ABSTRACT

Passive seismic imaging is the process of synthesizing the wealth of subsurface information available from reflection seismic experiments by recording ambient sound with an array of geophones distributed at the surface. Cross-correlating the traces of such a passive experiment can synthesize data that is immediately useful for analysis by the various techniques that have been developed for the manipulation of reflection seismic data. With a correlation-based imaging condition, wave-equation shot-profile depth migration can use raw transmission wavefields as input to produce a subsurface image. For passively acquired data, migration is even more important than for active data because the source wavefields are likely weak and complex which leads to a low signal-to-noise ratio. Fourier analysis of correlating long field records shows that aliasing of the wavefields from distinct shots is unavoidable. While this reduces the order of computations for correlation by the length of the original trace, the aliasing produces an output volume that may not be substantially more useful than the raw data due to the introduction of cross talk between multiple sources. Direct migration of raw field data can still produce an accurate image even when the transmission wavefields from individual sources are not separated. I illustrate the method with images from a shallow passive investigation targeting a buried hollow pipe and the water table reflection. The images show a strong anomaly at the 1 m depth of the pipe and faint events that could be the water table around 3 m. The images are not so clear as to be irrefutable. A number of deficiencies in the survey design and execution are identified for future efforts.

INTRODUCTION

Passive seismic imaging is an example of wavefield interferometric imaging. In this case, the goal is the production of subsurface structural images by recording the ambient noisefield of the Earth with surface arrays of seismometers or geophones. The images produced with this technique are directly analogous to those produced with the familiar conventional reflection seismic experiment. Within the exploration seismic community, the words imaging and migration are often used synonymously. Likewise, this paper presents the processing of passive seismic data as a migration operation.
The idea of imaging the subsurface using subsurface sources was first introduced by Claerbout (1968). That work provides a one-dimensional proof that the auto-correlation of time series collected on the surface of the Earth can produce the equivalent to a zero-offset time section. Subsequently, Zhang (1989), used planewave decomposition to prove the result in 3D over a homogeneous medium. Derode et al. (2003) develop the Green’s function of a heterogeneous medium with acoustic waves via correlation and validates the theory with an ultrasonic experiment. Wapenaar et al. (2004), through one-way reciprocity, prove that cross-correlating traces of the transmission response of an arbitrary medium synthesizes the complete reflection response, i.e. shot-gathers, collected in a conventional active source experiment. Schuster et al. (2004) show that the Kirchhoff migration kernel to image correlated gathers is identical to that used to migrate prestack active data when one assumes that impulsive virtual sources are located at all receiver locations. In summary, it is now well established that time differences calculated by correlation are informative about the medium between and below the receivers.

To distinguish data collected due to subsurface sources from conventional reflection seismic, the former are named transmission wavefields, $T$, and the latter reflection wavefields, $R$. The first section of this paper explains the basic kinematics of extracting $R$ from $T$. Next, I introduce the use of shot-profile wave-equation depth migration to create a subsurface structural image without first correlating the traces from the transmission wavefields.

One important attribute of truly passive data, is that the bulk of the raw data is likely worthless. Useful seismic energy captured in the transmission wavefield could include random distributions of subsurface noise, down-hole sources, or earthquake arrivals. Assuming they are not happening continuously, and not knowing when they occur, the passive seismologist must continuously record. The ramifications of processing an entire passively recorded data volume rather than individual wavefields from separate sources is explored through Fourier analysis.

Finally, results from a small field experiment are presented. Passive data were recorded over several days on clean beach sand with a 2x2 meter array of 72 geophones. A hollow pipe was buried beneath the array to provide an imaging target. Several data volumes from different times of the day and various pre-processing strategies were imaged to compare to an active survey collected at the same location.

### TRANSMISSION TO REFLECTION WAVEFIELDS

Figure 1, simplified for clarity, shows the basic kinematics exploited in processing transmission data. The figure includes two recording stations capturing an approximately planar wavefront emerging from a two-layer subsurface. Panel (a) shows the raypaths associated with the direct arrival and one reflected both at the free surface and a subsurface interface. The second travel path (labeled reflection ray) has the familiar kinematics of the reflection seismic experiment if a source were excited at receiver one. The transmission wavefield is depicted in panel (b). Wavelet polarity is appropriate for direct arrivals and reflection. The three main features of transmission wavefields can be appreciated here. First, the exact timing of the arrival is unknown. Second, the phase and duration of transmitted energy are unknown and likely com-
plicated. Third, if the incident wavetrain coda is long, arrivals in the transmission record can interfere.

Figure 1: (a) Approximately planar arrival with rays showing important propagation paths for passive imaging. (b) Idealized traces from a transmission wavefield. (c) Shot-gather (reflection wavefield) synthesized using trace $r_1$ as the source. Many details are neglected for simplicity. Hopefully, these are explained satisfactorily in the text. [NR]

Choosing trace $r_1$ as the comparison trace, panel (c) depicts the correlation spikes associated with the arrivals in the data panel (b), where $\otimes$ is correlation. A solid line with linear move-out is super-imposed across the correlated traces corresponding to the direct arrival recorded at each receiver location. The dashed line on panel (c) has hyperbolic moveout. However, no correlation peak exists on the $r_1 \otimes r_1$ trace under the hyperbola. Not drawn, the second arrival on $r_2$ will have a counterpart on $r_1$ from a ray reaching the free surface farther to the left of the model. In fact, the correlations produced from a single planewave will produce another planewave.

However, each planar reflection is moved to the lag-time associated with a two-way trip from the surface to the reflector. Correlation removes the wait time for the initial arrival and maintains the time differences between the direct arrival and reflections. Summing the correlations from a full suite of planewaves builds hyperbolic events through constructive and destructive interference. Analyzing seismic data in terms of planewave constituents is a commonly invoked tool in seismic processing. Summing the correlations from incident planewaves is a planewave superposition process.

