Source-Receiver Seismic Interferometry

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Summary

Seismic (or more generally, wavefield) interferometry allows receivers to assume the role of sources, or *vice versa*. Recorded waveforms (theoretically, Green's functions) can be constructed from new 'virtual' source positions at which only receivers have been physically located. To-date, however, forms of interferometry have only been derived to provide inter-receiver or inter-source Green's functions. For the first time we derive a form of interferometry that provides Green's functions on real-source to real-receiver paths, using only energy that has propagated from surrounding sources or to surrounding receivers. This allows various useful interferometric methods to be applied to (more conventional) source-receiver configurations for the first time.

Introduction

The field of seismic interferometry has grown dramatically since the inception of a consistent, 3-dimentional theory by Wapenaar (2004). It has lead to novel schemas for data acquisition using background noise sources (Campillo and Paul, 2003; Dragonov et al., 2006) and active sources (Bakulin and Calvert, 2006), for computational modeling of full waveforms (van Manen et al., 2005; 2006; 2007), and for noise removal (Curtis et al., 2006; Dong et al., 2006; Halliday et al., 2007, Halliday and Curtis 2008, 2009).

The above advances were made using a form of interferometry that uses cross-correlation, convolution or deconvolution to convert data recorded at a pair of receivers into *derived* data that would have been recorded had one of the two receivers been a source. We think of the derived data as the wavefield from a 'virtual' (imagined) source, since no real source need have been placed at either receiver location in order to obtain the new data. We refer to this method as *inter-receiver* interferometry.

Recently, Curtis et al., (2009) have shown that by applying reciprocity to the results of van Manen et al., (2005, 2006), Green's functions between pairs of source locations can be obtained by *inter-source* interferometry. Thus one constructs data as though one of a pair of sources had in fact been a receiver that recorded the signal from the other source. The authors used that theory to obtain seismograms from one earthquake recorded on a virtual seismometer constructed from, and at the location of, another earthquake deep in the Earth's subsurface.

This paper presents a third method to construct new data: *source-receiver interferometry*. Whereas the above studies convert receivers to virtual sources or sources to virtual receivers, this new method converts real sources to virtual sources, or real receivers to virtual receivers. Equivalently, it can be thought of as converting a real source and receiver pair to a virtual receiver and source pair, respectively. This is important as it allows various interferometric methods to be applied to (more conventional) source-receiver configurations, which has not been previously possible. Immediate applications include infilling new shot-receiver paths into an existing survey, assessing the character of passive background noise fields, or facilitating novel noise removal algorithms.

Result

Say a source and a receiver are placed at locations x_1 and x_2, respectively, in an acoustic medium. Let $G(x_1 \mid x_2)$ denote the Green's function (waveform) recorded at $\boldsymbol{x}_1$ due to a point, impulsive, volume-injection rate density source at x_2. Assume that receivers are located on a surrounding surface S_1, and that other impulsive sources are located on a surface S_2 surrounding the original receiver location x_2 (Figure 1A).

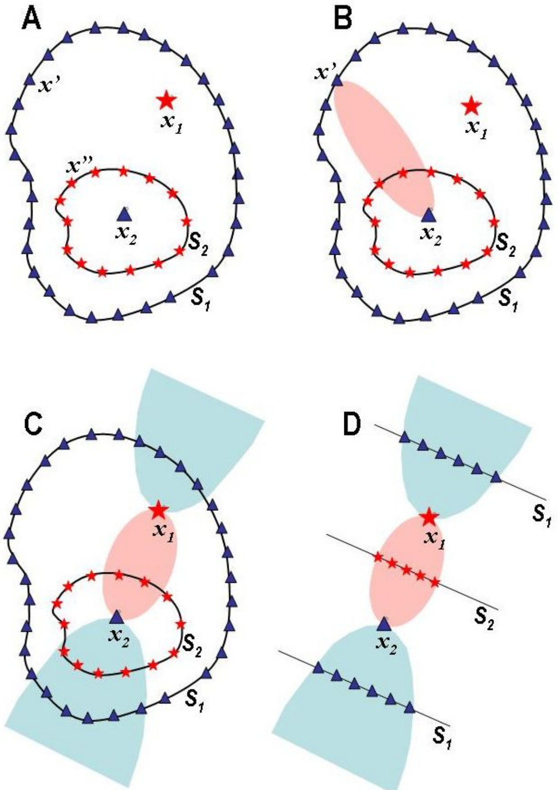

Figure 1. Geometries used. Triangles represent receivers, stars represent sources. S_1 and S_2 are closed lines in 2-dimensions, surfaces in 3-dimensions. Shaded hyperbolae show stationary phase regions for correlational interferometry, while full ellipses show the same for convolutional interferometry.

