Constructing new seismograms from old earthquakes:
Retrospective seismology at multiple length scales
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Abstract. If energy emitted by a seismic source such as an earthquake is recorded on a suitable backbone array of seismometers, source-receiver interferometry (SRI) is a method that allows those recordings to be projected to the location of another target seismometer, providing an estimate of the seismogram that would have been recorded at that location. Since the other seismometer may not have been deployed at the time the source occurred, this renders possible the concept of “retrospective seismology” whereby the installation of a sensor at one period of time allows the construction of virtual seismograms as though that sensor had been active before or after its period of installation. Here we construct such virtual seismograms on target sensors in both industrial seismic and earthquake seismology settings, using both active seismic sources and ambient seismic noise to construct SRI propagators, and on length scales ranging over 5 orders of magnitude from $\sim 40$ m to $\sim 2500$ km. In each case we compare seismograms constructed at target sensors by SRI to those actually recorded on the same sensors. We show that spatial integrations required by interferometric theory can be calculated over irregular receiver arrays by embedding these arrays within 2D spatial Voronoi cells, thus improving spatial interpolation and interferometric results. The results of SRI are significantly improved by restricting the backbone receiver array to include approximately those receivers that provide a stationary phase contribution to the interferometric integrals. Finally we apply both correlation-correlation and correlation-convolution SRI, and show that the latter constructs fewer non-physical arrivals.
1. Introduction

Traditional seismology uses the propagation of elastic waves from large earthquakes or explosions to infer earthquake or Earth properties and structure. This encouraged the deployment of seismometer networks worldwide, particularly in regions of high seismicity. However, earthquake seismogram data are only sensitive to earth properties on paths of energy propagation between earthquakes and seismometers. Recent developments in the field of seismic or wavefield interferometry (sometimes referred to as Green’s function retrieval) create new data types that are sensitive to a variety of different spatial volumes using the same seismometer networks and seismic source distributions (for reviews see Curtis et al. [2006]; Wapenaar et al. [2010a, b]; Galetti and Curtis [2012]). Here we investigate a new such method, on a range of spatial and temporal scales.

Seismic interferometry refers broadly to processes of cross-correlation, convolution or deconvolution that estimate the Green’s function between two receivers (inter-receiver interferometry), two sources (inter-source interferometry), or a source and a receiver (source-receiver interferometry). Inter-receiver interferometry [Weaver and Lobkis, 2001; Campillo and Paul, 2003; Wapenaar, 2003, 2004; van Manen et al., 2005, 2006; Wapenaar and Fokkema, 2006] allows one to construct the seismic signals that would have been recorded at one receiver location if an energy source had been fired at the location of the other receiver (the so-called “virtual source” location), whilst inter-source interferometry [Hong and Menke, 2006; Curtis et al., 2009; Tonegawa and Nishida, 2010; Poliannikov et al., 2012] allows one to construct the response from one of the sources that would have been recorded at the location of the other source (i.e., as if a “virtual receiver” had been
Since their inception these methods have been widely used in exploration seismics (e.g., Schuster [2001]; Schuster et al. [2004]; Bakulin and Calvert [2004]; Xiao et al. [2006]; Halliday et al. [2007, 2010, 2012]; Draganov et al. [2013]), and regional seismology (e.g., Shapiro and Campillo [2004]; Roux et al. [2005]; Sabra et al. [2005a, b]; Shapiro et al. [2005]; Stehly et al. [2007, 2008]; Wang et al. [2008]; Bensen et al. [2008]; Curtis et al. [2009]; Nicolson et al. [2012]). They have also been significantly advanced and tested in a number of other fields, including acoustics and ultrasonics [Cassereau and Fink, 1993; Roux and Fink, 2003; Weaver and Lobkis, 2001; Derode et al., 2003a, b], helioseismology [Rickett and Claerbout, 1999], structural engineering [Snieder and Safak, 2006], medical diagnostics [Sabra et al., 2007] and electromagnetics [Slob et al., 2007; Slob and Wapenaar, 2007].

