LONG-TERM EARTHQUAKE RECURRENCE CONSTRAINED BY TECTONIC SEISMIC MOMENT RELEASE RATES

BY IAN G. MAIN AND PAUL W. BURTON

In a previous paper, Main and Burton (1984a) applied Information Theory to the problem of seismic hazard estimation using an earthquake magnitude catalog and the long-term seismic moment release rate (or slip rate) of geologically observed faults. In this letter, we address the problem of estimating long-term earthquake recurrence rates and discuss some of the implications of the distribution in more detail, with the important addition of equations for the propagation of uncertainties in each of the parameters. A slightly more general derivation of the distribution is also given, because uncertainties in one of the parameters of the magnitude moment distribution can now be numerically accounted for.

Several authors have noted that the superposition of different earthquake populations can greatly affect estimates of long-term recurrence rates of large earthquakes from short-term catalogs, e.g., Bäth (1981), Singh et al. (1983), and Main and Burton (1984b). This is especially true in areas where the time constant of the largest events is greater than the current 80 yr or so of instrumentally recorded earthquake data. In order to extrapolate recurrence statistics beyond this time scale for long-term earthquake prediction, it is often necessary to incorporate geological and geophysical data on fault movements over much longer periods of time (e.g., Anderson, 1979). If constraints on the maximum fault area and the local slip rate are available from a local seismotectonic model, then this can greatly reduce the uncertainty of extrapolation from a short-term earthquake catalog. Information Theory is one method of directly incorporating such long-term information from the seismic moment release rate, thereby reducing and quantifying this uncertainty of extrapolation and providing a physical link between crustal deformation rates and seismic hazard estimation.

Long-term information on seismic moment release rates are available from geophysical plate tectonic models or from more direct geological and geophysical investigation of currently active faults (e.g., Anderson, 1979). It is sensible to combine this information with contemporary instrumental data to improve any extrapolation to longer period behavior from this currently short-term data set. This can be done via the concept of a seismic moment release rate $M_0$ defined by

$$ M_0 = \mu A \dot{s}, $$

where $\mu$ is the rigidity, $A$ is the seismogenic fault area, and $\dot{s}$ is the long-term slip rate. This information can be combined with that from an earthquake magnitude catalog via the concept of maximizing an entropy (Jaynes, 1957; Berrill and Davies, 1980), $S$, defined in the magnitude interval $(m_c, \omega)$ and the seismic moment interval $(M_{0c}, M_{0\omega})$ by

$$ S(m, M_0) = \int_{m_c}^{\omega} \int_{M_{0c}}^{M_{0\omega}} p(m, M_0) \ln p(m, M_0) dM_0 dm, $$

where $p$ is a probability density, subject to constraints on the average magnitude.
\( \langle m \rangle \) and the average seismic moment per event \( \langle M_0 \rangle \), with

\[
\langle M_0 \rangle = \frac{M_0}{N_T},
\]

if \( N_T \) is the average number of events above or equal to \( m_c \) per unit time in the catalog. (Here, we have made the physically reasonable assumption that events below \( M_{oc} \) contribute a negligible amount to the total seismic moment release.) If we approximate from empirical observation

\[
M_0(m) = 10^{(A + Bm)},
\]

where \( A(\Delta \sigma) = \delta A \) depends on the range of possible stress drops in the region (Kanamori, 1978; Singh and Havskov, 1980) and \( B = 3/2 \) (Kanamori and Anderson, 1975), then the resultant probability density is

\[
p(m) = \frac{\exp[-\lambda_1 m - \lambda_2 M_0(m)]}{Z},
\]

where \( Z \) is the normalizing integral. \( \lambda_1 \) and \( \lambda_2 \) are Lagrangian undetermined multipliers whose values are uniquely set by \( \langle m \rangle \) and \( \langle M_0 \rangle \) (Main and Burton, 1984a). The cumulative frequency per unit time is then

\[
N(x \geq m) = N_T \int_{m}^{\infty} p(x) \, dx = 1/T,
\]

where \( T \) is the average interoccurrence time of events of size \( m \) or larger. Note that in decadic terms, the Gutenberg-Richter b value is \( b = \lambda_1 \log_{10}(e) \).

