Comparative assessment of three approaches for deriving stream power plots along long profiles in the upper Hunter River catchment, New South Wales, Australia

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Abstract

The downstream distribution of stream power is derived and analysed for 11 different streams in the upper Hunter River catchment, Australia. Stream long profiles were produced in a GIS environment using DEM data and catchment area–discharge analysis. These profiles were analysed using three approaches, namely long profile smoothing, curve fitting and a theoretical model. The methodology for deriving stream power profiles using these three approaches is discussed. The long profile smoothing method provides a good approximation of the subcatchment variability in stream power trends. The curve fitting method shows that higher-order exponential curves provide a better fit for long profile data. For the streams of the upper Hunter River catchment, second-order exponential curves fit well with significantly less error. The curve fitting method predicts a bimodal (upstream and midstream) distribution of stream power, which is a deviation from our earlier understanding of a single midstream peak. The theoretical approach provides a mathematical expression of the observed bimodal stream power distribution. The bimodal distribution emphasises the erosion potential of headwater reaches. The resultant stream power distribution provides a catchment-scale characterisation of the distribution of available energy in any given system. Using these approaches, the variability of stream power in headwater reaches is explained by discharge variability, while variability in midstream and downstream reaches is related to high variability in channel gradient.

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1. Introduction

Channel appearance and behaviour are governed by the balance between driving and resisting forces. The rate of doing geomorphic work by stream water, commonly expressed as stream power, provides a measure of the driving force. The stream power of descending water is defined as the rate of energy conversion from potential to kinetic form (Bagnold, 1966). Considering the importance of the role of stream power in the functioning of fluvial systems, attempts have been made to determine the distribution of stream power either through point-location studies (e.g., Magilligan, 1992; Lecce, 1997; Fonstad, 2003) or through continuous distributions of stream power based on stream long profiles (e.g., Knighton, 1999; Finlayson et al., 2002; Fonstad, 2003; Reinfelds et al., 2004).
Theoretical approaches based on the exponential nature of longitudinal profiles predict the existence of stream power peaks in the midprofile region (Knighton, 1999). This understanding of stream power distribution contrasts with the general belief that stream power maxima are to be found in upstream regions where erosion is a dominant process. However, comparison of measured point data with continuous profile studies indicates a variable trend in observed stream power distributions ranging from multipeak, single peak, no peak (downstream decreasing trend), or random patterns (e.g., Magilligan, 1992; Lecce, 1997; Knighton, 1999; Finlayson et al., 2002; Fonstad, 2003; Reinfelds et al., 2004). The deviation of observed data from predicted patterns raises an important question—is it possible to define/predict patterns of energy availability along river channels?

In this paper, digital elevation model (DEM) data are used in a GIS environment to address this question. Earlier work on DEM analysis by Finlayson and Montgomery (2003) highlighted the effect of data resolution and projection on the stream power profile at the regional scale. The current work presents finer methodological details that can be applied to determine stream power plots at the catchment scale. In this study, three different approaches are used to appraise the stream power distribution along river channels in the upper Hunter River catchment, New South Wales, Australia, using catchment-scale 25 m DEM data, namely long profile smoothing, curve-fitting, and a theoretical model.

### 2. Stream power

The stream power per unit length or cross-sectional stream power \( (\Omega) \) is defined as

\[
\Omega = \gamma \cdot Q \cdot s
\]  

where \( \gamma \) is the unit weight of water (=9800 N/m\(^3\)), \( Q \) is discharge (m\(^3\)/s), and \( s \) is energy slope (m/m), which is considered equivalent to bed slope.

Eq. (1) indicates that there are two variable components of stream power, namely discharge (\( Q \)) and slope (\( s \)). A continuous distribution can be generated for both of these components to derive a continuous stream power profile. Channel slope profiles can be extracted from digital elevation data. A continuous distribution of discharge can be established on the basis of discharge–area relationships for a given study area. The discharge–area equation can be represented as

\[
Q = a \cdot A^b
\]  

where \( A \) is the contributing catchment area in square kilometer and \( a \) and \( b \) are empirical coefficients. Hence, stream power can be represented as

\[
\Omega = \gamma \cdot (a \cdot A^b) \cdot s.
\]  

Eq. (3) represents stream power as dependent on catchment area. Further, Knighton (1999) assessed stream power as a function of channel length based on the catchment area–discharge relationship of Eq. (2) and Hack’s (1957) relation. According to Hack’s relation, channel length in kilometers and catchment area in square kilometers are related as

\[
L = 1.4A^{0.6}.
\]  

From Eqs. (2) and (4),

\[
Q = c \cdot L^d
\]  

where \( c \) and \( d \) are empirical coefficients.

Also, channel slope between two points can be defined mathematically as

\[
s = -\frac{dH}{dL}
\]  

where \( dH \) and \( dL \) are height difference and channel length between the two points, respectively. Use of the negative sign (\( - \)) indicates the decrease in channel slope with respect to increase in length. Here, in exponential form, the elevation of any point on the long profile, \( H_n \), at distance \( L \), can be estimated as

\[
H_n = -H_0 \cdot e^{-\beta L}
\]  

where \( H_0 \) is the highest elevation point (at the origin of the channel) and \( \beta \) is profile concavity (i.e. the decay constant of the longitudinal profile exponential curve). Hence, slope will be given as

\[
s = H_0 \cdot \beta \cdot e^{-\beta L} = \alpha \cdot e^{-\beta L}.
\]  

Finally, stream power can be represented as a function of channel length, whereby

\[
\Omega = c \cdot L^d \cdot \alpha \cdot e^{-\beta L}.
\]  

Eq. (9) provides a continuous smooth pattern of downstream variation of stream power in the catchment. By contrast, Eq. (3) provides a continuous but stepped profile from irregular increases in catchment area as a result of tributary input. The advantage of the latter approach (Eq. (3)) is that discharge is approximated as a first dependent parameter of catchment area and provides a better approximation to real data. Hence, in this study, the initial stream power analysis, using a GIS-based approach, was carried out using Eq. (3).
the theoretical approach, approximation of stream power distribution was derived using Eq. (9).