Passive seismic imaging is predicated on raypaths bouncing every which-way from every direction. Cartoons depicting the transmission experiment always leave something out that causes an inconsistency that needs more raypaths and receivers to explain. Unfortunately the trend continues nearly forever. Such a complication arises with the inclusion of a second reflector. The two reflection rays correlate with each other with a positive coefficient. The two travel paths share the time through the shallow layer, so they correlate at a lag (or time difference), equal to the two-way travel time through the deep layer. This correlation is not a problem however. Part of the energy of the direct arrival will have made an intrabed multiple within the deep layer. This event has the opposite polarity compared to the direct arrival after once changing its propagation direction from $\uparrow$ to $\downarrow$. The delay of its arrival at receiver $r_2$ compared to the direct arrival at receiver $r_1$ is also the two-way travel time within the deep layer. This correlation thus has the same lag as the one between the reflectors and opposite sign. Therefore, the internal multiple cancels potential artifacts of the correlation. This
shows the importance of modeling transmission data with a two-way extrapolator. Without all possible multiples, correlation artifacts will quickly overwhelm the Earth structure within the correlated output.

Cross-correlation of each trace with every other trace handles the three main difficulties of transmission data: timing, waveform, and interference. First, the output of the correlation is in lag units, that when multiplied by the time sampling interval, provide the time delays between like events on different traces. The zero lag of the correlation is the zero time for the synthesized shot-gathers. Second, each trace records the character and duration of the incident energy as it is reflected at the surface. This becomes the source wavelet analogous to a recorded vibrator sweep. Third, overlapping wavelets are separated by correlation.

To calculate the Fourier transform of the reflection response of the subsurface, $R(x_r, x_s, \omega)$, Wapenaar et al. (2004), proves

$$2\Re\{R(x_r, x_s, \omega)\} = \delta(x_s - x_r) - \int_{\partial D_m} T(x_r, \xi, \omega)T^*(x_s, \xi, \omega) \, d^2 \xi,$$

where $^*$ represents conjugation. The vector $x$ will correspond herein to horizontal coordinates, where subscripts $r$ and $s$ indicate different station locations from a transmission wavefield. After correlation $r$ and $s$ acquire the meaning of receiver and source locations, respectively, associated with an active survey. The RHS represents summing correlations of windows of passive data around the occurrence of individual sources from locations $\xi$. The symbol $\partial D_m$ represents the domain boundary that surrounds the subsurface region of interest on which the sources are located. The transmission wavefields $T(\xi)$ contain the arrival and reverberations due to only one subsurface source. To synthesize the reflection experiment exactly, impulsive sources should completely surround the volume of the subsurface one is trying to image. Alternatively, many impulses can be substituted with a full suite of planewaves emerging from all angles and azimuths as in the kinematic explanation above.

**DIRECT MIGRATION**

Artman and Shragge (2003) show the applicability of direct migration for transmission wavefields. Artman et al. (2004) provide the mathematical justification for zero-phase source functions. Shragge et al. (2005) show results for the special case of imaging with teleseisms. Direct migration of transmission wavefields requires an imaging algorithm composed of wavefield extrapolation and a correlation based imaging condition. Shot-profile (Claerbout, 1971) wave-equation depth migration, described in Appendix A, fulfills these requirements.

Shot-profile migration uses one-way extrapolators to independently extrapolate up-going, $U$, and down-going, $D$, energy through the subsurface velocity model. $U$, extrapolated acausally, is a single shot-gather. $D$, extrapolated causally, is a wavefield modeled to mimic the source used in the experiment. I will define the causal extrapolation operator, $E^+$, and the acausal operator, $E^-$. Wavefields are extrapolated to progressively deeper levels, $z$, by recursive ap-
The imaging condition combines the two wavefields to output the subsurface image. The imaging condition for shot-profile migration is defined as the zero lag of the cross-correlation of the two wavefields

\[ i_z(x_r) = \sum_{x_s} \sum_{\omega} U_z(x_r; x_s, \omega) D^*_z(x_r; x_s, \omega) . \]  

The sum over frequency extracts the zero lag of the correlation. The sum over shots, \( x_s \), stacks the overlapping images from all the individual shot-gathers.

Extrapolation in the Fourier domain across a depth interval is a diagonal matrix whose values are phase shifts for each frequency-wavenumber component in the wavefield. Correlation of two equilength signals in the Fourier domain has one signal along the diagonal of a square matrix multiplied by the second signal vector. As such, the two diagonal square operations are commutable. This means that the correlation required to calculate the Earth’s reflection response from transmission wavefields can be performed after extrapolation as well as at the acquisition surface.

Table 1 pictorially demonstrates how direct migration of transmission wavefields fits into the framework of shot-profile migration to produce the 0\(^{th}\) and 1\(^{st}\) depth levels of the zero-offset image. The correlation in the imaging condition takes the place of preprocessing the transmission wavefield. The summations over shot locations and frequency in equation 4 are omitted to reduce complexity. Note however, that the sum over shot locations is the same as the integral over source locations in equation 1. Also, after the first extrapolation step, using the two different phase-shift operators, the two transmission wavefields are no longer identical, and can be redefined \( U \) and \( D \). This is noted with superscripts on the \( T \) wavefields at depth. The depth axis of the image is filled by recursive extrapolation of the wavefields followed by extracting the zero lag of the correlation.

Figure 2 compares the image produced by migrating raw transmission wavefields to one where synthesized shot-gathers by correlation were migrated. Transmission wavefields from 225 impulsive sources across the bottom of the velocity model, Panel (c), were modeled with a two-way time-domain extrapolation program. The time sampling rate was 0.004 s, and the source functions were Ricker wavelets with dominant frequency of 25 Hz. Panel (a) is the image created by correlating each transmission wavefield (implementing equation 1) and migrating the shot gathers by the algorithm described by the left column of Table 1. Panel (b) was produced by migrating each transmission wavefield directly and summing the images as described by the right column of Table 1. They are identical to machine precision.