Source-receiver interferometry (SRI) is a third form that acts rather differently from inter-receiver and inter-source interferometry. It can be derived either directly from representation theorems [Curtis and Halliday, 2010; Halliday and Curtis, 2010], or by combining the theories of inter-receiver and inter-source interferometry [Curtis, 2009; Curtis et al., 2012]. The result is that a Green’s function between a source and a receiver can be estimated from seismograms recorded on an array of other receivers (herein referred to as a backbone array) from a set of other sources. This has led to the development of new algorithms for imaging [Halliday and Curtis, 2010; Vasconcelos et al., 2010; Poliannikov, 2011; Poliannikov et al., 2012; Vasconcelos and Rickett, 2013; Ravasi and Curtis, 2013a, b; Vasconcelos, 2013; Ravasi et al., 2014], noise removal [Duguid et al., 2011], methods to correct for errors in inter-receiver and inter-source interferometry [King and Curtis, 2012; Meles and Curtis, 2014a] and methods to analyse and synthesise scattered wavefields (e.g.
Loer et al. [2014a]; Meles and Curtis [2014b], and Löer et al., manuscript submitted, 2014).

All of these methods involve using SRI for some form of spatial redatuming of recorded data.

Curtis et al. [2012] showed that SRI can also be used for a type of temporal redatuming: using recordings of earthquake sources made on a backbone seismometer array, virtual seismograms of the same source events can be constructed on receivers that were not deployed at the time of the event. Thus a form of ‘retrospective’ seismology is possible whereby one reconstructs the signals from an energy source at the location of a new seismometer, chosen retrospectively after that source occurred and after all energy from it has dissipated. One can thus use the benefit of hindsight of the source location or magnitude estimates to decide at which new locations to construct new (virtual) seismograms from that source.

This method may have significant implications within earthquake seismology: in the days following a large earthquake, temporary seismometers might be deployed closer to the earthquake epicentre to measure subsequent seismic activity in the area. Using a backbone array of seismometers that did record the earthquake energy, in principle one can spatially and temporally redatum the energy fluctuations from the main event onto the set of temporary seismometers. Curtis et al. [2012] demonstrated this idea by reconstructing seismic signals from two earthquakes in New Zealand on a set of temporary seismometers, some of which were not actively recording data when the earthquakes occurred. The new seismograms obtained were used to estimate seismic velocities in the vicinity of the seismometers and theory was presented that allows information about the source phase to be obtained from such virtual seismograms.
There are parallels between this method and the virtual earthquake approach of Denolle et al. [2013]. The latter authors use traditional inter-receiver interferometry and ambient noise to construct far-field surface wave seismograms that can be directly compared with real earthquake observations: surface impulse responses are constructed using the ambient seismic noise field and are then modified to correct for both depth and the double-couple focal mechanism of an earthquake [Denolle et al., 2013, 2014]. This turns one of the receivers in the pair into a virtual earthquake source. To test this method however, one requires a real earthquake source located close to the virtual source location. In contrast, SRI theory places no constraints on the locations of source and receiver pairs.

In this paper we test and extend the SRI method by reconstructing virtual seismograms (of both active seismic shots and earthquakes) with varying degrees of accuracy over a range of inter-receiver length scales spanning five orders of magnitude - from approximately 40 m to 2500 km. Each length scale is determined by the distance between a new “target” sensor and a backbone array of receivers that recorded the source energy. In one example we directly compare correlational and convolutional methods for SRI at the same sensors. We also use correlation-convolution SRI to extend the target sensor geometry, thus reaching length scales of up to 2420 km which allows the limitations of the SRI method to be assessed. At this largest length scale we show that the spatial extent of the backbone array significantly affects the accuracy of numerical approximations of interferometric integrals, and thus of SRI seismograms: longer arrays are not necessarily better.

2. Method
SRI may be applied in 2 steps [Curtis et al., 2012]: 1) An inter-receiver interferometry step constructs estimates of the Green’s functions between the backbone array seismometers and the target sensors; these Green’s functions are called the \textit{propagators} in SRI. 2) These propagators are used to project the recordings of the energy source on the backbone array to the location of the target sensor. In this section we describe inter-receiver and inter-source interferometry first, then how they are combined in SRI.

