherwise appear as acoustic radiation in the form of "bubble pulses" is then partially converted to heat at the source. The additional damping gives the far field signature a tendency to be minimum-phase (Safar 1976).

Any such tuning process is an attempt to suppress the "bubble pulses" in the tail of the wavelet. The "bubble pulses" themselves contain good coherent signal energy, but, because the phase spectrum of the far field wavelet is unknown, they cannot be used. In fact they are positively harmful and must be suppressed. Ideally the aim would be to produce a wavelet with one primary pulse and no bubble pulses. If a mechanism could somehow be found to enhance the primary pulse *at the expense* of the bubble pulses this would be perfect. In the absence of such a mechanism the aim is simply to prevent the tail-end acoustic energy from travelling vertically downwards.

It is ironic that, having increased the power of the array to the point where it is now adequate, ignorance of the phase spectrum forces the designer to prevent the tail-end energy from appearing in the downward-travelling wave. The design aim is to put all the energy that is required into the initial pulse, and to throw away everything that comes later. Considerable effort is being devoted to designing arrays which will throw away this unwanted energy. And since this constitutes the bulk of the energy available, the quantity of energy which could be used if only the phase were known is several times more than is required. Alternatively, if the phase were known, the array could be designed to do the same job with fewer guns and with less air.

It should be noted in passing that the peak amplitude and primary-to-bubble ratio of a tuned air gun array are parameters which are usually calculated from a broad band measurement of the far field wavelet. However, the frequency band of the reflection data is usually quite different. In particular, it has proportionately less high frequency energy than the broadband far field measurement, due to the high-cut filtering effect of the earth. Any such reduction in frequency bandwidth will increase the length of the wavelet in time, and, since the higher frequency energy is concentrated to the front of the wavelet, its attenuation by earth filtering has the effect of decreasing the primary-to-bubble ratio. It is thus meaningless to discuss the primary-to-bubble ratio without specifying the bandwidth. Over the received bandwidth the primary-to-bubble ratio of a tuned air gun array may, in some cases, be not very much greater than 1. Deconvolution is then *still* required and the minimum-phase assumption will be poor. This is confirmed by Hubbard (1978) who observed that even when the far field wavelet is short it is not minimum phase.

In summary, when the water is shallow it is not possible to measure this wavelet, and since its shape cannot be calculated, there are only three courses open:

1. To use a deep water measurement and then to hope that the wavelet generated in shallow water does not vary too much from this measurement.
2. To assume that the shallow water wavelet is minimum-phase, and to hope that the standard deconvolution method will work.
3. To forget all about deconvolution.

3. Use of Scaled Arrays

The approach I propose is to use two scaled arrays such that the far field wavelet $s_1(t)$ of the first is related to the far field wavelet $s_2(t)$ of the second by the scaling law

$$s_2(t) = \alpha s_1(t/\alpha), \tag{1}$$

where α is the scale factor and α^3 is the ratio of the energy in the second wavelet to the energy in the first. If the two arrays are used alternately and, for example, towed one behind the other, the shot locations can be made the same and the earth impulse response $g(t)$ will be the same for both shots. We may then deduce $g(t)$ from the two seismograms and the scaling law using the method of Ziolkowski, Lerwill, March and Peardon (1980). This is done by solving the following three simultaneous equations:

$$x_1(t) = s_1(t) * g(t) + n_1(t), \tag{2}$$

$$x_2(t) = s_2(t) * g(t) + n_2(t), \tag{3}$$

$$s_2(t) = \alpha s_1(t/\alpha). \tag{1}$$

In these equations $x_1(t)$ and $x_2(t)$ are the seismic signals generated by the first and second shots, respectively; $n_1(t)$ and $n_2(t)$ are additive uncorrelated noise signals; and the asterisk $*$ denotes convolution. These equations are discussed by Ziolkowski *et al.* (1980) and can be solved without making any assumptions about the phase spectrum of the source wavelet $s(t)$ or its relationship to the spectrum of the earth impulse response $g(t)$.

The scaling law (1) holds for "point" sources which generate spherically symmetric radiation in the useful bandwidth and have dimensions small compared with the shortest wavelength of the radiation. A derivation for point sources is given by Ziolkowski et al. (1980). The dimensions of arrays of such point sources are normally not small compared with the shortest wavelength of the radiation they generate, as we have seen, and this radiation is not normally spherically symmetric but has some azimuthal dependence. If the scaling law is to be applied to the radiation from arrays it is obviously crucial to take account of this azimuthal dependence.

