**Quantifying ice sheet uncertainty from observations using inverse methods**

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**Summary.** This project combines numerical modeling, optimization, and remote sensing products to improve glaciological inversion methods. It will provide valuable information regarding controls on ice flow and required accuracies of observations.

**Project Background**. The response of the world's ice sheets to a changing environment is a key ingredient in the understanding of past and future global climate change, due to their potential for rapid contributions to sea level change [Solomon et al, 2007]. The sensitivity of these ice masses to climate is to a large degree dependent on the behaviour of their fast flowing outlet glaciers. However, there remain significant gaps in our understanding of the dynamics of fast-flowing glacial ice, in part because computer models of ice sheets must represent physical properties which are unknown or difficult to measure. These unknown properties, such as bedrock topography and ice temperature, are often spatially variable, and hence there are an extremely large number of unknown inputs which affect the predictions derived from computer simulations. A popular technique for determining these unknown values is to use available observations, such as satellite-derived altimetry and surface velocities, to *invert* for these values; i.e. to find the values such that the model output best fits the observations. However, a question remains: to what degree are the estimated values *constrained* by those observations? That is, can we be confident that the observations yield accurate estimates for these hidden values? The importance of this question cannot be understated, as uncertainties in the estimates of these values translate to uncertainties in projected ice sheet behaviour, and in the contribution of ice sheets to sea level change.

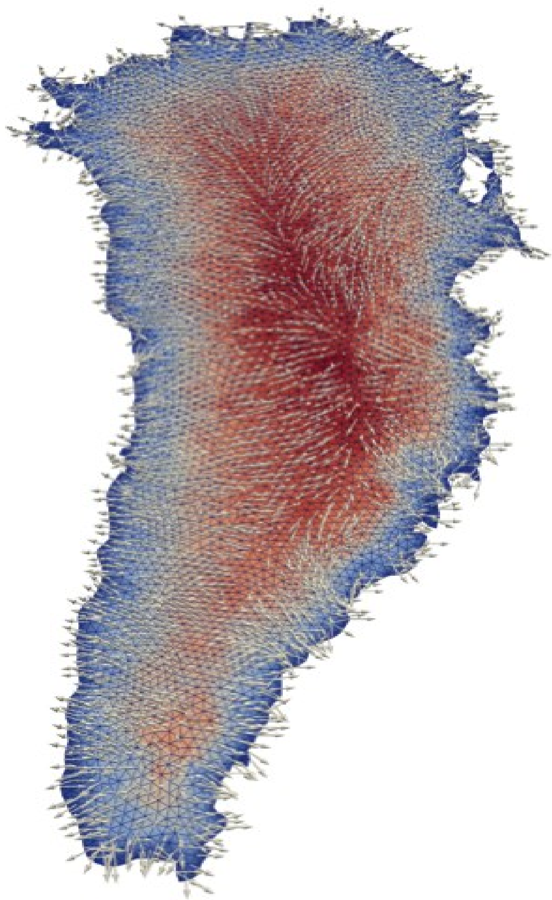

*Fig 1. Surface elevation (shading) and velocities (vectors) of the Greenland Ice Sheet modelled with a finite element method (triangles represent the finite element mesh). Courtesy Dr. P. Farrell*

**Aims and Methodology.** In recent years the preferred method for performing such large-scale parameter estimation problems for ice sheets has been a control method (MacAyeal, 1992). Conceptually no different than minimization of a function of a single variable, the method makes use of the **adjoint** of a model, a tool which allows for efficient calculation of gradients of complex codes. The method has gained popularity because of its tractability relative to other methods of inversion; yet in some ways it is limited. For example, it does not offer a comprehensive measure of how well the inversion is constrained by the data, and the possibility of multiple solutions – which could lead to different projections of ice sheet behavior – is not easily detected. The aims of this project are to address these and other shortcomings using sophisticated computational and mathematical tools. Using state-of-the-art finite element software, an advanced glaciological flow model will be generated, and high-level differentiation methods will be used to generate the required adjoint, as well as the **second order adjoint** (SOA), an invaluable tool in carrying out inversions efficiently, and in assessing their quality afterwards. The SOA is often difficult to generate and therefore seldom used, but can be generated efficiently using the methods proposed.

This new inverse model will be applied to fast-flowing outlet glaciers in Greenland and Antarctica which are currently undergoing rapid change, using both publicly available data, and also high temporal- and spatial-resolution data sets that will be generated by a cryosphere remote-sensing group in the School of GeoSciences. The sophisticated model will be able to take advantage of this high-quality data, and the inversions will enable improved understanding of the controls on glacier flow. A novel contribution of the model will be the ability to use the SOA to quantify the uncertainty in the results, and to identify areas in which improved observations would have the most informational content from an ice-dynamics perspective.

**Training**. The qualified student will join a large and internationally leading cryosphere group in the School of GeoSciences, and will interact strongly with experts in remote sensing and observation, with the potential to participate in ongoing fieldwork campaigns. They will be at the forefront of cutting edge glaciological flow modeling techniques, and trained in the use and interpretation of remote sensing products. Through the School of Mathematics and the co-supervision of Dr. Farrell they will be trained in sophisticated numerical and mathematical techniques including finite element methods and numerical large-scale optimization. The student will present the findings of the project at international conferences (e.g., EGU, AGU, SIAM) and submit the results to leading peer-reviewed journals

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