Using an analogy with statistical mechanics, it can be shown that the term in \( \lambda_2 \) represents a Boltzmann distribution of possible energy transitions and that in \( \lambda_1 \) corresponds to the geometric degeneracy of the source zone (Main and Burton, 1984a). If the density of the fault length distribution (Caputo, 1976) is given by \( d(1) \propto 1^{v-1} \) and \( M_0 \propto 1^3 \) for a self-similar model, then the b value \( b = BD/3 \) follows from equations (4) and (5) if we postulate \( D = v - 1 \) as the fractal dimension of the source zone (Aki, 1981; Mandelbrot, 1982; Main and Burton, 1984a). Thus, we expect \( 0.5 < b < 1.5 \) for corresponding dimension limits of one and three, respectively, if \( B = 3/2 \). This is supported by empirical observation, e.g., in the range of \( b \) values found in foreshock (0.35 – 0.61) and aftershock (0.63 – 1.13) sequences (Von Seggern, 1980). Thus, foreshocks appear to have a fractal dimension \( D \approx 1 \), and aftershocks \( D \approx 2 \), corresponding to stress concentration first at a point (or a few points) and after the main event on a broad plane. This observation is consistent with asperity or barrier models of earthquake rupture (e.g., see Aki, 1984).

The term in \( \lambda_2 \) allows a continuous range of different possibilities at high magnitudes. \( \lambda_2 > 0 \) is observed in the Central and Eastern Mediterranean (Main and Burton, 1984a) and in this paper for southern California, \( \lambda_2 = 0 \) in Greece (Báth, 1983), and \( \lambda_2 < 0 \) can occur if superposition is evident (Singh et al., 1983). By using the geological seismic moment release rate as a constraint, we can avoid
the bias inherent in choosing between the three possible types of behavior at the
highest magnitudes (Anderson and Luco, 1983), given that the maximum magnitude
may be estimated for the region of interest.

Figure 1 shows a frequency-magnitude plot for southern California for the time
interval 1932–1972, with a cut-off at $ML = 4.25$ in order to reduce problems of
incomplete reporting of smaller magnitudes (Main and Burton, 1984b). The right-
hand entry on the frequency graph corresponds to an average repeat time of $163 \pm
27$ yr found from trenching into the San Andreas fault at Pallet Creek (Sieh, 1978).

![Figure 1](example_url)

**Fig. 1.** Discrete frequency-magnitude statistics for southern California (1932–1972). The magnitude data
($ML$ and $Ms$) from the catalog are usually quoted to one decimal place, although the error is from about
$\pm 0.3$ to $\pm 0.5$ units. Note that this line is not due to an empirical fit to the data, but is inferred from the
parameters of Table 1. For $\lambda_s = 0$, the density and the discrete frequency ($F(m) = N_T \int_{-\infty}^{m_s} p(m) \, dm$,
with $\delta m = 0.1$) would form a straight line right up to the largest magnitudes; here, a slight curve down
at high magnitude is obtained.

(The uncertainty is a standard error in the mean). A seismic moment magnitude
range for the largest events of $M_w = 8.05 \pm 0.15$ results from the eight individual
interoccurrence times $t_i$ and equations (1) and (4) with $s = s(t = 3.2$ cm yr$^{-1}$) and
$\mu$, $L_{max}$, $W$, and $A$ being specified by the parameters in Table 1. The fault width $W$
is specified by the current seismogenic depth, and the characteristic fault length
$L_{max}$ of the major events is specified by the length of the currently locked (aseismic)
portion of the San Andreas fault in southern California, which last ruptured in 1857
with a magnitude $Ms > 7.8$ (Sieh, 1978; Abe and Noguchi, 1983). An estimated
upper bound $L_T$ to this length is assigned from the total length of the San Andreas
in southern California. A theoretical maximum possible magnitude of $M_w = 8.7$
results from a strike-slip model and $L_T$, $W$, $\mu$, of Table 1, so $8.2 < \omega < 8.7$ corresponds to $L_{\text{max}} < L < L_T$. One aspect of the magnitude uncertainty is immediately apparent on the diagram—the marked exaggeration of the points at half-magnitude intervals 4.5, 5.0, 5.5, etc.—which is due to a magnitude uncertainty of ±0.5 in the early years of the catalog. For example, almost all magnitudes in the catalog prior to December