Both inter-receiver and inter-source interferometry can be performed using cross-correlation (e.g. Campillo and Paul [2003]; Wapenaar [2003, 2004]; van Manen et al. [2005, 2006]; Wapenaar and Fokkema [2006]), convolution (e.g. Slob and Wapenaar [2007]; Halliday and Curtis [2009]) or deconvolution (e.g. Snieder and Safak [2006]; Snieder et al. [2006a]; Wapenaar et al. [2008]; Vasconcelos and Snieder [2008a, b]; Wapenaar et al. [2011]). Here we focus on methods of cross-correlation and convolution but reviews and derivations of all techniques can be found in Curtis et al. [2006]; Snieder et al. [2009]; Schuster [2009]; Wapenaar et al. [2010a, b] and Galetti and Curtis [2012].

\textbf{2.1. Inter-Receiver Interferometry}

Consider the geometry in Figure 1a where two receivers at $\mathbf{r}_1$ and $\mathbf{r}_2$ are surrounded by impulsive sources at positions $\mathbf{x}'$ on boundary $S'$. The responses from sources at $\mathbf{x}'$ are recorded at $\mathbf{r}_1$ and $\mathbf{r}_2$, cross-correlated and summed (integrated) over the boundary $S'$. This constructs an estimate of the Green’s function between points $\mathbf{r}_1$ and $\mathbf{r}_2$ [Wapenaar and Fokkema, 2006]. The Green’s function is the response that would have been recorded at receiver $\mathbf{r}_2$ if there had been an impulsive source at the location of receiver $\mathbf{r}_1$. The receiver at $\mathbf{r}_1$ is thus turned into a virtual (imagined) source. For waves propagating in
an acoustic medium this process is represented in the frequency domain by the following surface integral [Wapenaar and Fokkema, 2006]:

\[
G(\mathbf{r}_2, \mathbf{r}_1) + G^*(\mathbf{r}_2, \mathbf{r}_1) = \int_{S'} \frac{-1}{j\omega \rho(\mathbf{x}')}
\left( G(\mathbf{r}_2, \mathbf{x}') \partial_i G^*(\mathbf{r}_1, \mathbf{x}') - (\partial_i G(\mathbf{r}_2, \mathbf{x}')) G^*(\mathbf{r}_1, \mathbf{x}') \right) n_i' d^2\mathbf{x}'
\]  (1)

where \( j = \sqrt{-1} \), \( \omega \) is angular frequency, \( \rho(\mathbf{x}') \) is the mass density of the medium, \( n_j \) is the \( j \)th component of the normal vector on the boundary \( S' \) and \( \partial_k \) denotes a spatial derivative in the \( k \)-direction. \( G(\mathbf{r}_1, \mathbf{x}') \) and \( \partial_i G(\mathbf{r}_1, \mathbf{x}')n_i' \) represent the pressure at \( \mathbf{r}_1 \) due to monopole and dipole volume injection rate density sources at \( \mathbf{x}' \) on \( S' \), respectively, where the dipole sources are oriented normal to the boundary \( S' \). Boundary \( S' \) is arbitrary as long as it encloses \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \), and \( G(\mathbf{r}_2, \mathbf{r}_1) \) contains direct and (multiply) scattered wave contributions from inhomogeneities inside and outside of \( S' \). Einstein’s summation principle for repeated indices applies throughout. \( G(\mathbf{r}_2, \mathbf{r}_1) + G^*(\mathbf{r}_2, \mathbf{r}_1) \) is the homogeneous Green’s function; in the time domain \( G(\mathbf{r}_2, \mathbf{r}_1) \) represents the desired Green’s function from \( \mathbf{r}_1 \) to \( \mathbf{r}_2 \), while \( G^*(\mathbf{r}_2, \mathbf{r}_1) \) represents the complex conjugate of the same Green’s function, which in the time domain represents the same Green’s function but starting at zero time and extending in the negative time direction. Either positive or negative times of the homogeneous Green’s function thus represent the response recorded at \( \mathbf{r}_2 \) from an impulsive source at the location of \( \mathbf{r}_1 \). Note that in the frequency domain the definition of cross-correlation is the product of one term with the complex conjugate of another, as on the right side of Equation 1.
If the boundary \( S' \) only surrounds 1 receiver as in Figure 1b then a similar set of operations are carried out to estimate \( G(r_2, r_1) \), but instead of using correlation in this case the boundary source recordings must be convolved [Wapenaar and Fokkema, 2006]:

\[
G(r_2, r_1) = \int_{S'} \frac{-1}{j\omega \rho(x')} \left( G(r_1, x') \partial_i G(r_2, x') - (\partial_i G(r_1, x'))G(r_2, x') \right) n_i d^2x' \tag{2}
\]

where, as in Equation 1 terms such as \( G(r_1, x') \) are the Fourier transforms of causal time-domain Green’s functions \( G(r_1, x', t) \).