\( T \) is the superposition of \( U \) and \( D \). Extrapolating the transmission wavefield with a causal phase-shift operator propagates energy recorded at the free surface which reflects downward
Table 1: Equivalence of shot-profile migration of reflection data and direct migration of transmission wavefields. \( T(x_r, \xi, t) \) are the wavefields of equation 1. \( x_s \) has a similar meaning to \( \xi \). \( \sum_{x_s, \xi} \) and \( \sum_\omega \) produces the image \( i_z(x_r) \) for both methods. Only first and second levels of the recursive process are depicted.

Figure 2: (a) Correlation followed by migration. (b) Sum of all directly migrated transmission wavefields. (c) Velocity model used for modeling and migration.
to become the source function for later reflections. This allows the use of \( T \) in place of \( D \) in shot-profile migration. Extrapolating the transmission wavefield with an acausal phase-shift operator reverse-propagates the upcoming events reflected from subsurface structure. This allows the use of \( T \) in place of \( U \) in shot-profile migration.

In effect, the extrapolations re-datum the experiment to successively deeper levels in the subsurface at which the wavefields are correlated in accordance with the general theory of equation 1. The physics captured in the formulation of shot-profile migration remains the same regardless of the temporal or areal characteristics of the initial conditions in the wavefields.

The extraction of only the zero lag of the correlation for the image discards energy in the two wavefields that is not collocated. This includes energy that has been extrapolated in the wrong direction (since the same data is used initially for both \( U \) and \( D \) wavefields). Conveniently, the only modification needed to make a conventional shot-profile migration program into a transmission imaging program is to use \( T \) as \( D \) instead of a modeled wavefield.

Very important among the motivations for migrating transmission wavefields is the need to increase the signal-to-noise ratio of the output. If the experiment records only a small amount of energy, the synthesized gathers from correlation can be completely uninterpretable. The synthesized gather in the left panel of Figure 3 was produced the same 225 transmission wavefields used in Figure 2, though each was convolved with a random source function. The wavefields were individually correlated and summed. A few events centered around 4000 m, can be seen, but the gather is dominated by noise. In fact, this gather is full of useful energy hidden by the random source functions that the transmission wavefields were convolved with.

The right panel of Figure 3 shows an image produced by direct migration of the transmission wavefields followed by stacking the 225 images. An identical image, not shown, was produced by first correlating then migrating the synthesized gathers. Combining the weak redundant signal within each shot-gather with all of the others through migration produces a very good result despite the low quality of the shot-gather produced with the same data.

**TRULY PASSIVE DATA**

I differentiate passive seismic data from transmission wavefields when the timing of individual sources are unknowable and possibly overlapping. For passively collected transmission wavefields, the time axis and the shot axis are naturally combined. Field data can only be parameterized \( T_f(x_r, \tau) \) rather than \( T(x_r, \xi, t) \) as dictated in equation 1. Within \( \tau \), many individual sources, and the reflections that occur \( t \) seconds afterward, are distributed randomly across the total recording time. Both \( t \) and \( \tau \) represent the real time axis, though I will parameterize wavefields with the understanding that \( \max(t) \) is the two-way time to the deepest reflector of interest and \( \tau \) is the time axis from the beginning to the end of the total recording time. The difficulty in separating the data into constituent wavefields from a single source, parameterizing as function of \( \xi \), comes from not knowing the time separation between events and the possibility that they may overlap.

If it is impossible to separate field data into individual wavefields, transforming the long
data to the frequency domain has important ramifications on further processing. The definition of the discrete Fourier transform (DFT) for an arbitrary signal $f(\tau)$ can be evaluated for a particular frequency $\omega$,

$$F|_{\omega} = \text{DFT}[f(\tau)]|_{\omega} = \frac{1}{\sqrt{n_{\tau}}} \sum_{\tau} f(\tau) e^{-i \omega \tau}.$$  \hspace{1cm} (5)

where $n_{\tau}$ is the number of samples in $f$. The frequency domain dual variable of $\tau$ is $\omega$, and I will use $\omega$ for the frequency domain dual variable of $t$. If the long function $f(\tau)$ is broken into $N$ sections, $g_n(t)$, of length $\text{max}(t)$, the amplitude of the same frequency $\omega$ can also be calculated

$$F|_{\omega} = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \text{DFT}[g_n(t)]|_{\omega} = \frac{1}{\sqrt{N}} \text{DFT} \left[ \sum_{n=1}^{N} g_n(t) \right]|_{\omega}$$  \hspace{1cm} (6)

by simply changing the order of summation for convenience and scaling the result.

The only requirement for the equation above is that the two different length transforms contain the particular frequency being calculated (i.e. a frequency where $\omega = \phi$ is possible as dictated by the Fourier sampling theorem). The center equality of equation 6 (the sum of short transforms), shows that frequencies common to the two transforms are identical after a simple amplitude scaling. The right equality (the sum of time windows) shows that the long transform stacks the constituent windows at common frequencies. The two alternatives interpreted together show that subsampling the long transform aliases the time domain. This relationship between sampling and aliasing between a signal’s representation in two domains
is the Fourier dual to subsampling a time signal in order to reduce its Nyquist frequency at the risk of aliasing high frequencies.