4. Design of the Scaled Arrays

Consider the Fourier transform of equation (1):

$$S_2(f) = \alpha^2 S_1(\alpha f), \tag{4}$$

where f is frequency. Equation (4) states that the spectrum $S_2(f)$ is a shifted, amplified version of $S_1(f)$, where α is the shift factor and α^2 is the amplification factor. If α is greater than 1, $S_2(f)$ is shifted towards the lower frequencies relative to $S_1(f)$. Fig. 1 illustrates this frequency scaling.

It follows that—if the scaling law is to apply to the radiation seen at a given point in the far field of two scaled arrays—we must insist that the azimuthal dependence of amplitude and phase are preserved at the scaled frequencies.

Consider a two-dimensional array of limited extent lying in the plane $z = 0$, such that the origin 0 of the co-ordinate system lies within the array (see fig. 2). Let the array be contained within a rectangle defined by the lines

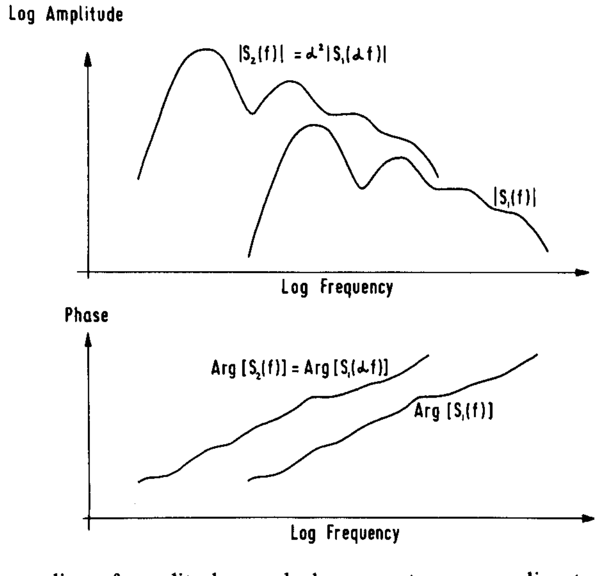

Fig. 1. Frequency scaling of amplitude—and phase spectrum according to (4) for $\alpha > 1$.

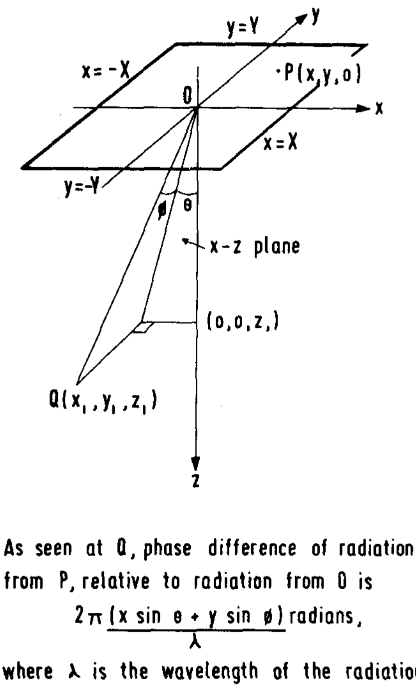

Fig. 2. Geometric relation of array at $-X \leq x \leq X$, $-Y \leq y \leq Y$, $z = 0$ and far-field observation point $Q(x, y, z)$. As seen at Q, phase difference of radiation from P, relative to radiation from O is $2\pi(x \sin \theta + y \sin \phi)$/radians, where λ is the wavelength of the radiation.

$x = \pm X$, $y = \pm Y$, and let the frequency of radiation be f with the amplitude and phase of the emission depending upon position $(x, y, 0)$ according to the complex-valued aperture function $h(x, y)$. This may be chosen arbitrarily within the rectangle we have defined, but is necessarily zero outside it.

Consider now a point Q in the far field of this array at a position (x_1, y_1, z_1) along an azimuth defined by the angles θ and ϕ as shown in fig. 2. The contribution at Q from the point P will be advanced or retarded in phase by $2\pi(f/c) \cdot (x \sin \theta + y \sin \phi)$ radians, where c is the velocity of sound. It is important to note that this expression is unaltered if the point P is shifted to the position $(\alpha x, \alpha y, 0)$, provided the frequency of the radiation is shifted to f/α. This is the key to the design of the scaled arrays.