### 2.2. Inter-Source Interferometry

The theory for inter-source interferometry is obtained by applying source-receiver reciprocity to Equation 1 [Curtis et al., 2009; Tonogawa and Nishida, 2010]. Consider now the geometry in Figure 1c where 2 impulsive sources \( s_1 \) and \( s_2 \) are surrounded by receivers at locations \( x \) on boundary \( S \). The responses from \( s_1 \) and \( s_2 \) are recorded at each \( x \), cross-correlated and the results summed (integrated) over all \( x \) on boundary \( S \). This constructs an estimate of the Green’s function between points \( s_1 \) and \( s_2 \). This Green’s function is the response from an impulsive source at \( s_1 \) that would have been recorded had a receiver been placed at \( s_2 \). The source at \( s_2 \) is thus used as a virtual receiver. For waves propagating in an acoustic medium this process is represented in the frequency domain by the following integral [Curtis et al., 2009]:

\[
G(s_2, s_1) + G^*(s_2, s_1) = \\
\int_{S} \frac{-1}{j\omega \rho(x)} \left( G^*(x, s_2) \partial_i G(x, s_1) - (\partial_i G^*(x, s_2))G(x, s_1) \right) n_i d^2x \tag{3}
\]
Similarly to the inter-receiver case, inter-source interferometry also takes a convolutional form when the boundary $S$ only surrounds one source as in Figure 1d:

$$G(s_2, s_1) = \int_S \frac{-1}{j \omega \rho(x)} (G(x, s_2) \partial_i G(x, s_1) - (\partial_i G(x, s_2)) G(x, s_1)) n_i d^2x$$  \hspace{1cm} (4)

### 2.3. Source-Receiver Interferometry

The three canonical source-receiver geometries for SRI are shown in Figure 2 [Curtis and Halliday, 2010]. In Figure 2a, a source $s$ and receiver $r$ are surrounded by a boundary of receivers $x$ on $S$ and a boundary of sources $x'$ on $S'$. For wave propagation in an acoustic medium we then obtain the exact SRI formula

$$G(r, s) + G^*(r, s) = \frac{-1}{j \omega \rho} \int_S \left\{ \frac{-1}{j \omega \rho} \int_{S'} \{ G^*(r, x') n_i \partial_i G(x, x') \right. $$

$$\left. - n_i \partial_i G^*(r, x') G(x, x') \} dS' \right\} n_i \partial_i G(x, s) $$

$$- n_i \partial_i \left[ \frac{-1}{j \omega \rho} \int_{S'} \{ G^*(r, x') n_i \partial_i G(x, x') \right. $$

$$\left. - n_i \partial_i G^*(r, x') G(x, x') \} dS' \right\} G(x, s) \} dS$$ \hspace{1cm} (5)

where $G(r, s)$ is the Green’s function in the frequency domain representing the pressure at $r$ due to a volume injection-rate density source at $s$. Note that all equations have been presented herein for waves propagating in an acoustic medium, thus ignoring the elastic nature of seismic waves. We also assume that the medium outside of each bounding surface ($S$ and $S'$) is approximately homogeneous and isotropic, and that the surface $S'$ is large and spherical [Wapenaar and Fokkema, 2006]. For the equivalent exact SRI equation in elastic media, and those equations pertaining to the other possible source-receiver geometries shown schematically in Figure 2 see Curtis and Halliday [2010].
We now discuss how Equation 5 can be modified for practical applications where acoustic
(and elastic) assumptions are often violated (e.g. boundaries $S$ and $S'$ are incomplete).
Assuming high-frequency wave propagation, locally planar wave fronts, and that surfaces
$S$ and $S'$ are large and approximately spherical, that the medium outside of the bounding
surfaces is homogeneous, and Sommerfield radiation conditions at the boundary surfaces,
Curtis et al. [2012] show that Equation 5 can be reduced to two equations:

$$G_H(x,r) \approx \frac{2jk}{\omega \rho} \int_{S'} G^*(r,x')G(x,x')dx'$$  \hspace{1cm} (6)

and

$$G_H(r,s) \approx \frac{2jk}{\omega \rho} \int_S G^*(x,s)G(x,r)dx$$  \hspace{1cm} (7)