Passive seismic field data, \( T_f(\tau) \), takes the role of the general signal \( f(\tau) \), and the \( g_n(t) \) are \( T(x_r, \xi, t)|_{\xi=n} \) (as well as background noise between sources). The center equality of equation 6 states that \( T_f(x_r, \omega) \) is a subset from \( T_f(x_r, \omega) \). The right equality states that

\[
T_f(x_r, t) = \sum_{\xi} T(x_r, \xi, t). \tag{7}
\]

With the relationships in equation 6, implementing a single correlation of passive field data does not produce \( R \), but some unknown quantity \( \tilde{R} \)

\[
\tilde{R}(x_r, x_s, \omega) = \frac{1}{N} T_f(x_r, \omega) T_f^*(x_s, \omega) \tag{8}
\]

where \( N \) is the number of length \( t \) time windows into which the total recording time can be divided, and \( T_f \) can be calculated with equation 7. The relationship is undefined at frequencies \( \omega \neq \omega \).

The left column of Figure 4 shows a processing flow of a simple 1D time domain signal with a zoomed in view of each trace (the first 32\textsuperscript{nd} of the axis) on the right. The top trace is the input signal \( T_f(\tau) \) with 4096 samples. It represents a long field recording including three transmission wavefields exploring a subsurface with a single layer. The middle trace is its autocorrelation calculated with all frequencies, \( \tilde{R}(\tau) = \text{DFT}^{-1}[T_f(\omega)T_f^*(\omega)] \). The bottom autocorrelation was calculated by first summing eight constituent windows of \( T_f(\tau) \) before DFT such that

\[
T_f(\omega) = \frac{1}{\sqrt{8}} \text{DFT}[\sum_{n=1}^{8} g_n(t)], \tag{9}
\]

where the number of samples in each \( g_n(t) \) was 512. An identical result can be produced by subsampling \( T_f(\omega) \) by 8. To facilitate plotting, the trace was padded with zeros.

Both versions of the output, middle and lower traces, have late correlation lags that do not correspond to physical raypaths to be found in \( R \). Although these could be removed in the Fourier domain by convolution with a sinc function, they are more efficiently windowed away in the time domain before further processing. From the bottom trace, notice that increasing the
decimation factor to 16 would alias the symmetric, acausal lags into the reflection response. To avoid this problem, the short time windows need to be more than twice as long as the time to the deepest reflection of interest. In this example the first two correlations of \( \tilde{R} \) equal \( R \). Producing the bottom trace involves \( O(10) \) less computations than the middle trace.

**Wavefield summation**

Unfortunately, the beginning of \( \tilde{R} \) is not always \( R \). The stacking of wavefields implicit in processing field data can be explored by considering field data composed of transmission wavefields \( a(x_r,t) \) and \( b(x_r,t) \) from individual sources. Like the input trace of Figure 4, they are placed on the field record at unknown times \( \tau_a \) and \( \tau_b \) such that

\[
T_f(\tau) = a \ast \delta(t-\tau_a) + b \ast \delta(t-\tau_b).
\]  

(10)

Correlation in the Fourier domain to implement equation 8 yields

\[
T_f^* T_f^* = AA^* + BB^* + AB^* e^{-i\omega(\tau_a-\tau_b)} + BA^* e^{-i\omega(\tau_b-\tau_a)}.
\]  

(11)

The sum of the first two terms is the result dictated by equation 1. The last two terms are cross talk. If \( |\tau_b-\tau_a| > \max(t) \), one term is acausal, and the other is at late lags that can be windowed away in the time domain. This was the case in the middle trace of Figure 4. If \( |\tau_b-\tau_a| < \max(t) \), the cross terms are included in the correlated gathers. In light of the previous section, the last scenario will be the predominant situation if the source delays \( \tau_a \) and \( \tau_b \) are truly random. Figure 4, however, is not a fair representation of field data because values for \( \tau_a, \tau_b \) were very carefully selected.

Redefine \( A \) and \( B \) as the impulse response of the Earth, \( I_e \), convolved with source functions, \( F \), which now contain their phase delays. As such, the cross terms of equation 11 are

\[
AB^* = (F_a I_e)(F_b I_e)^* = F_a F_b^* I_e^2 = F_c I_c^2.
\]  

(12)

Like the first two terms in equation 11, the cross terms contain the desired information about the Earth. However, the source function \( f_c \) included is not zero phase. These terms may be the other-terms or virtual multiples mentioned in Schuster et al. (2004). If the source functions are random series, terms with residual phase \( (F_c I_c^2) \) within the gathers will decorrelate and diminish in strength as the length of \( f \) and the number of cross terms increases. While we may hope to collect a large number of sources, it is probably unreasonable to expect many of them to be random series of great length. This is not probable if the noise sources have similar source mechanisms.

The inclusion of these cross terms in the correlation explains why \( \tilde{R} \) produced with equation 8 is not equivalent to \( R \) from equation 1. Virtual multiple events due to \( n_s \) sources will likely be more problematic than conventional multiples as every reflector can be repeated \( n_s!/(n_s-2)! \) times. The ratio of desirable zero phase terms, \( I_e \), to cross terms, \( F_j I_c^2 \), decreases as \( 1/(n_s-1) \) if the source terms contain the same wavelet.

Figure 5 shows the effect of the cross terms when calculating correlations. The figure is directly analogous to Figure 4, though with a more realistic input trace. First, there are
overlapping source-reflection pairs: the second source arrives at the receiver before the reflection from the first source. Second, the direct arrivals are spaced randomly along the time axis in contrast to Figure 4 where the direct arrivals were placed at samples 1, 513, and 2049. That contrivance allowed the summing of constituent time windows 256 or 512 samples long without introducing residual phase functions as described in equation 12.

Figure 5: Right column is 32x zoom of left beginning at $t = 0$. (top) Idealized signal of three identical subsurface sources. First two source-reflection pairs overlap. (middle) Autocorrelation. (bottom) Autocorrelation performed after stacking 8 constituent windows. Zero values are padded on the bottom trace to facilitate plotting.