3. Regional setting

The distribution of stream power along long profiles was explored for selected river systems in the upper Hunter River catchment of New South Wales, Australia (Fig. 1). The Hunter River at Muswellbrook drains a catchment area of 4480 km², with 10 major tributaries (Fig. 1; Table 1). The stream power distribution has been analysed for each of these streamlines. These rivers are characterised by gravel bed material load. Rainfall in the upper Hunter River catchment has a winter maximum, and averages around 600 mm/year.

Most of the upper Hunter River catchment comprises rugged and hilly topography, characterised by relief of around 500 m. Broadly, the catchment can be divided into eastern and western parts on the basis of topographic and geologic characteristics. The eastern tributaries originate from higher elevations of \( \approx 1200–1500 \) m, whereas western tributaries originate from elevations of \( \approx 800–1000 \) m.

Geologically, the upper Hunter River catchment is characterised by two distinct units separated by an inactive fault. The western Sydney Basin units are characterised by Permian sedimentary sequences including sandstones, shales, mudstones, and conglomerates. By contrast, the New England Fold Belt units in the east comprise folded and faulted Carboniferous rocks. In the headwaters of most streams, Tertiary basalt flows have capped the older rocks of the Sydney Basin and the New England Fold Belt units. The Sydney Basin units in the west produce flat topography in comparison to New England Fold Belt units in the east.

![Fig. 1. Streams and gauging stations in the upper Hunter River catchment.](image_url)

Table 1

<table>
<thead>
<tr>
<th>River</th>
<th>Channel length (km)</th>
<th>Altitude (m)</th>
<th>Catchment area (km²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hunter River (up to Muswellbrook)</td>
<td>223</td>
<td>1400</td>
<td>4480</td>
</tr>
<tr>
<td>Moonan Brook</td>
<td>27</td>
<td>1500</td>
<td>132</td>
</tr>
<tr>
<td>Middle Brook</td>
<td>34</td>
<td>1200</td>
<td>78</td>
</tr>
<tr>
<td>Isis River</td>
<td>78</td>
<td>1100</td>
<td>537</td>
</tr>
<tr>
<td>Pages River</td>
<td>105</td>
<td>1150</td>
<td>1192</td>
</tr>
<tr>
<td>Kingdon Ponds</td>
<td>53</td>
<td>760</td>
<td>380</td>
</tr>
<tr>
<td>Stewart Brook</td>
<td>36</td>
<td>1500</td>
<td>192</td>
</tr>
<tr>
<td>Davis Creek</td>
<td>39</td>
<td>1300</td>
<td>139</td>
</tr>
<tr>
<td>Dart Brook River</td>
<td>66</td>
<td>1020</td>
<td>804</td>
</tr>
<tr>
<td>Rouchel Brook</td>
<td>51</td>
<td>1300</td>
<td>432</td>
</tr>
<tr>
<td>Pages Creek</td>
<td>38</td>
<td>1360</td>
<td>178</td>
</tr>
</tbody>
</table>

4. Discharge data

Discharge data were obtained from the Pinneena database held by the NSW Department of Infrastructure, Planning and Natural Resources (DIPNR). The data include daily peak discharge values for 24 gauging stations in the upper Hunter River catchment (Fig. 1) and cover variable time spans ranging from a few years to more than 100 years of record. Variability in data distribution in time and space, the effect of secular climate change, and the influence of dams and weirs in the system limit the value of raw data. Hence, various practical steps were taken to obtain the best possible results. Filtered data were then used to generate discharge distributions with respect to catchment area.

Initially, discharge frequency/magnitude spectra were created using flood frequency and log Pearson III analyses. This approach deals with the effect of the variable distribution of data in time. In addition, the five gauging stations with <10 years of records were omitted from the analysis and consideration was given to the role of climatic variation in influencing the flood frequency analysis of the remaining 19 gauging stations. Since 1948, the climate in coastal NSW has been significantly wetter on average than in the period from about 1890 to 1948 (Kirkup et al., 2001). To analyse the impact of climate variation on flood frequency analysis, the return period floods were computed using full records (1907–present) and then using records from 1948–present. Empirical best-fit equations are almost identical, but the correlation coefficients are better (0.60–0.77) for the 1948–present data set in comparison to those for the entire 1907–present data (0.51–0.76). The lower value of the correlation coefficient for the larger data set highlighted the scattering effect of using data spread over two climate intervals. Hence, discharge data from 1948–present were used for flood frequency analysis. The resultant flood frequency curves were used to compute the 2-, 5-, 10-, 25- and 50-year return period floods for each of the 19 gauging stations.