We apply Huygens principle to obtain the far field radiation pattern by integrating the individual contributions across the face of the aperture. That is, the radiation amplitude and phase are given by the complex-valued function H:

$$H(\theta, \phi, f) = \int_{-X}^{X} \int_{-Y}^{Y} h(x, y) \exp \left\{ 2\pi i \frac{f}{c} (x \sin \theta + y \sin \phi) \right\} \mathrm{d}y \, \mathrm{d}x \tag{5}$$

(this argument is similar to Robinson's (1967, p. 44) but it has been extended to two dimensions).

Consider now the scaling of this array by a scale factor α, while simultaneously scaling the frequency by a factor $1/\alpha$. We begin by making substitutions into equation (5) as follows: $\nu = \alpha f$; $l = x/\alpha$; $m = y/\alpha$. This gives

$$H\left(\theta, \phi, \frac{\nu}{\alpha}\right) = \alpha^2 \int_{-(X/\alpha)}^{X/\alpha} \int_{-(Y/\alpha)}^{Y/\alpha} h(\alpha x, \alpha y) \times \exp\left\{2\pi i \frac{\nu}{c}(l \sin\theta + m \sin\phi)\right\} \mathrm{d}m\, \mathrm{d}l. \tag{6}$$

In this equation the variables ν, l and m are only dummies and could just as easily be replaced by f, x and y, respectively. Hence

$$H\left(\theta, \phi, \frac{f}{\alpha}\right) = \alpha^2 \int_{-(X/\alpha)}^{X/\alpha} \int_{-(Y/\alpha)}^{Y/\alpha} h(\alpha x, \alpha y) \times \exp\left\{2\pi i \frac{f}{c}(x \sin\theta + y \sin\phi)\right\} \mathrm{d}y\, \mathrm{d}x. \tag{7}$$

By comparing equations (5) and (7) we deduce the following scaling law for arrays: *If the dimensions of a two-dimensional array are scaled by a factor α and if the frequency of the radiation is scaled by a factor $1/\alpha$ then the radiation pattern of the scaled array has the same azimuthal dependence and α^2 times the amplitude at the scaled frequency.*

We could apply this to the example of a two-dimensional array of air guns. If the array operated at a depth d, and pressure p and consisted of a number of guns of volume V_1, V_2, V_3, etc., separated by distances r_1, r_2, r_3, etc., then a *scaled* array would operate at the same depth d and the *same* pressure p, but would consist of corresponding guns of volume $\alpha^3 V_1$, $\alpha^3 V_2$, $\alpha^3 V_3$, etc., separated by the corresponding distances αr_1, αr_2, αr_3, etc., as shown in fig. 3.

In summary, to apply the ideas of Ziolkowski et al. (1980) to *arrays* of sources, the arrays must be scaled. The idea presented here is valid for any two-dimensional array of non-infinite extent. The scaling of the arrays is to be performed in the following way: 1. the elements of the array are scaled by a factor α^3 by volume, by mass, or by energy (corresponding to a frequency scaling factor of $1/\alpha$); 2. the geometry of the array must be correspondingly scaled by a factor α; 3. no other parameter is to be changed.

5. Note on Element Interaction

In the absence of the other elements in the array, each element must radiate independently. In the case of a single air gun, for example, this radiation is

