Spatial variations of the fractal properties of seismicity in the Anatolian fault zones

Ali Osman Öncel\textsuperscript{a,b,*}, Ian Main\textsuperscript{b}, Ömer Alptekin\textsuperscript{a}, Patience Cowie\textsuperscript{b}

\textsuperscript{a} Istanbul University, Department of Geophysical Engineering, 34850 Avcılar, Istanbul, Turkey
\textsuperscript{b} Department of Geology and Geophysics, University of Edinburgh, West Mains Road, Edinburgh H9 3JW, UK

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Abstract

The Anatolian fault zones are seismically active strike-slip fault zones transcending the Anatolian plate in E–W and N–S directions. We investigate the spatial variations of seismicity along these zones in an attempt to investigate fault complexity along strike, quantified by the Gutenberg–Richter $b$-value and the fractal (correlation) dimension of earthquake epicentres, using the maximum likelihood method and the correlation integral, respectively. The investigation covers instrumentally recorded earthquakes of magnitude $M > 4.5$ occurring between 1900 and 1992. We find systematic spatial variations which may be related to structural or mechanical variability along strike. In particular the large change in strike at the northern apex of the North Anatolian Fault Zone is associated with the highest correlation dimension and lowest $b$-value for seismicity this century. The correlation dimension and $b$-value show a negative correlation with respect to each other, similar to results reported in other regional studies of Japan and southern California. This statistical correlation is stronger when more objective seismic zoning is carried out (based on number of events) rather than more subjective seismotectonic zoning in common use in seismic hazard analysis.

1. Introduction

The scale-invariant geometry of many natural fracture systems and earthquake populations suggests that fault structure and seismicity can often be described in terms of fractal sets (Mandelbrot, 1982). Examples include the fractal dimension of mapped fracture sets (Hirata, 1989a), the degree of fragmentation of the lithosphere (Turcotte, 1986) and the "roughness" of individual fault traces such as the San Andreas in California (Aviles et al., 1987; Okubo and Aki, 1987). Although not strictly a fractal dimension in the sense of Mandelbrot (1982), the Gutenberg–Richter $b$-value often used in seismic hazard analysis may also be at least indirectly related to a fractal dimension (Aki, 1981; King, 1983).

In principle the various fractal dimensions may be used as a quantitative measure of the degree of heterogeneity of seismic activity in fault systems. These in turn are controlled by the heterogeneity of the stress field and the pre-existing geological, mechanical or structural heterogeneity. However, fractal dimensions obtained by different methods generally reflect different aspects of the scale invariance, and need not be equal to or even positively correlated.

\* Corresponding author.

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with one another. For example, "the capacity dimension" $D_c$, estimated by box-counting methods (Feder, 1989), measures the space-filling properties of a fracture set with respect to changes in grid-scale (Hirata, 1989a). The power-law exponent $D$ of the fault length distribution, included in the more general definition of a fractal dimension by Turcotte (1989) and proportional to the seismic $b$-value, measures the relative proportion of large and small seismogenic faults (Aki, 1981; King, 1983) or cracks producing acoustic emission (Main et al., 1990). The correlation dimension $D_c$ (Grassberger and Procaccia, 1983) measures the spacing or clustering properties of a set of points, and has also been applied both to earthquake epicentres (Kagan and Knopoff, 1980; Hirata, 1989b) and to the hypocentre distributions of acoustic emissions in laboratory experiments (Hirata et al., 1987). This study examines the spatial variability of $b$ and $D_c$ in a zone dominated by a single type (strike-slip) of faulting, and compares the results obtained by the different methods. The zone of interest includes the Anatolian fault zones (AFZ) shown in Fig. 1.