The result desired by a passive seismologist trying to produce a zero-offset trace from $R$, bottom trace Figure 4, can not be produced from the top trace in Figure 5. The middle trace was correlated with input $T(\sigma)$. The bottom autocorrelation was computed after first stacking eight constituent windows. In contrast to the two autocorrelations in Figure 4, not even the early lags are the same. The presence of more aphysical correlations in the middle trace for this example has caused the symmetric peaks at late lags to be aliased into the early lags of the bottom result at this level of decimation. Both methods produce the wrong result at almost all times. They are however correct and identical at zero lag.

This analysis shows that without invoking long, purely random source functions, a single correlation of passive data will not model shot-gathers from a reflection survey. In the next section, 2D synthetic data are used to show the extent of the problem and the ability of direct migration to produce a correct subsurface image despite the summation of wavefields.

Examples with synthetic data

To demonstrate the effects of wavefield summation, two synthetic acoustic passive data sets are used. Transmission wavefields from 225 impulsive sources across the bottom of the velocity models were propagated with a two-way extrapolation program as before. To then simulate a passive recording campaign, a unique source function was convolved with each wavefield before summing all of them together. The total length of the source function trace mimics the duration of the recording campaign. The shape, time location, and length of the wavelet used within the source function trace, otherwise zero, embodies my assumptions on the nature of the ambient subsurface noisefield. The source functions incorporate many of the features of the toy example in Figures 4 & 5.

Figure 6 shows synthetic data from a model containing two diffractors in a constant velocity medium. Panel (a) is a transmission wavefield from a subsurface source below the far
left of the model, while the source in panel (b) was below $x = 5000$ m. Source wavelets were 50 ms long, with dominant frequency of 25 Hz, and placed on the time axis to align their direct arrivals. This experiment is the 2D analog to Figure 4. The total length of the source functions were the same as the modeled time axis. Panel (c) is the sum of all 225 similar wavefields from shots below the entire width of the acquisition. The coherent summation of the direct arrivals makes a strong planewave at $t \sim 0.6$ s, and the diffractors are well captured through constructive interference. Cross talk between the wavefields has been canceled through destructive interference. The summed wavefield below the strong planewave is the same as would be collected from a zero-offset reflection experiment. Correlation of the traces in panel (c) does not produce shot-gathers. Instead, each $x_s$ plane from $\bar{R}$ contains a series of planewaves that completely mask the diffractors. The slice from the correlated volume $x_s = x_r$ is however interpretable. The autocorrelation produces a result similar to panel (c) where the planewave is moved from $t \sim 0.6$ s to zero lag.

Figure 6: (a) Transmission wavefield from a source below 1200 m in a model containing two diffractors. (b) Transmission wavefield from source below 5000 m. (c) Sum of 225 modeled wavefields.

In contrast, Figure 7 shows synthetic data from the the same model with the addition of random phase delays for the source functions throughout the experiment. Panels (a)-(c) correspond to the same panels in the previous figure. The strong planewave and coherent diffractors from Figure 6 have been replaced by an uninterpretable superposition. Correlating traces from this wavefield returns $\bar{R}$ that is completely unusable rather than the reflection wavefield $R$. This is the 2D equivalent to Figure 5.

Data was also synthesized through a model containing two synclines. Source functions were identical to those used for the diffractor exercise above. Figure 8 shows summed wavefields with, panel (a), and without, panel (b), correcting for the onset time of the 225 subsurface sources. Again, correlating the superposition of wavefields, from either panel, returns an unusable data volume rather than shot-gathers from $R$. 
Figure 7: (a) Transmission wavefield from a source below 1200 m in a model containing two
diffractors. (b) Transmission wavefield from source below 5000 m. (c) Sum of all wavefields.

Figure 8: (a) Perfectly stacked shots from a double syncline model. (b) Stack of wavefields
with random onset times.
plane-wave migration
\[
\left( \sum_{x_s} U_{z=0}(x_r; x_s, \omega) \right) \otimes \left( \sum_{x_s} D_{z=0}(x_r; x_s, \omega) \right) = \left( \sum_{x_s} T_{z=0}(x_r, \omega) \right) \otimes T_{z=0}(x_r, \omega)
\]

wavefront imaging

\[
\left( \sum_{x_s} E^{-}(x_r; x_s, \omega) \right) \otimes \left( \sum_{x_s} E^{+}(x_r; x_s, \omega) \right) = \left( \sum_{x_s} T^{-}_{z=1}(x_r, \omega) \right) \otimes T^{+}_{z=1}(x_r, \omega)
\]

\[
\downarrow U_{z=1}(x_r, \omega) \otimes D_{z=1}(x_r, \omega) = T^{-}_{z=1}(x_r, \omega) \otimes T^{+}_{z=1}(x_r, \omega)
\]

Table 2: Equivalence between direct migration of passive field data and simultaneous migration of all shots in a reflection survey. Only first and second levels of the iterative process are depicted. \( \sum_{\omega} \) produces the image \( i_z \) for both methods.

**DIRECT MIGRATION OF PASSIVE DATA**

Using equation 8 to correlate field data (not being able to collect \( T \) as a function of individual source functions), we cannot process the result of the correlation with conventional reflection data tools. Without knowing the exact timing of all the source functions, it is not possible to completely eliminate all time delays as shown in equation 11. However, the field data can still be migrated with a scheme that includes extrapolation and a correlation imaging condition.

Shot-profile depth migration produces the correct image if the source wavefield, \( D \), is correct for the data wavefield, \( U \). Shot-profile migration becomes planewave migration if all shot-gathers are summed for wavefield \( U \), and a horizontal planar source is modeled for wavefield \( D \). The wavefield in Figure 6c could be successfully imaged with a planewave at \( t = 0.6 s \) modeled for the wavefield \( D \).