<table>
<thead>
<tr>
<th>TABLE 1</th>
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<tbody>
<tr>
<td>SUMMARY OF INPUT AND OUTPUT PARAMETERS OF THE SOLUTION FOR THE PARAMETERS OF EQUATION (5) FOR SOUTHERN CALIFORNIA DATA</td>
</tr>
<tr>
<td>(a) From the Earthquake Catalog 1932–1972 of Hileman et al. (1973)</td>
</tr>
<tr>
<td>$m$: 4.25</td>
</tr>
<tr>
<td>$\langle m \rangle$: 4.738 (0.014)</td>
</tr>
<tr>
<td>$N_T$: 25.512 (4.368) per year</td>
</tr>
<tr>
<td>(b) From a Tectonic Model (Anderson, 1979; Main and Burton, 1984a)</td>
</tr>
<tr>
<td>$\mu$: 3 $\times$ 10$^6$ bars (30%)</td>
</tr>
<tr>
<td>$L_T$: 670 km</td>
</tr>
<tr>
<td>$I_{\text{max}}$: 400 km (25%)</td>
</tr>
<tr>
<td>$\Delta \sigma_{\text{max}}$: 70 bars</td>
</tr>
<tr>
<td>$W$: 15 km (30%)</td>
</tr>
<tr>
<td>$s$: 5.5 cm yr$^{-1}$ (20%)</td>
</tr>
<tr>
<td>$\Delta \sigma$: 50 bars (40%)</td>
</tr>
<tr>
<td>$M_0$: 16 (8) $\times$ 10$^{18}$ Nm yr$^{-1}$</td>
</tr>
<tr>
<td>(c) Output Parameters</td>
</tr>
<tr>
<td>$\lambda_1(\sigma_{\text{st}})$: 2.041 (0.061)</td>
</tr>
<tr>
<td>$\lambda_2(\sigma_{\text{st}})$: 0.040 (0.078)</td>
</tr>
<tr>
<td>$\langle \sigma_{\text{st}}^2 \rangle$: (−0.00207)</td>
</tr>
</tbody>
</table>

Notes:
- $L_T$ is the length of the seismic zone studied, i.e. the total length of the San Andreas fault in southern California. This is regarded as an upper limited to $L_{\text{max}}$ in calculating the upper bound for $\omega$.
- $W$ is the seismogenic fault width.
- $L_{\text{max}}$ is the maximum length of possible fault break constrained by present bends and inhomogeneities. The assumption is that the northern boundary is constrained by the creeping segment of the San Andreas fault and the southern boundary by the termination of the quiescent zone where the deformation zone branches out and becomes more complex.
- $\sigma_{\text{st}}^1$, $\sigma_{\text{st}}^2$, and $\sigma_{\text{st}}^3$ represent the covariance error resulting from equivalent uncertainties in $\langle m \rangle$ and $\langle M_0 \rangle$, with $\sigma_{\text{st}}^2 = 0$ due to independence.
- $A$ is correct for S.I. units.
- $\Delta \sigma_{\text{max}}$ is the maximum stress drop from $\Delta \sigma_{\text{max}} = \Delta \sigma + \delta \Delta \sigma$.
- All uncertainties are given in parentheses, with uncertainties in $A$, $M_0$, and $\omega$ resulting from propagation of uncertainties in the parameters in the left-hand column of (b).

1943 are rounded to these levels, except for aftershocks of the March 1933 Long Beach earthquake.

Table 1 summarizes the input parameters from the earthquake catalog of Hilemann et al. (1973) and a plate tectonic model for the area (Anderson, 1979; Main
and Burton, 1984a) together with the parameters and uncertainties resulting from numerical solution of equations for the expectations \( \langle m \rangle \) and \( \langle M_0 \rangle \). Uncertainties (in parentheses) result from: an assumed resultant uncertainty of 50 per cent in \( M_0 \) attributed to uncertainties in the tectonic model; standard errors in the mean values \( \langle m \rangle \) and \( N_\tau \) calculated from the earthquake catalog; a possible error in \( A \) reflecting a range of stress drops 30 to 70 bars from Singh and Havskov (1980); and the range of possible maximum magnitudes. The covariance term in the error matrix resulting from uncertainties in \( \langle m \rangle \) and \( \langle M_0 \rangle \) reduces the overall error because it is negative. This represents mathematically one aspect of the reduction in the error of extrapolation associated with inclusion of \( M_0 \) as a direct constraint. Formally, the error is taken to be