The northward motion of the Arabian plate with respect to Eurasia gives rise to westward and eastward "escape" of the Anatolian blocks to give strike-slip motions, on the North Anatolian Fault Zone (NAFZ) in the north, on the East Anatolian Fault Zone (EAFZ) in the southeast, and on the Northeast Anatolian Fault Zone (NEAFZ) in the northeast (Ketin, 1969; McKenzie, 1972; Alptekin, 1973). Dextral slip on the NAFZ is known to have taken place since the Late Miocene (Şengör, 1979). The NAFZ is well defined morphologically from about 31°E, up to its junction with the EAFZ at Karlova, at 41°E (Allen, 1969). It has a major change in strike from an ENE to a WSW direction at 35°E, and a major splay at 37°E in the central part of the NAFZ (Fig. 1). The NAFZ has a total length of 1000–1100 km (Ambroseys, 1970; Allen, 1980) of which a total of 900 km (from Erzincan to the western end of the Mudurnu valley) has been broken by surface rupture this century. These events have occurred in a series of six large westward migrating earthquakes between 1939 and 1967 (Ambroseys, 1970; Barka, 1992). The measured offset of the fault also decreases westward before spreading into three strands in the Marmara and in the northern Aegean

![Fig. 1. Major tectonic map of Turkey and the surrounding regions. As a result of collision between the Arabian and the Eurasian plates the Anatolian and the Northeast Anatolian blocks are escaping sideways (arrows). The North Anatolian fault (NAF) and the East Anatolian fault (EAF), respectively, define the northern and southern east boundary of the westward moving Anatolian block (Barka and Kadinsky-Cade, 1988), and the Northeast Anatolian fault represents a northeastward extension of the EAF into northeastern Anatolia.](image-url)
sea regions, beginning from Adapazari, at 31°E (Barka and Kadinsky-Cade, 1988). Dextral strike-slip faulting along the NAFZ appears to continue eastward, beyond its triple junction (at 41°E) with the sinistral EAFZ, but is not as continuous as it is along the NAFZ (Jackson, 1992). The EAFZ starts from Karlova and extends with sinistral motion to the northern end of the Dead Sea Fault Zone (Lyberis et al., 1992). This paper presents the results of a study of a sequence of earthquakes that have occurred in the AFZ over a period from 1900 to 1992. These events are clearly related to the failure of the major faults, and hence provide an opportunity to examine the effect of spatial variations in fault zone complexity on the fractal properties of the seismicity in a zone dominated by a single type of faulting.

2. Method of analysis

We use the Gutenberg–Richter b-value and the fractal correlation dimension ($D_c$) of earthquake epicentre locations as independent measures of the scale-invariant properties of the size distribution and spatial clustering properties, respectively. The $b$-value is estimated by using the maximum likelihood method (Aki, 1965):

$$b = \frac{\log_{10} e}{\bar{m} - m_0}$$  \hspace{1cm} (1)

where $\bar{m}$ is the average magnitude and $m_0$ is the threshold magnitude for complete reporting of earthquake magnitudes. The 95% confidence limits for this estimate from a set of $n$ earthquakes are $\pm 1.9 b/\sqrt{n}$. The $b$-value ranges between 0.5 and 1.6 in the present study, implying a typical confidence limit of $\pm 0.1$--0.2 for a typical $n = 100$.

The fractal dimension based on the correlation integral (Grassberger, 1983; Grassberger and Procaccia, 1983) was used to estimate $D_c$. There are other approaches to the quantitative description of the spatial distribution of earthquakes, for example, the

![Map showing the epicentre distribution of earthquakes which occurred between 1900 and 1992 in Turkey. The data are split into five seismotectonic zones, labelled A–E. Justification for this structural zoning are given in the main text, Section 3.](image-url)
box-counting algorithm applied to a cloud of points (Sadowsky et al., 1984). However, the correlation integral method is often preferred due to its simplicity and reliability (Kagan and Knopoff, 1980; Hirata, 1989b; Rossi, 1990; Radulian and Trifu, 1991; Henderson et al., 1992). The correlation dimension $D_c$ is found from:

$$D_c = \lim_{r \to 0} \frac{\log C(r)}{\log r} \quad (2)$$

where $C(r) = N/n$ is the correlation integral, $N$ is the number of points in the particular analysis window separated by a distance less than $r$, and $n$ is the total number of points analysed. Here we also estimate the standard error $\sigma$ found by linear regression of $\log C$ against $\log r$. The percentage error in this calculation is in general smaller than that of the $b$-value because it is based on a regression involving $n(n - 1)/2$ individual two-point distances $r$ rather than $n$ magnitudes $m$. The angular distance $r$ in degrees between two events is calculated using the following formula (Hirata, 1989a,b):

$$r = \cos^{-1}\left[\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos(\phi_1 - \phi_2)\right] \quad (3)$$

where $(\theta_1, \phi_1)$ and $(\theta_2, \phi_2)$ are the colatitudes and longitudes of the two events. The correlation dimension is defined by the behaviour of the ratio $\log C(r)/\log r$ as $r$ tends to zero, and was estimated by fitting a straight line to a plot of $\log C(r)$ against $\log r$ (converted to a distance using $1^\circ = 111$ km) over a data range for the first 1.5 orders of magnitude for which the data were considered reliable. The lower bound may be determined by the epicentre resolution, and the upper bound by the influence of the finite size of the study zone (Kagan and Knopoff, 1980). In the present study the correlation plots in general exhibit scale invariance with parallel slopes in both the discrete and cumulative distributions between $0.7 < \log r < 2.2$, or $5 < r < 160$ km. These ranges were in part determined by using the central

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**Fig. 3.** Map of the location of seismograph stations in Turkey (after Sipahioglu and Alptekin, 1988).
difference formula, \( \frac{dC}{dr} = \frac{C(r + \Delta r) - C(r - \Delta r)}{2\Delta r} \), to obtain the local gradient \( \frac{dC}{dr} \). This difference procedure gave similar results to the overall line fit to Eq. (1).

3. The data

An earthquake catalogue for five major structural subdivisions of the AFZ, labelled A–E in Fig. 2, was compiled from the International Seismological Centre Data File and local catalogues for earthquakes recorded between 1900 and 1992. Zone A is the westward extension of the NAFZ, and includes the fault splays near the sea of Marmara and seismicity in the northern Aegean sea. Zone B is the central and eastern part of the NAFZ. It includes the major bend in the fault trace at the northernmost point of the NAFZ, and is bounded to the east by the junction with the EAFZ. Zone C is the eastward extension of

Fig. 4. Frequency–magnitude distributions for the five seismotectonic zones of Fig. 2. The dots represent the \textit{discrete} magnitude distribution, and the solid lines the \textit{cumulative} distribution.
Fig. 5. Diagrams showing the estimation of the correlation dimension by fitting a straight (dotted) line to the curve of log $C(r)$ against log $r$ (left-hand column) and local slope curves (right-hand column).
the NAFZ beyond this junction. Zone D is the EAFZ and zone E the NEAFZ defined above.

The earthquakes were observed by global networks including the WWSSN and its predecessors, and more recently by the national and local seismic networks shown in Fig. 3. The geographical distributions of these local and regional seismic stations are concentrated in northwestern Anatolia. Most of the (semi-permanent) stations in eastern Anatolia were set up after 1970 (Sipahioglu and Alptekin, 1988). The main improvement in seismological data in Turkey occurred after the WWSSN was set up in 1960. In order to assess the completeness of the catalogue, Fig. 4 shows the frequency–magnitude relation of events in the region with magnitudes greater than 3.0. The straight line on the discrete rather than cumulative distribution, plotted with a bin size of half a magnitude unit, indicates that the catalogue is substantially complete during the time period of interest for magnitudes above 4.5 (bin centred on magnitude 4.75). We used only the epicentral coordinates rather than the hypocentral ones in the analysis, partly because the seismogenic zone in this area is relatively shallow compared to its spatial extent, but mainly because, in common with many catalogues, the resolution of hypocentral depth is poor compared to that of the epicentre (Hirata, 1989b).