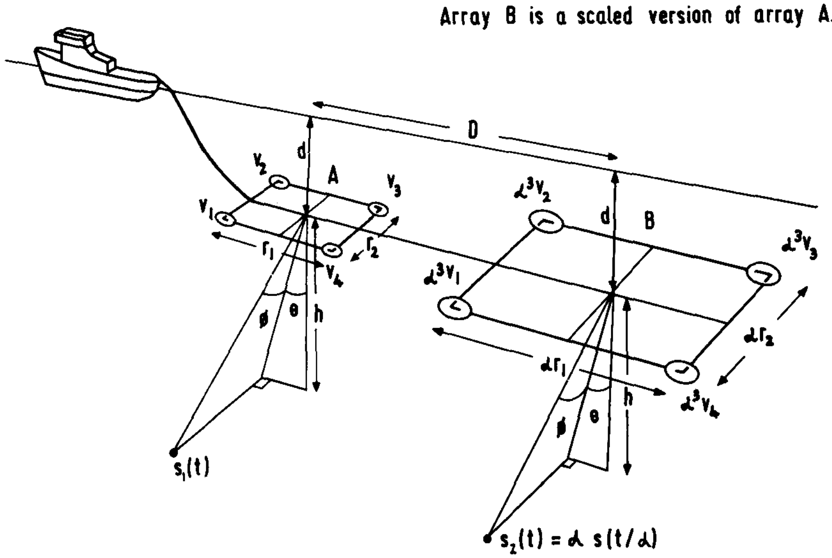

Fig. 3. Operation of scaled arrays *A* and *B* which are fired alternately. The spacing *D* is equal to the pop-interval to ensure that the arrays are fired once in the same place.

controlled by the manner in which the air escapes from the gun, and by the subsequent dynamics of the oscillating air bubble in a uniform medium whose ambient pressure varies only with depth. The extremely repeatable signature of an air gun at constant depth confirms this.

In the presence of the other elements, each element will behave differently, because the radiation from the other elements changes the ambient pressure in the medium in both time and space. In the case of air guns, this variation in ambient pressure will alter the dynamics of the bubble oscillation and hence the radiation it produces. This, in turn, will modulate the behaviour of the other elements, and so on. It follows that none of the elements in an array will behave independently.

The degree of interaction will vary with the size of the elements and their distance apart. The interaction will increase with an increase in the size of the elements and will increase with a decrease in the element separation. At any frequency, the interaction can be described as the change in amplitude and phase introduced into the radiation of a source element by the presence of the other elements. For our scaling law to work we naturally require that such interaction effects scale also.

There are two independent scaling operations. One is to scale the relative separations of the source elements by a factor α. The other is to scale the output energy of each element by a factor α^3 which leads to two simultaneous effects, as we have seen (Ziolkowski et al. 1980). First the radius of the equivalent cavity is scaled by a factor α; consequently the ratio of cavity radius to source separation will be the same in the scaled array, thus preserving the scaled geometry in two dimensions. Secondly, the radiated frequencies are scaled by a factor $1/\alpha$ and amplified by a factor α^2, without any change of phase; the corresponding wavelengths are therefore scaled by a factor α, and consequently the ratio of source separation to wavelength is the same at the scaled frequency.

The interaction effect at any frequency depends on the dimensions of the elements and their distance apart, relative to the wavelength. But, since the scaling of the dimensions of the elements and their distance apart is exactly the same as the wavelength scaling, it follows that the interaction effects between the elements in one array are identical with those in the scaled array at the scaled frequencies. The scaling law can therefore be used to relate the far field wavelets and the amplitude and phase spectra of the scaled arrays along any azimuth even when interaction effects are significant.

6. Limitations of the Scheme

For marine sources, and for buried sources on land, there is one effect which will not scale: the influence of the surface. The reflected radiation from the surface will change the ambient pressure around the source and thus modify the source radiation. Even a single source will be modulated by its own reflected radiation. Only the onset of the radiated signal will be uncontaminated by this surface modulation effect. Since the scaled sources must operate at the *same* depth, it is clear that the effect of the surface will *not* scale.

How important is this effect? It will obviously depend on the size of the source and on its depth, and it can be regarded as an interaction effect in exactly the same way as the inter-element interactions discussed above. In many cases, the source depth is large compared with the element separation and this surface reflection interaction will be smaller than the inter-element interaction. But this does not answer the question. In fact, the question cannot be answered unless we compare the radiation that is generated with, and without, this interaction being taken into account. This, and the full interaction between elements of an air gun array, is the subject of a separate investigation which might warrant a paper on its own. Nevertheless, we can get a feel for the magnitude of this effect by referring to Smith (1975).

Smith (1975) compared the shapes of measured air gun waveforms, in which this effect obviously comes into play, with theoretical air gun waveforms, in which this effect was not taken into account (because it was not appreciated at the time). For the large air guns used by Smith it is clear that the differences between the measured and predicted waveforms are small at normal operating depths. Even if *all* the differences between the measured and predicted waveforms can be ascribed to this one effect, we would conclude that it will probably have a small effect on the use of the scaling law to relate the farfield wavelets of scaled arrays.