Here, $G_H$ is the homogeneous Green’s function and is defined as $G_H = G + G^*$, where
* represents complex conjugation in the frequency domain or time-reversal in the time
domain. Equation 6 is applied first (step 1 described above) and is simply an approximate
representation of inter-receiver interferometry in Equation 1 [Wapenaar and Fokkema,
2006]. By isolating only the positive (or negative) times of $G_H(x,r)$, we identify an
estimate of $G(x,r)$, and doing this for all boundary locations $x$ we obtain the so-called
propagator Green’s functions that are used in step 2. Equation 7 then represents the cross-
correlation of these estimated propagators with the signals recorded from the real source at
$s$ by receivers at all $x$ on $S$. This is step 2 above, and represents an approximation to inter-
source interferometry (Equation 2). In the case represented by Figure 2b, convolution
should be used in place of correlation in Equation 7:
Equations 6 and 7 include integrations over all receiver and source positions on boundaries $S$ and $S'$, respectively. However, Snieder [2004] showed that only a small subset of boundary sources and boundary receivers are required to estimate the Green's function between 2 points provided that the medium is not too strongly scattering. These sources and receivers lie within regions of the boundaries at which the phase of the integrands in Equations 6 and 7 become approximately stationary. Schematic representations of these stationary phase regions are shown by thick grey lines in Figure 2a and Figure 2b.

Invoking the stationary phase approximation reduces the number of direct Green's function measurements that need to be made during practical applications of SRI. To reduce this number further we can replace the active sources at $x'$ on boundary $S'$ with mutually uncorrelated noise sources [Shapiro and Campillo, 2004; Wapenaar and Fokkema, 2006]. Equation 6 then becomes:

$$G_H(x, r) \approx C \nu^*(r) \nu(x)$$  

where $C$ is a constant and $\nu$ is the ambient field fluctuations recorded at $x$ and $r$. When recorded over long periods of time, these ambient field fluctuations may be considered to be a diffusive or random field, and cross-correlations of such fields have been found to result in a reasonable estimate of the Green's function [Lobkis and Weaver, 2001]. Applying Equation 9 in place of Equation 6 has thus become very attractive in regional seismology as it eliminates the need to measure the response from all sources on a boundary $S'$. 

$$G(r, s) \approx \frac{2jk}{\omega \rho} \int_S G(x, s)G(x, r)dx$$  

(8)

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individually, and thus studies no longer need to depend on suitably located earthquakes or other active sources [Shapiro and Campillo, 2004; Shapiro et al., 2005; Sabra et al., 2005a].

In practice, the energy released from an earthquake source has a characteristic source time signature \( T(\omega) \). Thus, boundary receivers at locations \( x \) actually record \( TG(x, s) \).

Furthermore, \( T \) need not be associated with a point source, but can be linked to a rupture sequence of point sources over a fault plane \( F \). Recordings \( R \) of such rupture sequences on backbone receivers at locations \( x \) can be described approximately by assuming linear additivity of the wavefields from each point source, and integrating over all point-source positions \( s \):

\[
R(x, \bar{s}) = \int_{s \in F} T(s)G(x, s)ds \tag{10}
\]

where \( \bar{s} \) is the set of point sources \( s \) on fault \( F \) that represents the earthquake rupture. Our goal in retrospective seismology is thus to estimate

\[
R(r, \bar{s}) = \int_{s \in F} T(s)G(r, s)ds \tag{11}
\]

where \( r \) represents the location of a target sensor that did not necessarily record the seismic energy from the earthquake. If we pre-multiply Equation 7 by \( T^*(s) \), integrate over point-sources \( s \) on fault plane \( F \), and apply source-receiver reciprocity we obtain