Secondly, the flood frequency curves from each gauging station were correlated with their respective catchment area to derive catchment area–discharge relations for the upper Hunter River catchment. The effect of civil engineering structures was then filtered out. The gauge downstream of Glenbawn Dam (Fig. 1) was removed from the analysis. The effect of small weirs was analysed for the remaining data set by comparing the discharge–area relation derived using all gauging stations (set I) with those derived using only gauging stations that are not downstream of weirs (set II). The discharge–area relation from set I data underestimated the discharge for lower return period floods by up to 40%. On the basis of this effect, the gauges positioned downstream of small weirs were also removed from the analysis. Finally, 14 gauging stations were chosen for derivation of the discharge–area relation.

In fitting curves to the discharge–area relationships, a linear relationship ($r^2=0.71–0.87$) is statistically better than the power function ($r^2=0.60–0.77$). However, as the linear equation does not intersect the origin, it provides nonsensical values for small catchment areas and the power fit is thus preferred. Notably, the difference between discharge values computed using linear and power equations remains around 10%, except for catchment areas of <300 km$^2$.

On the basis of this analysis, the discharge–area relations for different return periods were determined. In this manuscript, the 2-year return period flood is used to calculate stream power calculation, as this event is considered to represent the energy distribution within the channel (Wharton, 1995). The equation for the 2-year return period flood in the upper Hunter River system is

$$Q_2 = 1.21 \cdot A^{0.72}$$

where $Q_2$ is the discharge of the 2-year flow in cubic meters/second and $A$ is the catchment area in square kilometers.

5. Extraction of long profiles from the DEM

A digital elevation model (DEM) with a 25-m spatial resolution was obtained from NSW DIPNR and was used to derive stream long profiles and channel slope values. GIS analysis was carried out using ESRI ArcMap version 8.3 with the HydroTools and Spatial Analyst extensions. The process of retrieval of elevation data from along the stream network using the DEM is divided into two steps (Fig. 2).

The first step involved a general pre-processing operation and generation of thematic maps using the HydroTools extension. This includes removal of anomalous depressions from the land surface, definition of flow accumulation patterns, and ultimately a stream network. Depending on the quality and resolution of the DEM and the nature of the terrain, this will frequently be an iterative process. Given the coarse resolution of the available DEM and the extent of low relief floodplains in the study area, several iterations were required to digitally generate a stream network that could be matched to topographic maps and aerial photography. In the confined headwater valleys, the soft-
ware was easily able to define the stream network; however, on the broad alluvial flats that are characteristic of the lower regions of most of these catchments, the DEM-generated stream network was replaced by a stream network coverage digitised from 1:25,000 topographic maps (step-Ib).

The second step involved preparation of data for export in a format amenable to manipulation within a spreadsheet. All points on the DEM surface have values of elevation and flow accumulation (the number of cells that flow into any given cell) as a product of the hydrological pre-processing.

The points that lie along streamlines were incorporated into point coverages (i.e., thematic layers describing elevation and flow accumulation, respectively). The attribute tables of these coverages were linked and exported as an ASCII file containing both of these variables for every point along a given streamline. While the elevation is the object of interest for long profile construction, the flow accumulation value allows the data to be sorted in a spreadsheet and graphed in the appropriate downstream sequence. Importantly, flow accumulation is also a surrogate measure of catchment area and can be used to obtain values of this for use in Eq. (10).

The resultant long profiles are continuous, with values at intervals of 25 m (or 35.36 m for instances when the streamline moves diagonally) along the entire stream. However, the profiles contain many kinks, which are due to inherent noise in DEM data and are

Fig. 2. Flow chart showing the methodology of DEM analysis for long profile extraction.
not necessarily real (Fig. 3A). These anomalous kinks need to be removed before using the elevation profile to generate a stream power plot.

6. Channel slope

6.1. Long profile smoothing and slope measurements

A smoothing process was carried out to remove the kinks from the raw long profiles (Fig. 3A). All points with values greater than the minimum of preceding (i.e., upstream) values were selected and classified as anomalous (Fig. 3B). This process identified all the anomalous peaks in the long profile, but the potentially real knickpoints in the long profile remain unselected. The anomalous points are then given replacement values based on a decrease in elevation commensurate with the average gradient of the 10 preceding (i.e., upstream) points. The profiles thus generated were used to estimate slope values for stream power calculation. The corrected long profiles of the Hunter River and its tributaries are shown in Fig. 4.

The length of channel over which slope is measured is an important determinant of its value and hence of the resultant stream power values. Earlier works have highlighted the control of slope length on the stream power–geomorphic process relationship (Moore and Burch, 1986; Phillips, 1989). Bagnold (1966) suggested that for stream power computation, an appropriate channel length for slope measurement should be an average that is representative of channel characteristics, including all repetitive irregularities of slope, cross-section, and boundary conditions. In the case of the DEM-derived long profiles, slope values between successive points represent slopes over distances of only 25 m (when points are from adjacent DEM pixels) or 35.36 m (from diagonally contiguous pixels). These small channel lengths do not necessarily represent the average channel characteristics. Hence, an appropriate average of channel length was required for slope measurement. The continuous long profile data provide two options for computation of average slope: (i) for constant elevation difference, i.e., the “vertical slice” approach (e.g. Reinfels et al., 2004) or (ii) for constant length.
difference, i.e., the “horizontal slice” approach (e.g., Knighton, 1999).