\[
\partial N^2 = \left( \frac{\partial N}{\partial \lambda_1} \right)^2 \sigma_{\lambda_1}^2 + \left( \frac{\partial N}{\partial \lambda_2} \right)^2 \sigma_{\lambda_2}^2 + 2 \left( \frac{\partial N}{\partial \lambda_1} \right) \left( \frac{\partial N}{\partial \lambda_2} \right) \sigma_{\lambda_1 \lambda_2} + \left( \frac{\partial N}{\partial A} \right)^2 \delta A^2 + \left( \frac{\partial N}{\partial N_\tau} \right)^2 \delta N_\tau^2 + \left( \frac{\partial N}{\partial \omega} \right)^2 \delta \omega^2,
\]

(7)

with a similar expression for the error in \( m(T) \) defined by equation (6), with \( N \) replaced throughout equation (7) by \( m(T) \). We have neglected the contribution of covariance terms involving \( A \), \( N_\tau \), and \( \omega \) and other higher order terms here and used the symbol \( \sigma \) to distinguish errors resulting from equivalent uncertainties in \( \langle m \rangle \) and \( \langle M_0 \rangle \) from uncertainties in \( A \), \( N_\tau \), and \( \omega \), which are characterized by the symbol \( \delta \).

Figure 2 shows the cumulative frequency relationship with errors in \( N(m) \) computed using this equation. Even though superposition of two characteristic earthquake populations is possible (arbitrarily designated "large" and "intermediate"-sized events seem to follow different distributions above and below \( M_S = 6.7 \)), the extrapolation still agrees with the medium-term data from trenching. It is interesting to note that the \( b \) value of the "large" earthquakes is 0.51 and that of the smaller ones is 0.89, corresponding speculatively to fractal dimensions of just over 1 and just under 2, respectively. Thus, the larger events appear to respond to fractures that are essentially one-dimensional, representing the boundary interaction between two thin plates at the San Andreas transform. This is also consistent with a fault model with constant width, where \( L \) is the only degree of freedom and \( L > W \) (Scholz, 1982). Using this model, the slip \( s = \alpha L \), and so

\[ M_0 = \alpha \mu W L^2, \]

(8)

which implies that, given \( W \approx 15 \text{ km}, \mu \approx 3 \times 10^{10} \text{ Nm}^{-2}, \alpha \approx 1.25 \times 10^{-5} \) for strike-slip earthquakes (Scholz, 1982), and \( A \) and \( B \) of the moment magnitude relation of Table 1, the transition magnitude of 6.7 \( M_S \) observed in Figure 1 corresponds to a fault length of 36 km.

Thus, the "intermediate"-sized events probably reflect smaller local readjustments to these larger plate rupturing stresses, with \( L < 36, W < 15 \), and since \( D < 2 \), these may occur in a broad deformation area around the main fault. This is apparent from the spread of epicenters in southern California as well as from these theoretical considerations. In this interpretation, the contribution of the fractal dimension corresponding to depth may be quite small, but this may be a reflection of the relatively simple geometry of a transform-type active boundary and need not apply.
to more complex inter- or intraplate seismic zones. It is also interesting to note that, for an aspect ratio $L/W \approx 2$ (Purcaru and Berckhemer, 1982) for the smaller shocks, the upper bound to such regularity would occur at $L \approx 30$ km, given $W \approx 15$ km. This is very near to the inferred break of slope in the magnitude distribution corresponding to $L \approx 36$ km. Thus, the position of the observed break of slope is consistent with Purcaru and Berckhemer's (1982) observation that large strike-slip earthquakes are different from other classes in that they have a markedly variable aspect ratio. In this case, this is a result of length being the only geometric degree of freedom for the largest events.