In operation, there are always small variations, for example, in the depth of the source, which affect the far field wavelet slightly. These small variations can be considered as a source of noise which will be small in normal operation. The wavelet recovery process via the scaling law equations is robust in the presence of noise, as we showed in the previous paper (Ziolkowski et al. 1980). If one of the elements in an array ceases to work altogether, the corresponding element in the scaled array can be switched off, and the two arrays will once again be scaled correctly.

In practice, then, the problems which are foreseen at this stage are slight. However, it must be observed that the idea presented here has never been put into practice.

7. Conclusions and Applications

The ideas of Ziolkowski et al. (1980) were conceived only for point sources; that is, for sources whose far field radiation has spherical symmetry. The idea presented here permits the technique to be extended to situations where arrays of point sound sources are used, particularly in the marine environment.

The principal advantage of this technique is that it allows the seismologist to find the far field waveform of an array in any azimuth, whatever the phase spectrum of the wavelet and whatever the degree of interaction between different elements of the array. This can be done even in situations where it would be impossible to measure it. It follows that all the normal wavelet processing techniques may be used on the data using a known wavelet, instead of one derived from a set of statistical assumptions of dubious validity.

Furthermore, since the array designer is now freed from the constraint that the far field wavelet must be either short, or minimum-phase, or both, he may now design the array to satisfy only his other criteria. For example, he could design the array to produce a wavelet which has maximum power over a given bandwidth, or one which has maximum signal energy in a particular direction, or whatever.

He may, on the other hand, still desire to have a short minimum-phase wavelet. In the author's opinion, the air gun does not naturally satisfy this desire. On the contrary, as we have seen, conventional tuning of air gun arrays forces the bulk of the signal energy to appear in directions other than the vertical. Obviously this energy must go somewhere, and in areas of structural complexity it is clear that off-angle signal energy can be scattered back to the recording hydrophones. The signal shape of these side reflections will not be the same as for reflections from vertically below, as was noted earlier. Therefore, any signal processing which works well on energy which is transmitted and received vertically will *not* work well on these side reflections.

This point is particularly important for three-dimensional surveys, where beamforming techniques can be used to focus on reflected energy from a particular azimuth—whether from vertically below or from some other direction. With an array of conventional design the signal shape is known usually only in one direction—the vertical. Using the design method described here it is possible to determine the signal shape in *any* direction. Thus in three-dimensional surveys this method has the added advantage over other methods that all reflected energy can be deconvolved with equal felicity, regardless of the azimuth of its arrival.

The scaling law theory is not perfect, however. As we saw from section 6, the reflected acoustic radiation from the surface must modulate the behaviour of the source. This effect, though likely to be quite small at normal operating depths, does not scale, and will therefore be a source of error.

Finally it must be emphasized that the idea discussed here is theoretical. It has not been tested. If it works, better use can be made of available signal energy than is being done at present, especially at the high frequency end of the spectrum where the source arrays can have their maximum directivity.

Acknowledgments

I owe a great debt to Dr Andrew Stacey of BNOC, who has consistently encouraged my work on air guns for more than a decade. This particular idea emerged at the end of a long conversation over a very memorable dinner when we were both struggling to avoid imminent influenza. I am also very grateful to Dr Chris Walker, and Dr Neil Goulty of National Coal Board, Mr Elio Poggiagliolmi of Entee and Messrs. Lloyd Peardon, Derek March and Bill Lerwill of Seismograph Service Ltd. for reading the manuscript in its various drafts and for making many useful criticisms and suggestions.

Appendix A

The Near Field and Far Field of an Array

An array is a source whose dimensions are of the same order or larger than a wavelength of its radiation. The radiation of such an array is usually azimuth-dependent. Furthermore, in any azimuth, the geometrical effects are also significant at short ranges, since radiation from different parts of the array can arrive at the same point with phase delays which depend on the range. The far field of such an array we define as the region in which the shape of the wavelet depends only on azimuth and not on the range. (We are ignoring all non-elastic effects.)