Stream power is usually defined in terms of unit length and, according to this definition, a value of stream power represents the total power available in a defined unit length of channel. Further, stream power characterises the ability of a river to carry out its geomorphic work, which depends upon the dissipation of energy per unit length. Hence, while stream power may be estimated on the basis of change in channel elevation, i.e., slope, the dissipation of energy that it expresses occurs over a defined channel length. Both these criteria suggest that the channel length used for stream power estimation must be constant throughout a stream power longitudinal profile. Hence, in this analysis, average channel slope was measured using the “horizontal slice” approach.

Slopes were calculated over channel lengths of 0.5, 1 and 2 km to test the best representation of average channel characteristics (Fig. 5). The slope value for each pixel was determined by considering its change in elevation from the pixel located at the respective distance upstream. Channel lengths of only 0.5 km still contain much local variation, while lengths of 2 km are considered to average out some important information. Hence, slope measurements were based on 1 km chan-

Fig. 4. Long profiles of Hunter trunk stream and its tributaries; the shape (steepness) is variable for different streams, which will provide variable stream power distribution in the catchment.

Fig. 5. Determination of average channel length for slope measurements; the average slope of 1 km provides better results; 0.5 km average slope contains local noise while 2 km average slope masks much detail.
nel lengths. In reality, of course, the distance between points in the data set is not constant, given that points are separated by distances of either 25 or 35.36 m. Hence, some error is associated with channel length characterisation, as the channel length for slope measurement varies in the range of 0.85–1.15 km with an

Fig. 6. Exponential curve fitting to long profiles and associated error. (A) Error associated with curve fitting of first-, second-, third-, and fourth-order exponential curves to long profiles of different streams of the upper Hunter River catchment. In all streams, the curve fitting error is less for higher-order exponential curves in comparison to first-order exponential curve. (B) Graphical representation of long profiles and curve fitting of different orders of exponential curve for the Hunter trunk stream. Higher-order curves have a better fit in headwater as well as midstream reaches.
average of 1.0 km. However, in most cases the value of channel length lies around 1.0 km and the error computed through the root mean square method amounts to ~ 4% (0.041 km). The error in channel length characterisation does not cause error in the slope values. However, as stream power dissipation is defined for the unit length, it generates some error in stream power data. In this instance, the error is around 40 m per 1 km length and may be neglected in stream power computation.

6.2. Curve fitting and slope measurements

Curve fitting was the second approach applied for slope measurement. This approach provides a mathematically defined estimate of channel slope and hence stream power. However, it has the disadvantage that real knickpoints cannot be easily maintained within the data set.

Earlier work on long profiles has suggested different long profile forms result from a range of different controls (Hack, 1957; Shepherd, 1985). Based on sediment transport and discharge equations, theoretical models of long profile shape take an exponential form (Morris and Williams, 1997). This has been supported by laboratory and field studies (Leopold and Langbein, 1962; Morris and Williams, 1997). In this study, exponential curves were fitted to the untransformed, DEM-generated, long profile data.

First-order exponential curves can be expressed as in Eq. (7); higher-order exponential curves can be expressed as

\[ H = -H_1 e^{-\beta_1 L} - H_2 e^{-\beta_2 L} - ... - H_n e^{-\beta_n L}. \]  

The higher-order exponential curves indicate that stream long profiles are controlled by more than one set of external controls or different stages of landscape evolution. Mathematically, this implies that different parts of the long profile are characterised by a different decay rate, which highlights the physical heterogeneity of the river basin and channel processes.

The goodness of fit of curves was assessed through the curve fitting error, which is the average of squared values of the difference between DEM elevation values and those predicted by the fitted exponential curve. We observed that first-order equations do not fit the long profiles well, while the second- and higher-order exponential curves fit very well (Fig. 6A). For the Hunter River itself, the closeness of fit is also shown (Fig. 6B). The higher value of average curve fitting error for first-order exponential curves indicates that these are a poor fit to the longitudinal profiles, whereas the second- or higher-order curves are characterised by very low values of average curve fitting error and show good fit with the longitudinal profile.

The initial increase in the order of curve fitting improves the degree of fit. However, a subsequent increase in the order of the exponential curve results in little improvement in the fit. The curve fitting error is normalised with the relative-relief of particular subcatchments to provide uniformity among all the subcatchments. The normalized value of difference in the curve fitting value of two successive orders provides a threshold of 0.1 m with which to determine the order of fit required for any channel. For the streams of the upper Hunter catchment, the second-order exponential curves were used in further analyses.

Slope at any point on the long profile can be expressed as the tangent of the fitted curve at that point and thus channel slope was computed for first order as

\[ s = -\frac{dH}{dL} = H_0 \beta e^{\beta L} \]  

and for second order as

\[ s = -\frac{dH}{dL} = H_1 \beta_1 e^{-\beta_1 L} + H_2 \beta_2 e^{-\beta_2 L} \]  

or

\[ s = x_1 e^{-\beta_1 L} + x_2 e^{-\beta_2 L}. \]

Then, these point estimates of slope were averaged over a 1 km upstream length (i.e., an average of 37 upstream points, as previously outlined). Finally, on the basis of Eqs. (3) (10) and (12) or (14), the stream power per unit length (1 km) was computed.

7. Methods for deriving stream power profiles

7.1. Long profile smoothing

Points in the long profiles are arranged in the sequence of increasing flow accumulation. Thus, each point on the long profile is associated with its upstream catchment area. These values of catchment area have been used to derive estimates of discharge (Eq. (10)). The discharge for a 2-year return period flow was determined for each point on the long profile. Stream power for each point was computed as a function of discharge and slope and then plotted against distance to obtain the stream power profile.

