Green’s function representations for electromagnetic interferometry

Applications for Ground Penetrating Radar

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Claerbout (1968) showed that the autocorrelation of an acoustic transmission response recorded in a one-dimensional configuration at the pressure free surface yields the reflection response. Weaver and Lobkis (2001) showed that the autocorrelation function of an acoustic field response is the wavefield response of a direct pulse-echo experiment in a three-dimensional configuration. The condition is that the wave field is diffuse, which in their case was generated by thermal noise. Based on the diffusivity of the wave field, many authors showed similar results also for crosscorrelations in open and closed configurations. Essentially Claerbout and Weaver and Lobkis showed the same result, but Claerbout showed it for a one-dimensional configuration and did not need a diffuse wave field. Later it was shown by Wapenaar (2002) and Derode (2003) that Claerbout’s principle could be extended to arbitrary three-dimensional media.

Here we derive similar representations of electromagnetic Green’s functions in open configurations for non-conductive media and for conductive media. Since usually interferometric techniques rely on conservation of total wave energy, cross-correlation type techniques cannot be used for recordings of wave phenomena where a substantial part of the wave energy is converted into heat. For electromagnetic waves in conductive media with relaxation, wave energy is dissipated, while for diffusive electromagnetic fields and stationary currents, the wave energy is zero. We show here that for both types of applications exact Green’s function representations can be obtained by convolving two recordings at two different locations using the reciprocity theorem of the time-convolution type. Hence we extend the notion of interferometry introduced by Schuster (2001) to include generating new responses from correlation of a signal with a time-reversed signal, which is represented by a time-convolution. For both types of electromagnetic interferometry we make simplifications to modify the representations such that they can be used for interferometry. We discuss the effects due to the simplifying assumptions and due to the presence of absorption and illustrate these discussions with numerical examples at the end of the paper.

Introduction

Correlation type interferometry

\[ \mathcal{J} \left( x, \omega \right) = \delta \left( x-x_0 \right) \delta \left( \omega \right) \]

and must be neglected.

Localizing the electric field receiver positions:

\[ J_{x}^{E_1} (x, x') = 0 \]

\[ J_{x}^{E_2} (x, x') = 0 \]

Definition of Green’s functions:

\[ E_{x}^{E_1} (x, x') = G_{x}^{E_1} (x, x') \]

\[ H_{x}^{E_1} (x, x') = G_{x}^{E_1} (x', x) \]

\[ E_{x}^{E_2} (x, x') = G_{x}^{E_2} (x, x') \]

\[ H_{x}^{E_2} (x, x') = G_{x}^{E_2} (x', x) \]

Convolution type interferometry

\[ \mathcal{R} \left( x, x', \omega \right) = \frac{1}{\mu} \left( \mathcal{E}^{E_1} (x, x') \right) \]

\[ \mathcal{R} \left( x, x', \omega \right) = \frac{1}{\mu} \left( \mathcal{E}^{E_2} (x, x') \right) \]

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Approximations for electromagnetic interferometry

Configuration I:

\[ G_{x}^{E_1} (x, x') \]

\[ G_{x}^{E_2} (x, x') \]

\[ G_{x}^{E_1} (x, x') \]

\[ G_{x}^{E_2} (x, x') \]

\[ G_{x}^{E_1} (x, x') \]

\[ G_{x}^{E_2} (x, x') \]

Convolution type interferometry

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Effects of approximations in configuration II:

The medium between the boundaries can be heterogeneous and anisotropic. All numerical examples are computed for this three-layer configuration.

Approximations for electromagnetic interferometry

We showed relationships that can be used to reconstruct the electric field Green's function at point A as if from a source at point B from crosscorrelations of wavefield quantities observed at A and B due to sources on a closed surface around the points, or when one of the points is outside the domain. The relationships were derived using a two-way correlation type wavefield reciprocity theorem. In the case with relaxation phenomena, the convolution type interferometry was used when one of the observation points is inside and the other outside the domain spanned by the boundary.

We showed that suitable approximations can be made and with numerical modeling results that for low loss media correlation type interferometry can be used, where some artefacts can occur that are similar to the artefacts caused by the dipole-to-monopole approximation. Approximations in convolution type interferometry always lead to artefacts in the time window of interest and must be used together with correlation type interferometry to identify the unwanted events in the final results.

Conclusions

References