This method can introduce cross talk between the individual experiments while attempting to process them all together. Complete areal distribution of sources will cancel the cross talk through destructive interference and synthesize a zero-offset data acquisition. The information lost in this sum is the redundancy across the offset axis. For simple structural imaging, the zero-offset image is satisfactory, though for more complicated mediums and further processing a full plane-wave migration can be implemented (Sun et al., 2001; Liu et al., 2002).

Table 2 pictorially demonstrates how direct migration of field passive data fits into the framework of plane-wave migration analogous to Table 1. Moving the sum over shots in the imaging condition, equation 4, to operate on the wavefields rather than their correlation, changes shot-profile migration to planewave migration where a horizontal planar source is modeled for wavefield \( D \). As shown above, the Fourier transform of passive field data similarly sums the shot axis. The source function built by this action will have some temporal topography in this case as compared to planewave migration for the controlled experiments.

Figure 9 shows zero-offset images produced by direct migration of the data shown in Figure 8 respectively. The quality of panel (a) is equivalent to a zero-offset reflection migration. Dipping reflectors and sharp changes in slope produce migration tails. Panel (b) is a remarkably clean image given the appearance of the input data, though not as high quality as its counterpart. A faint virtual multiple mimicking the first event can be seen at \( z = 350 \) m. The
physical multiple from the first reflector at $z = 485$ m is very dim (which actually highlights the second reflector that gets masked in the previous panel).

![Figure 9](image)

Figure 9: (a) Zero-offset image produced by direct migration of data in Figure 8a. (b) Zero-offset image produced by direct migration of data in Figure 8b.

The length of the source functions used to synthesize the data were equal to the modeling time of the transmission wavefield. Thus, summing the wavefields as dictated by equation 7 does not allow for the requirement that the wavefields should be at least twice as long as $\max(t)$ to avoid aliasing the symmetric acausal lags. The short time axis also allows late time events to wrap around the time axis when applying the random phase delays. For these reasons, the image in Figure 9b represents a worst-case scenario of careless data preparation.

FIELD EXPERIMENT

Cross-correlating seismic traces of passively collected wavefields has a rich history pertaining to the study of the sun (Duvall et al., 1993). On Earth thus far, only two dedicated field campaigns to test the practicality of passive seismic imaging can be found in the literature: Baskir and Weller (1975), and Cole (1995). Neither experiment produced convincing results. With the hope that hardware limitations or locality could explain their lack of success, I conducted a shallow, meter(s) scale, passive seismic experiment in the summer of 2002. Seventy-two 40 Hz geophones were deployed on a 25 cm grid on the beach of Monterey Bay, California linked to a Geometrics seismograph. The experiment was combined with an active investigation of the same site using the same recording equipment and a small hammer (Bachrach and Mukerji, 2002). A short length of 15 cm diameter plastic pipe was buried a bit less than one meter below the surface. The array was approximately 100 meters from the water's edge. The water table is approximately three meters deep. The velocity of the sand, derived from the active survey, was a simple gradient of 180 to 290 m/s from the surface to the water table, and then 1500 m/s.
Figure 10 shows the time-migrated active source image with a clear anomaly associated with the hollow pipe and the water table. A simple RMS gradient velocity to the water table was used for imaging. The high quality of the beach sand allowed usable signal to as high as 1200 Hz for that survey.

Figure 10: In-line, x, and crossline, y, time migrated active seismic image. The hollow pipe causes an over-migrated anomaly at 12ms, 19m in the inline (X) direction. A strong water table reflection is imaged at 28ms. After Bachrach, 2003.

Passive data was collected over the course of two days two weeks later. Due to the limitations of the recording equipment, only one hour of data exist from the campaign. The seismograph was only able to buffer several seconds of data in memory before writing to a file. The time required to write, reset and re-trigger happened to be about 5 times greater than the length of data captured depending on sample rates. Data was collected at several sampling rates. Through the course of the experiment, we found it possible to fly a small kite (plastic grocery bag) that would continuously move the triggering wire over the hammer plate to trigger the system automatically as soon as it was ready to record. The individual records were then spliced together along the time axis to produce long traces. The gaps in the traces do not invalidate the assumptions of the experiment as long as the individual recordings are at least as long as the longest two-way travel time to the deepest reflector.

Because the array was only eight by nine stations, shot-gathers produced by correlation, even when resampled as a function of radial distance from the center trace, had too few traces to find consistent events. Migrating the data, as described above, provides both signal to noise enhancement, as well as interpolation. In this case, five empty traces were inserted between the geophone locations for processing, as shown in Figure 11. As the data are extrapolated through the velocity model, the energy on the live traces moves laterally across to fill the empty traces. After the distance propagated is approximately equal to the separation between live traces, wavefronts have coalesced into close approximations to their expression if the data
were first interpolated.

![Figure 11: A small time window of in-line and crossline sections of a raw passive transmission wavefield inserted on a five times finer grid for migration.](image)

Data were collected to correspond to distinct environmental conditions through the course of the experiment. Afternoon data was collected during high levels of cultural activity and wind action. Night data had neither of these features, while morning data had no appreciable wind noise. In all cases the pounding of the surf remained mostly consistent. By processing data within various time windows, it was hoped that images of the water table at different depths could be produced. However, given the 2 m maximum offset of the array, ray parameters less than 17° from the vertical would be required to image a 3 m table reflector at the very center of the array. Very little energy was captured at such steep incidence angles. Had we not been so careful not to walk around the array during recording, this might not have been the case.

Figures 12-14 show the images produced during the different times of the day. Approximately five minutes of 0.001 seconds/sample data were used to produce each image by direct migration. Usable energy out to 450 Hz is contained in all the data collected. Abiding by the 1/4 wavelength rule, and using 200 m/s with 400 Hz, the data should resolve targets to ∼0.125 m. Other data volumes corresponding to various faster and slower sampling rates were processed, though these results are the most pleasing.