To get an idea of the sort of distances involved, we consider the simple case of a uniform line array of length D, emitting radiation of wavelength λ. If we consider a point P on the axis of the array (see fig. A1) at a range R, we will just observe destructive interference between the end of the array A and the center O, when the difference in travel paths $AP - OP = \lambda/2$. The far field region clearly exists at ranges greater than R, where

$$\left(R + \frac{\lambda}{2}\right)^2 = \left(\frac{D}{2}\right)^2 + R^2.$$

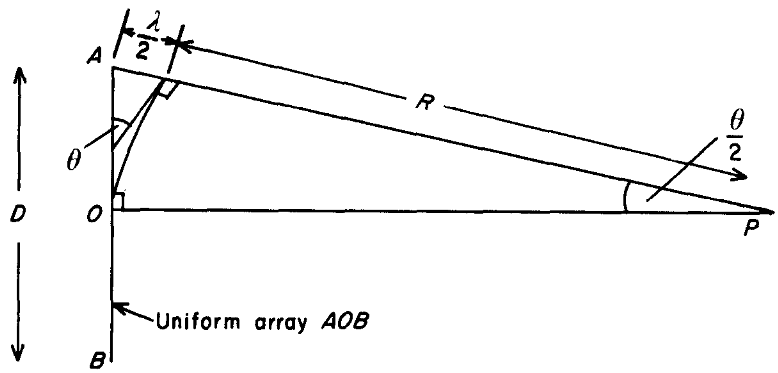

Fig. A1.

Making the approximation that $\lambda^2/4$ is small compared with the other terms, we find that

$$R = \frac{D^2}{4\lambda}.$$

In general, arrays with dimension D which emit radiation of wavelength λ will have a far field region which exists at ranges r of the order $r > D^2/\lambda$. We can also see that the beamwidth of this radiation is of the order of λ/D radians.

These two results are well known and are reproduced here for reference only. We observe that if D is small compared with λ, the beamwidth is large and the directional effect of the source is negligible. Consequently, the radiation must be spherically symmetric.

Appendix B

The Near Field and Far Field of a Point Source

We define a point source as one whose dimensions are small compared with the smallest wavelength of its radiation. The radiation pattern of such a source is spherically symmetric. At some small distance from most seismic sources the particle motions become small enough to be described adequately by linear elastic theory. In a fluid the pressure p in this linear region must satisfy the spherical wave equation

$$\frac{\partial^2(rp)}{\partial t^2} = c^2 \frac{\partial^2(rp)}{\partial r^2} \tag{B1}$$

where r is distance from the source, t is time and $c = \sqrt{k/\rho}$ is the velocity of sound in the fluid and k is its bulk modulus and ρ is its density.

Equation (B1) has a general solution which includes both converging and diverging waves. We ignore the possibility of converging waves. Therefore the general solution of equation (B1) is

$$p = \frac{1}{r} g'\left(t - \frac{r}{c}\right) \tag{B2}$$

where g' is the time derivative of an unknown function g. The radial particle acceleration is given by

$$a_r = -\frac{1}{\rho}\frac{\partial p}{\partial r}.$$

Hence

$$a_r = \frac{1}{\rho r^2} g'\left(t - \frac{r}{c}\right) + \frac{1}{\rho c r} g''\left(t - \frac{r}{c}\right). \tag{B3}$$

The radial particle velocity u_r is the time integral of the acceleration:

$$u_r = \int a_r \, \mathrm{d}t = \frac{1}{\rho r^2} g\left(t - \frac{r}{c}\right) + \frac{1}{\rho c r} g'\left(t - \frac{r}{c}\right). \tag{B4}$$

There are two terms in this expression for particle velocity; the first $(1/r^2)$ must dominate when r is small—that is in the *near field*—while the second $(1/r)$ must dominate when r is large—that is in the *far field*. We note that in the far field, where the first term is negligible, the pressure and particle velocity are related by the simple expression

$$u_r = \frac{p}{\rho c}. \tag{B5}$$

The terms "near field" and "far field" as described here are rather vague and need to be better defined. In order to see better what these terms mean we need to transform equation (B4) to the frequency domain. With the substitution $\tau = t - r/c$ (delayed time), we can rewrite equation (B4) as

$$u_r(\tau) = \frac{1}{\rho r^2} g(\tau) + \frac{1}{\rho c r} g'(\tau). \tag{B4}$$

If we now define $G(f)$ as the Fourier transform of $g(\tau)$, such that

$$g(\tau) = \int_{-\infty}^{\infty} G(f)\, e^{2\pi i f \tau}\, df, \tag{B6}$$

we may differentiate this with respect to τ to find

$$g'(\tau) = \int_{-\infty}^{\infty} 2\pi i f G(f)\, e^{2\pi i f \tau}\, df. \tag{B7}$$