\[
\int_{s \in F} T(s)G_H(r, s)ds \cong \frac{2jk}{\omega \rho} \int_S \int_{s \in F} [T(s)G(x, s)]^*G(x, r)dsdx
\]
This shows that if we calculate the right side of Equation 12 by cross-correlating the propagators $G(x, r)$ with the event recordings $R(x, \bar{s})$ on the backbone array at location $x$, integrate over the backbone array, and apply source-receiver reciprocity we obtain the integral of $T(s)^* G_H(r, s) = T(s)^* G(r, s) + T(s)^* G^*(r, s)$ over the fault plane $F$ (left side of Equation 12). At positive times we therefore obtain the integral of $T(s)^* G(r, s)$: this is not the true response of an earthquake point source $s$ at receiver $r$ as $G$ is convolved with the time-reverse of the source time signature $T$, rather than with $T$ itself as required by Equation 11. However, at negative times we obtain the integral of $T(s)^* G^*(r, s)$: after time-reversal (complex conjugation), this gives the desired $T(s) G(r, s)$ integrated over fault plane $F$ or $R(r, \bar{s})$ in Equation 11. Thus, when using correlation-correlation SRI, we use only the acausal part of reconstructed signals to approximate the seismogram responses from earthquake sources at target seismometers $r$. Nevertheless, we will discuss the geometry necessary to construct both the causal and acausal sides of the homogeneous Green’s function, and the benefits of such a geometry, in Section 5 below.

A similar analysis for correlation-convolution SRI in Equation 8 shows that the correct seismogram is created by evaluating

$$
\int_{s \in F} T(s) G(r, s) ds \approx \frac{2jkr}{\omega \rho} \int_{s \in F} \int_{x \in F} T(s) G(x, s) G(x, r) ds dx
$$

$$
\approx \frac{2jkr}{\omega \rho} \int_{x \in F} R(x, \bar{s}) G(x, r) dx
$$

(13)
Thus, convolutional interferometry on the right of Equation 13 constructs the desired, one-sided estimate of $R(r, \bar{s})$ in Equation 11 directly, without the need for time-reversal (complex conjugation).

In practice, all information in the above equations are discretized in space such that the continuous surface integral over boundary $S$ is approximately represented by a summation over seismometers at locations $x_i \in S$. Each seismometer occupies the space $\Delta x_i$, which, in 1 dimension, is the sum of half distances to the next seismometer on either side (see Figure 3a), whilst in 2 dimensions, $\Delta x_i$ represents the surface area of the portion of space closer to that seismometer than neighbouring seismometers (see shaded polygons in Figure 3b). We can thus re-write Equations 12 and 13 as:

$$R(r, \bar{s}) = \int_{s \in F} T(s)G(r, s)ds \approx \frac{2jk}{\omega \rho} \sum_{i=1}^{n} R^*(x_i, \bar{s})G(x_i, r)\Delta x_i$$

(14)

and

$$R(r, \bar{s}) = \int_{s \in F} T(s)G(r, s)ds \approx \frac{2jk}{\omega \rho} \sum_{i=1}^{n} R(x_i, \bar{s})G(x_i, r)\Delta x_i$$

(15)

where $R(x_i, \bar{s})$ is the earthquake recording at the seismometer at location $x_i$, and the same discretization (summation notation $\sum_{i=1}^{n} \cdots \Delta x_i$) can be applied to all other continuous integrals over boundary $S$ described above. Thus we evaluate all integrals using a summation over available receivers, but when we discuss the theory above in the text we continue to refer to integration as this is the underlying correct operation.

The extent to which this discretization affects the final SRI seismograms is dependent on the frequency content of the data and the spatial sampling of seismometers on the receiver.
boundary $S$. Despite all of these assumptions (i.e., restricted boundaries, stationary phase approximations, discretization of the integrals, etc.), Curtis et al. [2012] have shown that the SRI method is able to reconstruct reasonably accurate earthquake seismograms retrospectively. Herein we test this method further by performing SRI over 3 different length scales, and include both active source and earthquake seismology examples.