Pre-processing in most cases consisted of a simple bandpass to eliminate electrical grid harmonics, as the higher octaves carry the only useful signal considering the low velocity of the beach sand and the small areal extent of the array. Figure 12 is the image produced mid-day. The two panels are the x and y sections corresponding to the center of the buried pipe. Figure 13 was produced with data from around midnight. The image planes are the same as for the previous figure. One dimensional spectral whitening was also tried, though the simple application remained stable only during the night acquisition. Figure 14 was produced
with the whitened version of the data used for the previous figure. Notice the instability at shallow depths before the wavefront healing has interpolated across the empty traces. Data collected in the morning did not yield appreciably different images from the night data to warrant inclusion.

Figure 12: Migrated image from passive data collected during the windy afternoon. In-line and crossline depth section extracted at the coordinates of the buried pipe. [daybp] [NR]

All output images contain an appreciable anomaly at the location of the buried pipe. Complicating the interpretation of the results, the ends of the pipe were not sealed before burial. After two weeks under the beach, it is impossible to know how much of the pipe was filled, which would destroy the slow, air-filled target. Future experiments would also incorporate target with a severe angle that could clearly stand out.

In the whitened night data image and the bandpassed day data image, there is hint of a reflector at depth that could be the water table. High tide on that day was at 4:30 in the afternoon, and fell to low tide at about 8:30 pm, and thus the relative change of this hint of a reflector is consistent. However, due to the limitations of the array discussed above, and the lack of strength and continuity along the crossline direction, I do not hold this to be a very reliable interpretation.

Discussion

Draganov et al. (2004) systematically explores the quality of a passive seismic processing effort as a function of the number of subsurface sources, the length of assumed source functions, and migration. Rickett and Claerbout (1996) identify increasing the signal-to-noise ratio by the familiar $1/\sqrt{n}$ factor where $n$ can be time samples in the source function, or number of subsurface sources captured in the records. Migration sums the information each shot-gather contains about a specific subsurface location to the same point in the image. Therefore, while
Figure 13: Migrated image from passive data collected during the night. In-line and crossline depth section extracted at the coordinates of the buried pipe.

Figure 14: Migrated image from passive data collected during the night. One dimensional spectral whitening applied before migration to the same raw data used in Figure 13. In-line and crossline depth section extracted at the coordinates of the buried pipe.
a correlated gather may not look like it contains useful information, migrating that data can produce interpretable results.

The direct controls available to increase quality of passive seismic effort are the length of time data is collected, and the number of receivers fielded for the experiment. If the natural rate of seismicity within a field area is constant, accumulation of sufficient signal dictates how long to record. Not surprisingly, increasing the total length of time of the source traces for the synthetic data described above does not change the quality of the output. If all the sources are used with the same source functions, this only adds quiet waiting time between the events that contributes neither positively nor negatively to the output. This experiment implies a changing rate of seismicity. When interpreting the increase in signal by factor $1/\sqrt{t}$ with application to short subsurface sources, $t$ represents the mean length of the source functions rather than total recording time. Assuming some rate of subsurface sources associated with each field site, the total recording time will control the quality of the output by $1/\sqrt{n_s}$ where $n_s$ is the number of sources captured.

Another method to increase the quality of the experiment is to migrate more traces. Migration facilitates the constructive summation of information captured by each receiver in the survey. Therefore, more receivers sampling the ambient noisefield results in more constructive summation to each image location in the migrated image. In this manner migration increases the signal-to-noise ratio of a subsurface reflection by the ratio $1/\sqrt{r}$, where $r$ is the number of receivers that contain the reflection. This allows the production of very interpretable images despite the raw data or correlated gathers showing little promise.

Definitive parameters for the numbers of geophones required, and sufficient length of time to assure quality results for a passive seismic experiment are ongoing research topics as few field experiments have yet been analyzed. It is clear however, that an over-complete sampling of the surficial wavefield is required, and that the length of time required will be dictated on the activity of the local ambient noisefield. Considering the layout of equipment, over-complete sampling means that more receivers are better, and areal arrays are much better than linear ones. This can be understood by considering a planewave propagating along an azimuth other than that of a linear array. After the direct arrival is captured, the subsequent reflection path pierces the Earth’s surface again in the crossline direction away from the array. With a 2.5D approximation, the apparent ray parameter of the arrival will suffice given an areally consistent and planar source wave. Because the true direct arrival associated with a reflection travel-path was not recorded, the possibility of erroneous phase delays and wavelets could distort the result.

**CONCLUSION**

If time-localized events are present, such as teleseismic arrivals, one can process small time windows when sure of significant contribution to the image. Without knowledge of if or how many sources are active within the bulk of passive data, long correlations of the raw data are an inevitable approximation, equation 8, to the rigorous derivation, equation 1. Fortunately, first aliasing the short time records reduces the computation cost for a DFT by $1/n_\tau$ where $n_\tau$
is the number of samples in the long input trace. The trace length will be $O(10^7)$ for just one day of data collected at 0.004 s sampling rate.

The aliasing implicit in not separating individual wavefields for processing sums the source functions within the output. This superposition does not produce $R(x, x', \omega)$ by correlation if source functions are not random. Instead, passive data can only be processed by direct migration with an algorithm that can accept generalized source functions (parameterized by space and time), and uses a correlation imaging condition. Both of these conditions are enjoyed by shot-profile migration. Direct migration also saves substantial computation cost since $O(n^2)$ traces are produced by cross-correlating $n$ receivers from a passive array.