4. Results

We first obtained the correlation dimensions ($D_c$) of the distributions of earthquake epicentres and the $b$-values for the five subdivisions of the AFZ shown in Fig. 2, in order to investigate possible spatial variability due to the major structural properties of the AFZ along strike. This exercise was carried out in order to examine the effects of seismotectonic zoning on the analysis. In the first columns of Fig. 5, the fractal distributions of the earthquake epicentres of AFZ generally exhibit scale invariance in the cumulative statistics between $0.7 < \log r < 2.2$, or $5 < r < 160$ km, giving regression coefficients of the order $r = 0.98-1$ in the cumulative data over this range (1.5 orders of magnitude). The second column shows the local slopes from the central difference analysis for the AFZ. The largest variation of the

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**Fig. 6.** Diagrams showing the summary results obtained for the AFZ for zones A–E. A = west of the extension of the NAFZ; B = North Anatolian Fault Zone; C = eastward extension of the NAFZ; D = East Anatolian Fault Zone; E = Northeast Anatolian Fault Zone. (a) Correlation plots for the range 5 km $< r < 160$ km. (b) Spatial variability in $b$ and $D_c$, with error bars at one standard deviation. (c) Linear regression (dotted line) of $b$ on $D_c$. 

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local slopes is found for zone B (second row of Fig. 5), which also shows a systematic curvature in the cumulative curve. For consistency, all data were analysed over the same length scale ($5 < r < 160$ km).

In the correlation plots for zones B and D (rows 2 and 4 of Fig. 5), there appears to be a systematic break in slope observed about 1.3 ($r = 20$ km). In these cases we may therefore define two populations for (1) $r < 20$ km and (2) $20$ km $< r < 160$ km, with $D_c(1) > D_c(2)$. This may be an artefact of the smaller length scale range being inherently more sensitive to the use of epicentral data in a catalogue of shallow crustal earthquakes. Alternatively, it may be due to a real change in scaling properties, perhaps related to fault segmentation on this length scale, ultimately controlled by the finite width of the seismic zone. This is at least plausible since many earthquake scaling properties also show breaks in slope at length scales corresponding to the seismogenic width. In either case the observation is of greater clustering in population (1). This is not just due to the sampling effect of having a smaller length scale than population (2), but is manifest in the greater degree of scale-invariant clustering (higher $D_c$) in population (1). A linear regression found $D_c(1) = 1.98$ [$D_c(2) = 1.63$] for zone B and $D_c(1) = 1.73$ [$D_c(2) = 1.43$] for zone D. Earthquakes spaced at a distance $r > 160$ km are sensitive to the edge effect of the aerial extent of the catalogue, and at $r < 5$ km to errors in location, and hence plot in general off the main trend in Fig. 5. These data are not used in the subsequent analysis.

The collected results for the range $5 < r < 160$ km are shown in Fig. 6. The individual correlation plots for the different seismotectonic zones A–E, showing a statistically significant systematic spatial variability, are shown in Fig. 6a. A plot of the different correlation dimensions and $b$-values, together with errors expressed at one standard deviation, is shown in Fig. 6b. The negative correlation apparent in Fig. 6b between $b$ and $D_c$ is confirmed by regression in Fig. 6c. In common with previous studies (Hirata, 1989b; Henderson et al., 1992) this correlation is weak but negative ($r = -0.64$), consistent with a greater degree of clustering being

![Map showing zoning defined by the area containing one hundred earthquakes along the NAFZ, starting from the west. Each area is defined bounded along strike by a line of longitude and has the same width.](image_url)

Fig. 7. Map showing zoning defined by the area containing one hundred earthquakes along the NAFZ, starting from the west. Each area is defined bounded along strike by a line of longitude and has the same width.
Fig. 8. Magnitude–frequency distributions for the subdivisions of Fig. 7.
Fig. 9. Diagrams showing the estimation of the correlation dimension by fitting a straight (dotted) line to the curve of log $C(r)$ against log $r$ for the seven zones of Fig. 7.
associated with a lower $b$-value. The negative correlation would be much weaker without zone B, but nevertheless the spatial variability of $D_c$ (Fig. 6a, b) is statistically significant in all the zones.