The main channel and major tributaries of the upper Hunter River catchment are characterised by a variable
pattern of stream power (Fig. 7). The maxima of stream power either lie in the headwaters or in midstream locations or, in some instances, a well-defined maximum does not exist. On the basis of these observations, the distribution of stream power in these streams is divided into five classes (Table 2a).

The Hunter trunk stream and most of its tributaries are characterised by multiple peaks. The Hunter River profile includes a narrow peak in the headwaters and a broad peak midstream. This broad, midstream peak results from regular input from large tributaries and occasional local slope variation in bedrock-controlled reaches. Similarly, Stewarts Brook has a peak in the headwaters as well as the midcatchment region. Pages Creek and Davis Creek contain peaks in headwater, midcatchment, and downstream locations. The latter peak is broad and is locally accentuated by a narrow peak in channel slope. Dart Brook does not contain well-defined peaks in its headwaters, but peaks occur in midcatchment and downstream areas.

Moonan Brook and Middle Brook are characterised by maxima in their headwater region, followed by a declining trend consisting of small peaks. A greater rate of decrease in stream power is noted for Moonan

Fig. 7. Downstream variation in stream power and its components using the long profile smoothing method. (A) Hunter River, (B) Pages Creek, (C) Stewarts Brook, (D) Moonan Brook, (E) Isis River, (F) Pages River, (G) Rouchel Brook, (H) Davis Creek, (I) Middle Brook, (J) Kingdon Ponds, and (K) Dart Brook. The stream power distribution of streams are characterised by variability in terms of the number of peaks, their location, and nature. u/s=upstream peak, m/s=midstream peak, and d/s=downstream peak.
Brook, which rises at a higher elevation (1500 m) in relation to Middle Brook (1200 m).

The Isis River exhibits peaks in midcatchment. The stream power distribution along Kingdon Ponds and Rouchel Brook is almost constant with no apparent peaks. These profiles contain several smaller peaks corresponding to reaches with higher channel slopes.

Table 2a
Appearance of stream power profiles along the tributary to its confluence with the trunk stream using the smoothing method

<table>
<thead>
<tr>
<th>Nature of profile</th>
<th>Name of channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream peak</td>
<td>Moonan Brook, Middle Brook</td>
</tr>
<tr>
<td>Midstream peak</td>
<td>Isis River</td>
</tr>
<tr>
<td>Downstream peak</td>
<td>Hunter River (u/s, m/s), Pages Creek (u/s, m/s, d/s); Stewarts Brook (u/s, m/s); Davis Creek (u/s, m/s, d/s); Dart Brook (m/s, d/s), Pages River (m/s, d/s)</td>
</tr>
<tr>
<td>Multiple peaks</td>
<td>Rouchel Brook, Kingdon Ponds</td>
</tr>
<tr>
<td>No distinct peak</td>
<td></td>
</tr>
</tbody>
</table>

u/s=upstream, m/s=midstream, and d/s=downstream.
Along all streams, the distribution of stream power closely matches the slope profile, highlighting the importance of subregional variation in channel slope. In most cases, the stream power profile does not follow the theoretical distributions produced by Knighton (1999), where single midcatchment maxima of stream power occurred. This may be a question of scale (i.e., that theoretical distributions should be applied only to full profiles, extending all the way to a common base level). To investigate this possibility, the distribution of stream power for each channel was extended to Muswellbrook, which incorporates some part of the Hunter River (Fig. 8).

The combined stream lengths were characterised by different distribution patterns to those demonstrated in Fig. 7 and Table 2a. However, significant variability exists along different tributaries (Table 2b). Only the Hunter River and Moonan Brook are characterised by multiple peaks. Four of the remaining channels are characterised by midcatchment peaks, whereas in the other five cases channels have peaks in their downstream region. In each of these latter cases, the downstream

Fig. 8. Downstream variation in stream power and its components using the long profile smoothing method with profiles extended to Muswellbrook. (A) Hunter River, (B) Pages Creek, (C) Stewarts Brook, (D) Moonan Brook, (E) Isis River, (F) Pages River, (G) Roucel Brook, (H) Davis Creek, (I) Middle Brook, (J) Kingdom Ponds, and (K) Dart Brook. The distribution of the extended profile is characterised by variability in terms of number of peaks, its location, and nature. u/s=upstream peak, m/s=midstream peak, and d/s=downstream peak.
peaks occur along the Hunter trunk stream itself. Thus, extending the stream power profile for all tributaries to a common base level does not provide a pattern similar to the theoretical profile of Knighton (1999). This deviation is further analysed through other approaches.