The long-term average repeat time of the largest events ($M_w \geq 7.9$) from the parameters of Table 1 and equation (6) is once every 157 yr, with an error range from possible variations in $\Delta s$, $W$, $L$, $s$, and $N_T$ of 87 to 281 yr, which corresponds quite well to the range of interoccurrence times found on trenching (55 to 275 yr with an average of 163 yr from Sieh, 1978). This good agreement, which is also

![Cumulative frequency-magnitude statistics.](image)

*Fig. 2. Cumulative frequency-magnitude statistics.* The cumulative frequency $N(x \geq m)$ is compared with the data. The most obvious improvement on Figure 1 is the natural smoothing and normalizing of the data points due to the combination of information at every point. $N \pm \delta N$ is given by equations (6) and (7) with the input parameters of Table 1. Large ($L$) and intermediate ($I$) earthquakes seem to follow different distributions as indicated by the dotted lines. Despite this apparent superposition, the long-term prediction of the largest events (whose observed occurrence is represented by the right-hand box) is still correct within the calculated uncertainty. This is important since the extrapolation of the short-term statistics, when constrained by the tectonic model, agrees with the medium-term hazard rate found by direct trenching in this case. The three types of information represented on this diagram are therefore compatible within their errors, implying a process which is highly stationary when several cycles are considered.
reflected in Figure 2, suggests that long-term prediction of seismicity rates by this method could usefully be applied where direct trenching is impossible, but information on slip rates is available. As a cautionary note, the range of times quoted takes no account of the fraction of the moment release rate due to creep, and as such they will underestimate the interoccurrence times in areas where this is important. For the present, the technique will be limited to plate boundaries, or intraplate areas with previous large-scale rupture. The good agreement for southern California reflects the fact that the fault here, as in many seismic gaps throughout the world, seems to have been "locked" in position since the last catastrophic event in 1857 (Sieh, 1978), so creep is probably relatively unimportant in this area.

Table 2 shows the magnitudes associated with average repeat times $T$ from equations (5) and (6)—the kind of information useful in earthquake hazard reduction through design engineering and insurance and regional planning. The incorporation of the information on $M_0$ and careful analysis of all sources of error by the method presented here should quantitatively improve confidence in the design loadings of buildings and communications systems whose lifetimes are expected to exceed the timespan of the available earthquake catalog.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$M_T$</th>
<th>$\sigma_{M_T}$</th>
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<tbody>
<tr>
<td>1</td>
<td>5.831</td>
<td>(0.089)</td>
</tr>
<tr>
<td>2</td>
<td>6.165</td>
<td>(0.092)</td>
</tr>
<tr>
<td>5</td>
<td>6.597</td>
<td>(0.096)</td>
</tr>
<tr>
<td>10</td>
<td>6.912</td>
<td>(0.098)</td>
</tr>
<tr>
<td>20</td>
<td>7.208</td>
<td>(0.098)</td>
</tr>
<tr>
<td>50</td>
<td>7.556</td>
<td>(0.099)</td>
</tr>
<tr>
<td>100</td>
<td>7.777</td>
<td>(0.128)</td>
</tr>
<tr>
<td>200</td>
<td>7.960</td>
<td>(0.213)</td>
</tr>
</tbody>
</table>

Finally, the statistical average interoccurrence time could be used in conjunction with the deterministic concepts of a seismic gap (McCann et al., 1979) and potential maximum strain energy release (Shimazaki and Nakata, 1980; Makropoulos and Burton, 1983) in order to forecast the likelihood of large future earthquakes for specific fault segments. Recently, Sykes and Nishenko (1984) and Jacob (1984) used average recurrence intervals and standard deviations from historical records or palaeoearthquake studies in order to estimate time-varying probabilities of rupture and hence the current hazard for the San Andreas fault and the Aleutian arc, respectively. This paper has shown specifically how the short-term historical record, which is often a starting point for these probabilistic models, might be improved by the addition of a deterministic constraint imposed by the longer term slip rate, in areas where trenching is difficult or impossible.

ACKNOWLEDGMENTS

This work was supported by the Natural Environment Research Council and is published with the approval of the Director of the British Geological Survey (NERC). We thank John Anderson for providing much of the information on the tectonic model for southern California and for some helpful suggestions and comments on the manuscript.