The Appendix employs both the inter-source and inter-receiver methods, and both correlational and convolutional interferometry, to prove the following, new interferometric formula for the homogeneous Green's function between x_1 and x_2:

$$G^h(x_1 \mid x_2) \approx \iint_{S_1 S_2} \frac{C}{R(x_1, x_2)} G^*(x_1 \mid x') G(x' \mid x'') G(x'' \mid x_2)\, dx''\, dx' \qquad \ldots(1)$$

Here, $R(x_1, x_2) = \rho(x_1)\rho(x_2)$ where ρ is the mass density, $G^h(x_A \mid x_B)$ is the homogeneous Green's function defined as $G^h(x_A \mid x_B) = G(x_A \mid x_B) - G^*(x_A \mid x_B)$ where * denotes complex conjugation, C is a constant, and terms in equation (1) are written in the frequency domain where angular frequency ω has been omitted for notational simplicity. The Green's function between x_1 and x_2 is found by taking G^h at positive times.

The relation in equation (1) synthesizes the Green's function on the x_1-x_2 source-receiver path using only Green's functions from and to surrounding sources and receivers, respectively. It is approximate because only monopole (e.g., explosive) sources are included. An exact relationship would require dipolar (strain rather than displacement) sources which are generally not available in practical surveys. Also, the Green's function estimates obtained are diffraction-limited in the sense defined by van Manen et al. (2005); this only affects results if locations x_1 and x_2 are less than a wavelength apart.

It is not usual for surveys to acquire data using sources or receivers located in closed surfaces such as S_1 and S_2 in Figure 1A. Snieder (2004) showed that for homogeneous background media, the integrand of correlational interferometric integrals such as the integral over receivers on S_1 in equation (1), only integrates constructively over parts of S_1 spanned by open, hyperbolic shapes such as those depicted in Figure 1C (Halliday and Curtis, 2009). A similar analysis for convolutional interferometry shows that the integral over S_2 is only constructive over parts of S_2 spanned by the central, elliptical region in Figure 1C. Hence, provided the medium is not too heterogeneous in practice we can limit the source and receiver geometry to that shown in Figure 1D. For this geometry, Figure 2 shows the various sets of Green's functions used in equation (1) to create an approximation to the Green's function between x_1 and x_2 (taking the mirror image of Figure 2 about surface S_2 accounts for the other section of S_1 shown in Figure 1D).

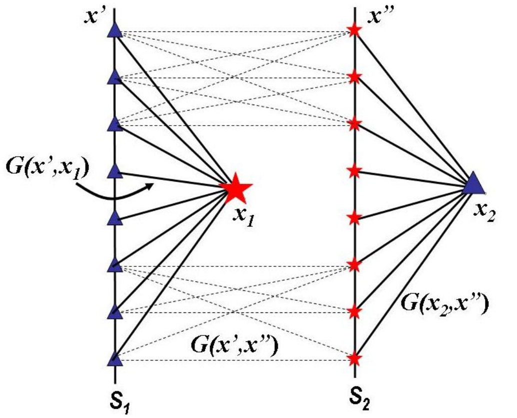

Figure 2. Graphical representation of the three measured sets of Green's functions used in equation (1). Symbol key as in Figure 1.

It is also possible to reverse the role of sources and receivers in all of the above. In that case, all sources in Figures 1 and 2 should be swapped for receivers and vice versa. The form of equation (1) remains unchanged.

Discussion

There are several applications for which it might be useful to turn a real source into a virtual source. First, notice that the source-receiver record between x_1 and x_2 is obtained using all of the other source-receiver Green's functions illustrated in Figure 2. This means that if the latter records are available, the source-receiver record between x_1 and x_2 can be synthesized without having to measure it directly. This might occur, for example, if the receiver at x_2 had not been activated when the shot at x_1 was fired, but had been activated by the time shots on surface S_2 were fired. Thus, new source-receiver paths can be added to existing surveys.

Second, there are situations where records from both an active and a virtual source enable methods that are otherwise impossible. An example is the ground roll removal method of Curtis et al., (2006), Dong et al., (2006), Halliday et al. (2007). These methods make use of the fact that interferometric estimates of Green's functions between two points, synthesized using energy from surrounding surface sources, tend to be dominated by surface waves (ground roll). If a real source-receiver record also exists between the same two points, the interferometric estimate can be adaptively subtracted from the real record to leave a reflection record with ground roll removed. This method clearly requires both virtual and real sources from the same point (x_1), recorded at the same receiver (at x_2). To obtain such records using either inter-source or inter-receiver interferometry would require the co-location of receivers at every source point, which is not economical in land seismic surveys. Using source-receiver interferometry, however, the virtual-source record can be synthesized from the real source using records that might usefully be recorded in a land seismic survey anyway.