7.2. Curve fitting

Two patterns of stream power distribution were obtained based on the calculation of slope using first- and second-order fitted exponential curves (Fig. 9). In

Table 2b

Appearance of stream power profiles for channels extended to Muswellbrook using the smoothing method

<table>
<thead>
<tr>
<th>Nature of profile</th>
<th>Name of channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream peak</td>
<td>Isis River, Pages Creek (m/s broad peak), Stewarts Brook, Pages River, Rouchel Brook, Davis Creek, Kingdon Ponds, Middle Brook, Dart Brook, Hunter River (u/s, m/s broad peak), Moonan Brook (u/s, m/s)</td>
</tr>
<tr>
<td>Midstream peak</td>
<td></td>
</tr>
<tr>
<td>Downstream peak</td>
<td></td>
</tr>
<tr>
<td>Multiple peaks</td>
<td></td>
</tr>
<tr>
<td>No distinct peak</td>
<td></td>
</tr>
</tbody>
</table>

u/s=upstream, m/s=midstream, and d/s=downstream.
all cases, the first-order approximation of slope underestimates stream power in headwater reaches and overestimates it in the midcatchment region. This result suggests that the long profiles of all streams are characterised by more than one value of decay constant. Headwater parts of the channels are characterised by higher decay constant (and hence higher decay rate) compared to midcatchment and downstream parts of the channel. Hence, the decay rate of an assumed single exponential curve for the whole long profile lies between the decay rate of headwater and midstream reaches. Similar differences were observed by Knighton (1999) in the streams of the Trent River catchment in England. By contrast, the second-order approximation of stream power distribution fits well with the broad pattern of measured data from the DEM. Hence, the second-order exponential curve provides a better approximation to the slope data, generating a good fit with the broad pattern of measured stream power using the DEM. The minor fluctuations in the stream power profile reflect the effect of tributary contributions and subregional slope variations.

Fig. 9. Stream power distribution in the upper Hunter River catchment using the three approaches: long profile smoothing, curve fitting and theoretical model. The stream power distribution based on second-order exponential curves provides a broad pattern that is similar to the stream power profile after using the smoothing method. (A) Hunter River, (B) Pages Creek, (C) Stewarts Brook, (D) Moonan Brook, (E) Isis River, (F) Pages River, (G) Rouchel Brook, (H) Davis Creek, (I) Middle Brook, (J) Kingdon Ponds, and (K) Dart Brook.
Using the results from the second-order exponential curve fitting method, the distribution of stream power peaks was examined (Table 3). Not unlike the results from the long profile smoothing method, the distribution of stream power peaks has a variable pattern along different streams (Table 3).

**Table 3**
Appearance of stream power profiles using the second-order exponential curve fitting method

<table>
<thead>
<tr>
<th>Nature of profile</th>
<th>Name of channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream peak</td>
<td>Hunter River, Pages Creek, Moonan Brook, Middle Brook</td>
</tr>
<tr>
<td>Midstream peak</td>
<td>Isis River</td>
</tr>
<tr>
<td>Downstream peak</td>
<td>–</td>
</tr>
<tr>
<td>Multiple peaks</td>
<td>Pages River (u/s, m/s, d/s), Stewarts Brook (u/s, m/s), Roucnel Brook (u/s, d/s), Davis Creek (u/s, m/s), Dart Brook (m/s, d/s), Kingdon Ponds (u/s, d/s)</td>
</tr>
<tr>
<td>No distinct peak</td>
<td>–</td>
</tr>
</tbody>
</table>

*Fig. 9 (continued).*

u/s=upstream, m/s=midstream, and d/s=downstream.
Fig. 10. Theoretical downstream stream power distribution for streamlines of the upper Hunter River catchment. The distribution was based on large/near infinite channel lengths to assess the broad-scale pattern. The figure highlights the control of tributary length on the occurrence of predicted peaks in the stream power distribution.
When the distribution patterns derived using the second-order curve fitting method (Table 3) are compared with the distribution obtained from the smoothing method (Table 2a), the stream power distribution patterns for six streams (Moonan Brook, Middle Brook, Isis River, Davis Creek, Stewarts Brook, and Dart Brook) are similar, while only the upstream peaks in the stream power profile of the Hunter River and Pages Creek are consistent. Dissimilarities along the remaining three streams likely reflect the effect of subregional slope variations that occur when using the smoothing approach. For example, the subregional variability along Rouchel Brook causes dissimilarities in the pattern of peaks using the two approaches. In this case, the curve fitting method defines a broader distribution of stream power peaks, which was not clear in the earlier approach because of subregional variability. In the remaining two rivers, the Pages River and Kingdon Ponds, the curve fitting approach produced a slightly higher value for the first peak in comparison to the smoothing method.

7.3. Theoretical model

On the basis of its better fit, the second-order approximation of slope has been used to refine Knighton’s (1999) theoretical model of stream power. Thus, combining Eqs. (1), (5) and (14)

\[ \Omega = \gamma \cdot C \cdot L^d \cdot \left( x_1 e^{-\beta_1 L} + x_2 e^{-\beta_2 L} \right). \]  

(15)

Eq. (15) was used to generate a continuous smooth stream power profile. The relation between catchment area and channel length was explored for all streams of the upper Hunter River catchment and found to show deviation from Hack’s relation (Hack, 1957). Therefore, the catchment area–channel length relations for individual streams were generated. The resultant smooth profiles of stream power are plotted for first-order as well as second-order exponential curves (Fig. 10). Distributions were extrapolated to an effectively infinite length (500 km) to obtain the general theoretical distribution of stream power in different channels irrespective of their length.

The plots show significant deviation in the trend of stream power based on the second-order approximation of slope against that based on the first-order approximation. In general, the first-order stream power distribution suggests only single maxima at midstream locations, while the second-order curve shows the existence of two peaks, one in the headwaters and another at midcatchment.

In general, the midcatchment peak is much broader (80–100 km) than the headwater peak (typically only a few kilometres). The midcatchment peak generally occurs around 50–70 km downstream of channel origin. Thus, the actual length of the river is a major control on the appearance of stream power profiles. As all the tributaries join their respective trunk streams either before attaining a peak or just at the peak, a well-developed midcatchment peak cannot be observed on some tributaries (Fig. 10). However, significant variability exists in these results.