Fig. 10. Diagram showing the summary results obtained for zones 1–7 along the NAFZ, presented in the same way as in Fig. 6.

In an attempt to investigate a more objective approach than the structural zoning of Fig. 2, the NAFZ was next divided into seven spatial subdivisions defined solely by the number ($n = 100$) of earthquakes in each, numbered from 1 to 7 along the W–E trend in Fig. 7. The frequency–magnitude distributions for the seven subdivisions are shown in Fig. 8. These also suggest that the individual catalogues are reasonably complete for earthquakes greater than 4.5 this century. The fractal dimensions of the earthquake epicentres and the $b$-values were calculated in the same way for the subdivisions of NAFZ, based on the plots shown in Fig. 9. The curves show here for zones 2, 3, 4 and 5 also show

Fig. 11. Plots of the relation between the seismic event rate density $N$ [$n = 100$ events divided by the size lateral distance (km) of the variable subdivisions] and the correlation dimension (a) and seismic $b$-value (b).
evidence of a break in slope at about 20 km, with local slopes changing from local values of \( D_c(1) \) of 1.85, 1.99, 2.12, 1.77 to regional values \( D_c(2) \) of 1.28, 1.15, 1.38, 1.22, respectively. \( D_c(1) \) is greater and \( D_c(2) \) smaller than the fractal dimension \( D_s \) of 1.42, 1.53, 1.71, 1.52 found, respectively, for zones 2, 3, 4 and 5 for the range \( 5 < r < 160 \) km. The individual correlation plots for the latter scale range are shown in Fig. 10a. The spatial variability of \( b \) and \( D_c \), again with an apparent negative correlation, is shown in Fig. 10b. Zone 4, containing the bend in the strike of the NAFZ, shows the largest correlation dimension and the lowest \( b \)-value. In this case a more significant negative correlation coefficient of \( r = -0.85 \) was calculated from the overall data (Fig. 10c).

Individual regression plots of the density of seismic events \( N \) [i.e., \( n = 100 \) events divided by the size lateral distance (km) along strike of the variable subdivisions] vs. \( D_c \) and \( b \) are shown in Fig. 11a and b, respectively. \( N \) is negatively correlated with \( D_c \) (\( r = -0.78 \)) and positively correlated with the \( b \)-value (\( r = 0.96 \)). Thus a high \( b \)-value or a low correlation dimension are associated with greater spatial clustering defined in terms solely of the density of events along strike. These observations are statistically comparable with the observed negative relation between \( D_c \) and \( b \) (\( r = -0.85 \)) observed in Fig. 10c.

5. Discussion

The ultimate cause of faulting and seismic activity on the Anatolian fault zones is the continuum northward movement of the Arabian plate towards Eurasia. As a result of this, the Anatolian block rotates and “escapes” to the west (Fig. 1). The geometry of the fault zones takes the form of a complex zone of segments and branches at different orientations to this stress field, in turn resulting in along-strike variations in normal stress which may be the ultimate cause of the along-strike variability in the seismicity statistics presented here. This interpretation is at least plausible because Velde et al. (1993) have already shown a systematic variation of the fractal dimension (calculated by a Cantor’s dust analysis) with confining pressure (and hence normal stress) in experimental studies (0.5–5 kbar) of fracturing in Barre Granite. The fractal dimension for all of the fractures in their samples increased systematically with confining pressure, and was associated with more distributed damage. In addition to this, they reported that if a high enough confining pressure is applied for a long period of time, the rock would show creep and plastic deformation with a fractal pattern of fractures. Hirata et al. (1987) showed that once creep is initiated the fractal (correlation) dimension of the associated acoustic emissions decreases in time through the three stages of primary, secondary and tertiary creep. Thus we have no unique interpretation of our results. They may either reflect long-lived spatial variations in normal stress, or simply different snapshots in time of an evolving process with much longer repeat times than the current timespan of the instrumental catalogue. In particular we see no systematic changes associated with the “westward migration” of seismic activity this century or the westward decrease in fault offset. However, it is interesting to note that the highest fractal dimension determined in our study (Fig. 10b) occurs in zone 4, where the trace of the NAFZ has its largest change in strike, and also splits into two main branches. This zone is also associated with a zone of creep behaviour reported by Dewey (1976) near to Çankiri.