3. Application 1 at Engineering Seismology Scale

We applied correlation-correlation SRI to construct the seismogram between an active source and a number of receivers in a small exploration or engineering scale seismic experiment performed in the field beside Schlumberger Gould Research in 2010 [Duguid et al., 2011]. The acquisition geometry is shown in Figure 4: active seismic sources consisting of an accelerated weight drop, were placed at intervals of 4 m along the running-track shaped boundary $S'$. This boundary encloses a grid of receivers at locations $r_i$, some of which we use as target sensors, and a receiver line $S$ which acted as the backbone array in this case. Active shots were also recorded at all receiver positions from a source at location $s$. Our goal is to construct seismograms from source $s$ on target receivers $r_i$ using SRI, and compare the results to the real recordings.

We constructed the seismograms between source $s$ and receivers $r_i$ by applying SRI using Equations 6 and 7, and without using the direct recordings of the source on $r_i$. Thus we simulate the case where the source at $s$ was fired before or after the period during which the receivers at $r_i$ were installed and activated, and hence where the source was only recorded on the backbone receiver boundary $S$. This was achieved by first using seismic energy propagating from sources on boundary $S'$ to estimate the Green’s function propagators between $r_i$ and each $x$ on $S$ using inter-receiver interferometry in Equation
6, thus turning receivers $r_i$ into virtual sources recorded by receivers at $x$ on $S$. We then redatum the signals from the backbone array $S$ to the target sensors $r_i$ using inter-source interferometry in Equation 7, turning the virtual sources at $r_i$ into virtual receivers.

The active-source data was recorded at 250 Hz over a time period of 4 s in a field adjacent to Schlumberger Gould Research (SGR) in July 2010. As different types of geophones were deployed during acquisition (with responses centred at 4.5 Hz, 10 Hz and 14 Hz in the target sensors, and at only 4.5 Hz at receiver line $S$), transfer functions from 4.5 Hz to 10 Hz, and from 14 Hz to 10 Hz were estimated from the recorded data and applied to the 4.5 Hz and 14 Hz data before any subsequent processing [Duguid et al., 2011]. In order to ensure coherency in the frequency content across all receivers, the data from boundary $S'$ was filtered between 8 Hz and 22 Hz before the inter-receiver step. In accordance with the stationary-phase principles of Snieder [2004], only a subset of boundary sources that were assumed to provide a constructive contribution to the integrand in Equation 7 were used to construct the inter-receiver Green’s functions (see the small/black stars in Figure 4a). Before the inter-source step, a second filter (15.5 Hz - 20 Hz) was applied to the signals recorded by the backbone array, and to those recorded from the active source $s$ at the receivers $r_i$. This was the only frequency band with significant energy that overlapped between all recorded signals, which thus limits the results to a narrow frequency band.

The results of SRI for the 6 receivers $r_i$ are shown in Figure 5: all seismograms are cut to 1.4 s in length and are normalised to their absolute maximum amplitude. The dominant arrival is the emerging surface wave (ground roll) which can be seen to move out from the source for increasingly distant receivers. The match is not perfect, and this is likely partly because the equations derived herein for correlational interferometry assume a non-
attenuating medium which is an approximation. Also, similar weight drop sources were used at locations $x'$ and $s$: thus in the first step of interferometry both Green's functions on the right of Equation 6 are in fact convolved with the source time function and the result on the left will therefore be multiplied by the source power spectrum. This extra factor is then multiplied into the SRI result in Equation 7, but will not be present in the real recording. Of course, since we have both the real recordings and the SRI seismograms at receivers $r_i$ in this case, in principle we could divide one by the other to obtain the source power spectrum [Behura and Snieder, 2013]. We do not implement this here as first it assumes a non-attenuative medium, and second we focus on testing the case where we do not have any direct recordings at $r_i$. Finally, in this controlled small scale example the SRI reconstructions are narrow-band signals with a low frequency content compared to other industrial surveys (15.5 Hz - 20 Hz). This has implications for the use of such SRI seismograms for subsurface imaging as spatial resolution will be low.

Nevertheless, in all cases the match between the real and SRI traces is reasonably good, showing the reliability of the method in a controlled experiment and when an ideal geometry of sources and receivers is available. In the next sections, we apply the principles of SRI and retrospective seismology to less controlled scenarios in earthquake seismology, showing the potential of this method when the distribution of sources and receivers is far from perfect.