Three patterns of peaks can be detected: the presence of only a partially developed first peak (Isis River and Pages River), a partially developed second peak (Dart Brook, Middle Brook, and Rouchel Brook) or the presence of two equally well-developed peaks (Pages Creek, Stewart Brook, and Kingdon Ponds). Overall, streams with peaks in their headwater reaches are more common. Six channels (namely the Hunter River, Moonan Brook, Rouchel Brook, Davis Creek, Dart Brook, and Middle Brook) are characterised by this pattern.

The theoretical profile (Fig. 10) provides an indication of the broad character of stream power distribution. Profiles based on this theoretical distribution were compared with the stream power distribution obtained from the smoothing and curve fitting methods (Fig. 9). The second-order theoretical distribution provides a much better approximation of the stream power distribution that was generated using the long profile smoothing method. The theoretical stream power profiles also match well with stream power profiles produced using the curve fitting method. The streams that do not follow this trend are Kingdon Ponds and Dart Brook, in which the theoretical profile provides a better approximation of the profile generated using the smoothing approach. In these latter cases, the trend produced using the curve fitting method sits higher on the plot.

8. Discussion

8.1. Comparison of approaches

Results from the three approaches used to derive stream power distributions along long profiles produced broadly similar outcomes. However, some noticeably different trends emerge associated with the position of stream power maxima. The stream power profile produced through the smoothing process requires the least amount of transformation of data and provides a good approximation of stream power trends obtained from
DEM data. Apart from the catchment-scale pattern of stream power distribution, subregional variability is clearly shown using this method.

The stream power profile obtained using the curve fitting approach generates a profile through approximating the long profile variation by fitting different orders of exponential curves. It requires a greater amount of transformation in the data and provides a generalised pattern of stream power distribution. While this method masks any subregional slope variation, it does pick up broad peaks in the stream power trend. These broad patterns are useful for analysing catchment/subcatchment scale variability in stream power. In the upper Hunter River catchment, second-order curves provided a better approximation of the stream power trends. However, second-order curve fitting may not apply elsewhere, as the degree of fit may be different in different landscape settings. Therefore, the validity of the order of best-fit curve should be examined on a catchment-by-catchment basis.

The theoretical method also provides a broadly smooth pattern, but it removes almost all fluctuation due to subregional slope and tributary inflow effects. The approach represents the distribution in a mathematical way by providing equations of a generalized stream power distribution for any given river. As the theoretical stream power distribution based on second-order exponential curves matches well with the stream power distribution through the smoothing approach, it has significant potential in the mathematical modelling of catchment scale channel processes and morphology in settings that are characterised by limited regional variability.

The choice of method used to examine stream power trends along long profiles should be framed in terms of the resolution and purpose for which the data will be used. For example, if broad catchment–catchment patterns are being examined, a theoretical or curve fitting approach may be chosen such that trends in peak location can be isolated. Alternatively, if reach-scale river character and behaviour are being examined, the long profile smoothing approach may be considered more appropriate as the position of peaks along different rivers likely reflects the location of erosion, transportation, and deposition processes along the channel.

8.2. The relative role of discharge and slope in explaining estimated patterns of stream power distribution

The initial evaluation of stream power profiles and their downstream variability suggest the non-existence of any holistic rules defining the distribution of stream power. This variable trend in stream power profiles is similar to trends observed from other river basins in different geological settings (Lecce, 1997; Knighton, 1999; Fonstad, 2003; Reinfelds et al., 2004). Superposition of fitted curves based on theoretical analysis does however show some broader scale patterns.

Seemingly, stream power distribution along river courses in the upper Hunter River catchment is a combination of patterns at two scales, namely a broader pattern at the catchment scale and subregional variations along each river course. Stream power variability reflects a combination of these two patterns. The broader pattern is commonly characterised by two peaks, with one in the headwater region and a second peak in the midcatchment region. The bimodal distribution of stream power is associated with second order exponential long profiles. The magnitude and position of these peaks vary from river to river. This will have a strong influence on the capacity for geomorphic work and adjustment along these rivers.

### Table 4

<table>
<thead>
<tr>
<th>River</th>
<th>Discharge (m³/s)</th>
<th>Slope (1-km average value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hunter River</td>
<td>134.86 63.20</td>
<td>0.00809 0.01889</td>
</tr>
<tr>
<td>Pages Creek</td>
<td>29.42 14.28</td>
<td>0.01526 0.01796</td>
</tr>
<tr>
<td>Moonan Brook</td>
<td>24.60 11.26</td>
<td>0.04096 0.05922</td>
</tr>
<tr>
<td>Stewart Brook</td>
<td>32.45 14.80</td>
<td>0.01431 0.01668</td>
</tr>
<tr>
<td>Isis River</td>
<td>71.58 34.21</td>
<td>0.00717 0.00621</td>
</tr>
<tr>
<td>Pages River</td>
<td>107.03 62.86</td>
<td>0.00599 0.00600</td>
</tr>
<tr>
<td>Rouchel Brook</td>
<td>48.98 32.03</td>
<td>0.01156 0.01220</td>
</tr>
<tr>
<td>Davis Creek</td>
<td>26.01 12.69</td>
<td>0.01636 0.01947</td>
</tr>
<tr>
<td>Kingdon Ponds</td>
<td>53.05 22.54</td>
<td>0.00454 0.00517</td>
</tr>
<tr>
<td>Middle Brook</td>
<td>18.89 6.75</td>
<td>0.01263 0.01457</td>
</tr>
<tr>
<td>Dart Brook</td>
<td>58.81 36.15</td>
<td>0.00595 0.00837</td>
</tr>
</tbody>
</table>