Our study has also highlighted the problems caused by multifractal behaviour involving systematic changes in scaling behaviour, possibly due to fault segmentation controlled by the finite width of the seismogenic zone. Other examples of this type of behaviour have been noticed in geology, including Orford and Whalley (1983), who observed the existence of a “bifractal” character of the shape of sedimentary particles. They addressed three types of particle shape with respect to their fractal character. Type I has a unique fractal dimension related to a regular shape. Type II corresponds to convex fractal structure due to more irregular structure \( D(1) < D(2) \), where population 1 is the smaller range. Type III exhibits a concave nature due to particles with a very marked irregular edge \( D(1) > D(2) \). In some parts of the Anatolian fault zones we adopted a similar bifractal interpretation of the results and computed two fractal dimensions, with \( D(1) > D(2) \). \( D(1) \) is a “textural” fractal dimension sensitive to
the smallest scale-length of the studied object, whereas $D(2)$ is a “structural” fractal reflecting the macro-scale features of the object. The results presented here in Section 4 also suggest a bifractal interpretation, with a systematic increase in structural irregularity from zones 1 to 4 and decrease from zones 4 to 7 along the NAFFZ.

Our results are directly consistent with those of Hirata (1989a,b), who showed a negative correlation ($r = -0.77$) between the $b$-value and the fractal dimension of epicentres, measured by the same two-point correlation dimension in the Tohoku region of Japan over a period of 55 years (1926–1986). A similar result was obtained by Henderson et al. (1992) for the Riverside catalogue in southern California, for earthquakes greater than local magnitude 1.3 during the time period of 1970–1990. In the present study, we found that the same negative correlation is stronger ($r = -0.85$) for the objective zoning scheme based solely on the number of events, rather than the more subjective seismotectonic zoning scheme ($r = -0.64$). In contrast, Guo and Ogata (1995) found a positive correlation of $+0.59$ between $b$-value and fractal dimension using aftershock data. We note that their results are concerned with the behaviour of aftershock events and spurious. Since their results were produced by removing mainshocks from the catalogue, our results can not be compared with their results directly. This may be due either to differences in analysis (such as removing aftershocks), or to differences in the physical nature of the clustering (see fig. 2 in Main, 1992).

Finally there are possible artefacts included in the present results due to systematic temporal variations in the pattern of earthquake location and station coverage which are difficult to assess. However, any such fluctuations are likely to have affected the results in a similar way for each zone, and we have attempted to minimise such artefacts by choosing the largest events which are more likely to be completely reported. In the present comparative study the effect of any such artefacts is therefore more likely to affect the absolute values of the parameters rather than their relative values. Nevertheless, a more detailed study based on a more dense network with relatively uniform detection properties would be useful in future to investigate these problems further.

6. Conclusion

We have analysed seismicity data covering a period between 1900 and 1992 to study the spatial distribution of the Gutenberg–Richter $b$-value and the fractal (correlation) dimension of the earthquake epicentres in the Anatolian fault zones. We used the maximum likelihood method and the correlation integral method, and have concluded that there is a negative correlation between the $b$-value and the fractal dimension of earthquake epicentres along the Anatolian fault zones, similar to results presented elsewhere. The negative correlation is most significant when an objective zoning scheme is used ($r = -0.85$). The spatial variations of the latter parameters along the Anatolian fault zones indicate some important variability of seismotectonic properties along strike. These do not appear to be related systematically either to the westward migration of seismicity or the reported decrease in fault offset to the west. The EAFZ and the NAFFZ have an apparent bifractal character possibly related to the finite width of the seismogenic zone, but the greatest change in the fractal properties occurs in zone 4 at the northern apex of the NAFFZ, where the fault strike changes most rapidly.

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