A third application is to create a new method to characterize the effectiveness of seismic interferometry. For a particular source and receiver geometry on surfaces S_1 and S_2 we might wish to assess the extent to which interferometry could be used to synthesize new records. If a real source and receiver were placed at x_1 and x_2 respectively, the recorded trace from the real source could be compared to that constructed by interferometry using equation (1). Similarly to above, the only previously existing way to make a direct comparison would be to co-locate a receiver with the source and use inter-receiver interferometry to construct the comparison record; however, such comparisons would be affected by differences between the radiation pattern of the source, and the spatio-temporal sensitivity of the receiver. Using the new method we remove any such effects since the same source is used for both compared records.

While these are all useful applications, the field of seismic interferometry is developing rapidly. It is likely that many others will be invented once this new method is publicly available.

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Appendix: Proof of Equation (1)

The proof of equation (1) consists of two main steps: first, we use convolutional form of inter-source interferometry to turn the receiver at x_2 into a virtual source. Second, we use the correlational form of inter-receiver interferometry to turn the real source at x_1 into a virtual receiver. In both steps we invoke the far-field, monopole source approximation common to most current interferometric applications. The proof is given in the frequency domain.

In the experiment in Figure 1A in an acoustic medium, the following Green's functions (dropping the angular frequency for notational simplification) are recorded directly: $G(x'|x_1)$, $G(x'|x'')$, and $G(x_2|x'')$. By invoking source-receiver reciprocity we also obtain $G(x_1|x')$, $G(x''|x')$, and $G(x''|x_2)$.

Step 1: Receiver at x_2 → virtual source recorded on S_1

Using the convolutional form of inter-receiver interferometry of Slob et al (2007), Slob and Wapenaar (2007), and Halliday and Curtis (2009), we obtain the Green's functions between each receiver at *x'* on S_1 and a virtual source at location x_2 by

$$G(x'|x_2) = G(x_2|x') = \int_{S_2} \frac{1}{\rho}\left[G(x_2|x'')\nabla G(x'|x'') - \nabla G(x_2|x'')G(x'|x'')\right]\cdot n_{S_2}\, dx'' \qquad \ldots(2)$$

where n_{S_2} is the unit normal vector to S_2. We now invoke the normal-incidence approximation on surface S_2: assuming that the waves that arrive at *x'* and x_2 leave sources on S_2 at angles close to normal to S_2, equation (2) simplifies to

$$G(x'|x_2) = G(x_2|x') \approx \int_{S_2} \frac{C_1}{\rho}\left[G(x'|x'')G(x_2|x'')\right] dx'' \qquad \ldots(3)$$

for constant C_1. This approximation would not hold for waves propagating from S_2 along paths far from the direction of the $x' - x_2$ axis. However, a similar (stationary phase) analysis to that in Snieder (2004) shows that for media in which most energy from the virtual source would travel to x_2 approximately along the $x' - x_2$ ray path, such far-from-ray waves tend to cancel destructively within the integration of equations (2) and (3), and hence contribute negligibly to the resulting Green's function; the dominant contribution comes from sources on S_2 within an elliptical region around the $x' - x_2$ ray path as in Figure 1B (e.g., Halliday and Curtis, 2009). For such constructively-interfering paths the normal-incidence approximation is reasonable.

Note that an alternative interpretation of Step 1 that leaves the equations unchanged is that the receiver at x_2 is left as a receiver, but that each receiver on S_1 has been turned into a virtual source.

Step 2: Source at x_1 → virtual receiver

Using the correlational form of inter-source interferometry from Curtis et al. (2009), the homogeneous Green's function between a (virtual) source at x_2 and the (real) source at x_1 is then obtained from

$$G^h(x_1|x_2) = G^h(x_2|x_1) = \int_{S_1} \frac{1}{\rho}\left[G(x_2|x')\nabla G^*(x_1|x') - \nabla G(x_2|x')G^*(x_1|x')\right]\cdot n_{S_1}\, dx' \qquad \ldots(4)$$

where n_{S_1} is the unit normal vector to S_2. Invoking the normal-incidence approximation on S_1 results in

$$G^h(x_1|x_2) = G^h(x_2|x_1) \approx \int_{S_1} \frac{C_2}{\rho} G(x_2|x')G^*(x_1|x')\, dx' \qquad \ldots(5)$$

for constant C_2. Equation (5) is invoked using the estimate of $G(x_2|x')$ from equation (3).

In the running interpretation, both the receiver at x_2 has been turned into a virtual source, and the source at x_1 has been turned into a virtual receiver. In the alternative interpretation given above, the real source at x_1 has been turned into a virtual source that is recorded at the real receiver at x_2.

Substituting equation (3) into equation (5) and invoking source-receiver reciprocity gives equation (1), for $C=C_1 C_2$.

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