* Ratio = standard deviation * 100/mean.
Fig. 11. Downstream distribution of variability in channel slope, discharge, and stream power. (A) Hunter River, (B) Pages River, (C) Dart Brook, and (D) Rouchel Brook. High stream power variability in headwater reaches is mainly due to high discharge variability, while in midstream and downstream reaches it is mostly due to high slope variability. HV=high variability and LV=low variability.
The bimodal distribution of stream power based on second-order exponential long profiles is a deviation from earlier understanding of single peak (midcatchment) stream power distributions (Knighton, 1999; Fonstad, 2003). The single peak stream power distribution was based on the generation of first-order exponential long profile curves (Knighton, 1999; Fonstad, 2003). However, current work shows that higher-order exponential curves provide a much better curve fitting to the long profiles and reduces curve fitting error significantly. The upstream peak in the resultant bimodal distribution highlights the erosion potential of headwater reaches, whereas the position of midcatchment peaks correlates well with the single peak stream power profiles based on first-order exponential curves.

Numerous narrow peaks at a scale of 1–5 km also characterise the stream power distribution, and these often exceed the magnitude of broader predicted peaks. These narrow peaks, which cause large variability in the appearance of the stream power distribution, closely follow the local slope.

Mathematically, Eq. (1) suggests that the changes in either discharge or channel slope should have an equal effect on variations in stream power. However, the similarity between the pattern of the stream power and slope profiles (Fig. 7) suggests that channel slope overrides discharge as the dominant control.

Variability in discharge and slope was analysed through computation of mean and standard deviation values of slope and discharge (2-year return period flood) for all channels. Statistical analysis shows a large scattering in slope data in comparison to discharge data (Table 4). Standard deviations of slope data are 86–233% of mean values, compared to 35–65% of mean values for discharge data. In this analysis, the significant variability in slope values is considered to be responsible for the variability in the stream power distribution. Admittedly, the use of catchment area–discharge relationships to derive discharge tends to average out any effects of variability in topography, surface roughness, or infiltration. Hence, discharge variability simply reflects variation in tributary contributions.

To examine this further, confluence points were examined in greater detail. Tributaries caused an increase of 14–83% in the discharge of the receiving stream. Relative increase in discharge is much higher (64–84%) where streams of broadly comparable discharge come together, i.e., Hunter–Pages River, Pages River–Isis River, and Rouchel Brook–Davis Creek. However, slope variability up- and downstream of confluences is only on the order of 1–3% of mean value. Thus, stream power will increase abruptly at confluence points largely from discharge variation. Therefore, along any long profile, the relative role of discharge and slope in explaining the variability in stream power is dependent on position along the long profile. At confluences, jumps in the stream power plot are a function of discharge contributions from the tributary system; whereas between confluences, variation in stream power is due to local-scale slope variability.

To examine this between-confluence variability in greater detail, downstream changes in slope variability were assessed along the Hunter trunk stream and three of its tributaries, namely Pages River, Dart Brook, and Rouchel Brook. In general terms, variability in slope along long profiles is much more pronounced than variability in discharge (Fig. 11). In contrast to intuitive expectations, slope variability is less pronounced in upstream reaches of all the streams. However, the percent variation in channel slope in areas of steeper, more rugged terrain is lower than in downstream reaches. This zone of lower variability in slope extends from around 8 (Rouchel Brook) to 30 km (Dart Brook) from source and corresponds with a minimum slope of around 0.01 m/m for the Hunter River and 0.007 m/m for the tributaries (i.e., percent slope variation is more pronounced along slopes <$0.01$ or <$0.007$ m/m for the trunk and tributary streams, respectively). On the other hand, discharge variability is more pronounced in upstream parts of the catchment, likely reflecting the large number of first-order tributaries. The occurrence of first-order tributaries is governed by a threshold of source area. As shown by Montgomery and Dietrich (1988), the threshold for requisite source area increases with decrease in local gradient. Thus, the effect of first-order tributaries will be less or non-existent in less steep downstream terrain, whereas it will be higher in steep headwater terrain and will cause large discharge variability.

Stream power variation can therefore be explained by variability in discharge in upstream areas and slope variability in midcatchment and downstream areas. The transition in this pattern occurs at slopes <$0.007$ m/m. In addition, large jumps in stream power at confluence points are due to sudden increases in discharge related to tributary contribution.

9. Conclusions

The methods used here to derive stream power plots are generic and can be applied in any setting. Results from the upper Hunter River catchment indicate that different approaches should be used for different purposes. Second-order exponential curve fitting and the-
oretical approaches successfully predict the broad-scale patterns in stream power distribution. These can be used for catchment/subcatchment-scale analysis/comparison and modelling work, respectively. In this instance, stream power peaks generally occur at upstream and midcatchment positions. As the midcatchment peak theoretically lies at a certain distance downstream, tributaries shorter than this particular length are not characterised by this peak.

The stream power distribution generated via the long profile smoothing approach provides high-resolution analysis of flow energy, which could be used to study reach-scale channel processes. This local-scale variability can be explained by variations in discharge and slope at different positions along the long profile.

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