By inspection, we immediately identify the Fourier transform of $g'(\tau)$ as $2\pi i f G(f)$. We may now transform equation (B4) to yield

$$\begin{aligned} Ur(f) &= \frac{1}{\rho r^2} G(f) + \frac{1}{\rho c r} 2\pi i f G(f) \\ &= \frac{1}{\rho r} G(f) \left[\frac{1}{r} + \frac{2\pi i}{\lambda}\right]. \end{aligned} \tag{B8}$$

This last expression tells us that the two terms are 90° out of phase with each other: the far field particle velocity term $(1/r)$ is in phase with the pressure, while the near field particle velocity term $(1/r^2)$ lags the pressure by 90°. We also see that the two terms are equal when

$$r = \frac{\lambda}{2\pi}.$$

Thus the "near field" and "far field" are regions which are frequency—or wavenumber—dependent. The true "far field" is the region in which the pressure and particle velocity are linearly related and in phase with each other. This region exists at distances r greater than about one wavelength. At distances less than a wavelength, the relation between pressure and particle velocity is clearly more complicated—this is the near field.

If we measure the pressure wave $p(\tau)$ we will measure the same thing, apart from the amplitude scaling factor $1/r$, at all ranges r for which this linear theory applies. If we measure the particle velocity $u_r(\tau)$, its shape will vary substantially at ranges r less than a wavelength. The far field particle velocity function is the derivative of the very near field function.

REFERENCES

BERKHOUT, A.J. 1970, Minimum phase in sampled signal theory, Ph.D. Thesis, Koninklijke Shell Exploration and Production Laboratory, Rijswijk, Netherlands.

GILES, B.F. 1968, Pneumatic acoustic energy source, Geophysical Prospecting 16, 22–53.

GILES, B.F. and JOHNSON, R.C. 1973, System approach to air-gun array design, Geophysical Prospecting 21, 77–101.

HUBBARD, T.P. 1978, The advantages of continuously recording far-field signatures, presented at the 40th meeting of the EAEG in Dublin. Preprint published by Seismograph Service (England) Limited, Holwood, Keston, Kent, England.

KRAMER, F.S., PETERSON, R.A. and WALTER, W.C. 1968, Seismic energy sources handbook, Bendix United Geophysical Corporation, Pasadena, Calif.

LARNER, K., HALE, D., MISENER, S. and HEWLETT, C. 1979, Desired characteristics of an air-gun source, presented at 10th Offshore Technology Conference, Houston, Texas (2nd May, 1979), preprint published by Western Geophysical, Houston, Texas.

NOOTEBOOM, J.J. 1978, Signature and amplitude of linear gun arrays, Geophysical Prospecting 26, 194–201.

ROBINSON, E.A. 1967, Statistical communication and detection with special reference to digital data processing of radar and seismic signals, Charles Griffin & Co., London.

SAFAR, M.H. 1976, Efficient design of air-gun arrays, Geophysical Prospecting 24, 773–787.

SAFAR, M.H. 1977, Reply to A. Ziolkowski's comments, Geophysical Prospecting 25, 188–193.

SMITH, S.G. 1975, Measurement of air gun waveforms, Geophysical Journal of the Royal Astronomical Society 42, 273–280.

ZIOLKOWSKI, A.M. 1970, A method for calculating the output pressure waveform from an air gun, Geophysical Journal of the Royal Astronomical Society 21, 137–161.

ZIOLKOWSKI, A.M. 1971, Design of a marine seismic reflection profiling system using air guns as a sound source, Geophysical Journal of the Royal Astronomical Society 23, 499–530.

ZIOLKOWSKI, A.M. 1977(a), Comments on "Calibration of marine sources using a hydrophone of unknown sensitivity", Geophysical Prospecting 25, 186–188.

ZIOLKOWSKI, A.M. 1977(b), Comments on "The radiation of acoustic waves from an air-gun", Geophysical Prospecting 25, 560–563.

ZIOLKOWSKI, A.M., LERWILL, W.E., MARCH, D.W. and PEARDON, L.G. 1980, Wavelet Deconvolution using a Source Scaling Law, Geophysical Prospecting 28, 872–901.