Migrating all sources at the same time removes the redundant information from a reflector as a function of incidence angle. This makes velocity updating after migration impossible. At this early stage, I contend that passive surveys will only be conducted in actively studied regions where very good velocity models are already available. If this becomes a severe limit, the incorporation of plane-wave migration strategies can fill the offset dimension of the image.

Finally, moving the extraction of the reflection response from the transmission wavefield down to the image point during migration also introduces the possibility for more advanced imaging conditions, such as deconvolution, and other migration strategies, such as converted mode imaging.

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APPENDIX A

MIGRATION

Migration produces a subsurface image from many seismic experiments collected on a conve-
nient datum (usually the surface of the Earth). Each shot collected in a survey carries redundant information about subsurface reflectors. Collapsing this redundancy to specific locations in the subsurface by migration makes a structural image beneath the survey. For this reason, the words imaging and migration are used interchangeably. Of the many migration strategies available, this discussion centers on shot-profile depth migration. Depth migration is a cascade of extrapolation and imaging.

Extrapolation

The wave equation describes the propagation of seismic energy through a medium. The scalar simplification of the equation describes the propagation of compressional waves through an acoustic medium. While this simplification is not necessary, it is an established, robust, and convenient framework for this discussion. A wavefield is extrapolated from an initial condition to a close approximation of its state at a different location or time using a one-way propagator derived in Claerbout (1985).

Despite the fact that energy within the medium freely propagates in all directions, the Fourier solution to the wave equation can most easily be implemented as the cascade of two phase-shift operators that both handle lateral propagation, while individually accounting for either positive or negative propagation in a third dimension. These are the unitary, causal and acausal single square root operators, $E$, so named after their form

$$E^{+1} = e^{-ik_z \Delta z} \quad \text{and} \quad E^{-1} = e^{ik_z \Delta z} \quad \text{(A-1)}$$

where

$$k_z = \sqrt{(\omega s)^2 - k_x^2} \quad \text{(A-2)}$$

In the above equations, $\Delta z$ is the depth interval across which we are extrapolating the data, $k_z$ is the wavenumber in the depth direction, $k_x$ is the horizontal wavenumber calculated from the data, and $s$ is the slowness model of the subsurface. Because $E$ is a unitary operator\textsuperscript{1}, conjugation changes the propagation direction from causal to acausal or vice versa. These simple operators are precise for only laterally invariant media. More advanced propagators are extensively discussed in the literature, and do not change the discussion herein. Migrations in this paper use the phase-shift plus interpolation (PSPI) algorithm (Gazdag and Sguazzero, 1984).

\textsuperscript{1}This is strictly true only for propagating wavefields. Non-propagating harmonics, or standing waves, would cause a problem, but are not recorded by the geophones.
The goal of migration is to approximately reverse the seismic experiment with a double extrapolation process. The up-coming energy of a single shot-gather, \( U_{z=0} \), is the \( j \)th shot-gather from the total reflection experiment located at \( x_{sj} \) transformed to wavenumber, \( k_r \):

\[
U_{z=0}(k_r; x_{sj}, \omega) = R(k_r, x_s = x_{sj}, \omega). \tag{A-3}
\]

Each gather is iteratively extrapolated by \( E^{-1} \) to all desired depth levels \( z > 0 \)

\[
U_{z+1}(k_r; x_{sj}, \omega) = E^{-1} U_z(k_r; x_{sj}, \omega). \tag{A-4}
\]

The phase-shift of \( E^{-1} \) subtracts time from the beginning of the experiment and collapses curvilinear wavefronts in order to model the wavefield as if it were collected at a deeper level.

The down-going energy for a particular shot is a modeled wavefield, \( D_{z=0}(x_r; x_{sj}, \omega) \), of zeros with a single trace source wavelet (at time zero) at the source location \( x_{sj} \). This wavefield is transformed to wavenumber and extrapolated with the causal phase-shift operator \( E^+ \) to all desired levels \( z > 0 \)

\[
D_{z+1}(k_r; x_{sj}, \omega) = E^+ D_z(k_r; x_{sj}, \omega). \tag{A-5}
\]

The phase-shift adds time to the onset of experiment corresponding to the travel time required for the energy of the source to reach progressively deeper levels of the Earth across all \( x_r \) locations. If an areal source, such as a length of primachord or 30 Vibroseis trucks, were used instead of a point source, \( D_{z=0} \) should be modeled to reflect the appropriate source function.

This double extrapolation process is performed for each individual shot experiment to all depth levels of interest. Instead of reducing the complexity and volume of the original data, the process greatly increases the volume by maintaining the separation of up-coming and down-going energy through all depth levels for all time for all the receivers recording each shot.

**Imaging**

The imaging aspect of migration compares the energy in the \( D \) and \( U \) wavefields at each subsurface location to output a single subsurface model (Claerbout, 1971). The operator used to accomplish this goal is called the imaging condition. While different migration schemes require subtly different imaging conditions, the following discussion focuses on the one required for shot-profile depth migration.

Reflectors are correctly located in the image, \( i_z(x, h) \), at every depth level \( z \) as a function of horizontal position, \( x \), and offset, \( h \), when energy in the two wavefields is collocated. This condition maps energy to the image when the source has reached the location where a reflection was produced. Last, the entire model space is populated by summing the results of all the images produced in this manner by each shot collected in the survey (Rickett and Sava, 2002)

\[
i_z(x, h) = \delta_{x,x_r} \sum_{x_{sk}} \sum_{\omega} U_z(x_r + h; x_{sk}, \omega) D_z^*(x_r - h; x_{sk}, \omega). \tag{A-6}
\]
The Kronecker delta function indicates that the surface coordinates of the wavefields, $x_r$, are also used for the image. Notice that the zero lag of the correlation is calculated by summing over frequency. Evaluating the imaging condition at each depth to which the data and source wavefields have been extrapolated produces a structural image of the subsurface corresponding to changes in material properties.