4. Applications in Earthquake Seismology

We now apply correlation-correlation SRI and correlation-convolution SRI in two earthquake seismology settings using ambient wavefield fluctuations recorded on a backbone array of seismometers $x$ and on target sensors $r$, as described by Equations 9, 12 and 13.
We reconstruct the seismograms from 2 earthquakes retrospectively, on 2 target sensors (seismometers) that were deployed and then removed before the earthquakes occurred, and thus which did not record the events. To test the robustness of the method, we also reconstruct the event seismograms on up to 7 other target sensors that were operational at the time that the earthquakes occurred, and compare the reconstructed seismograms with those actually recorded. The quality of the match between the real and SRI seismograms constructed at any one target sensor is quantified by calculating the correlation coefficient over the lengths of the traces displayed in Figures 8a, 10, 12 and 14.

4.1. Spatial Sampling and Station Recording Criteria

A search of the dates of installation of all USArray stations was carried out to determine the number of seismometers deployed at the time of a catalogue of earthquakes. Of these seismometers, those that satisfy the following spatial criteria were selected for the backbone seismometer array (see Figure 6 for a schematic illustration of these criteria):

1. The backbone seismometer array consists of at least 2 parallel lines of approximately regularly spaced seismometers (using a laterally extended band of seismometers rather than a single line compensates to some extent for the lack of seismometers at depth, which otherwise may lead to spurious arrivals due to inter-mode surface wave correlations [Halliday and Curtis, 2008]).

2. A great circle path (GCP) from the earthquake epicentre intersects the seismometer array approximately perpendicularly - at a local intersection angle of between 70° and 110°.

3. The same GCP intersects at least 1 target seismometer that did not record the event.
4. Seismometers in the backbone array are operational at the time the event occurred, and all seismometers have at least 6 months in common of ambient seismic noise recordings.

For backbone and target seismometers that satisfy the above criteria, instrument response files and up to 2 years of vertical component, daily ambient noise records sampled at 4 samples per second were downloaded from the IRIS database. Ambient noise was downloaded from January 2009 to December 2010 depending on the deployment history of the seismometers selected. Both summer and winter noise data was thus downloaded, reducing the bias in ambient wavefield directivity from temporal (seasonal) dependencies.

In the figures herein we schematically represent the noise sources by small black stars in the nearby oceans for simplicity, but note that their actual origin can be far further afield [Shapiro et al., 2006; Yang and Ritzwoller, 2008; Kedar, 2011; Zeng and Ni, 2014]. Earthquake recordings made at the selected backbone seismometers were also downloaded. Each earthquake trace began at the origin time of the event and was 8000 s in length.

4.2. Data Processing and Methodology

Processing of the ambient seismic noise data and earthquake recordings is carried out following the methods of Bensen et al. [2007]. For each daily noise record and each earthquake recording made on the backbone seismometer array, the instrument response, mean and trend are removed and bandpass filters with corner frequencies at 0.01 Hz and 1 Hz are applied. This frequency range is kept as broad as possible at this stage to ensure that no important frequencies are lost early on in the noise processing sequence.

Inter-Receiver Interferometry Step
Temporal normalisation is applied to each daily noise record to reduce the effects of large amplitude signals on the cross-correlations. Such signals are caused by earthquakes, non-stationary noise sources near to the station and instrumental irregularities [Bensen et al., 2007]. We use the 1-bit normalisation method which replaces all positive amplitudes with +1 and all negative amplitudes with −1. This is the most aggressive temporal normalisation method described by Bensen et al. [2007] as it removes all amplitude information by keeping only the sign of the raw signal. It has been shown to improve the signal-to-noise ratio (SNR) of ambient noise Green’s function estimations [Derode et al., 2003b], and has been used in a number of ambient seismic noise studies (e.g. Shapiro and Campillo [2004]; Shapiro et al. [2005]; Stehly et al. [2007]; Nicolson et al. [2012, 2014]).

Finally, spectral normalisation (whitening) is applied to the filtered, 1-bit normalised noise records. This targets the spectrally biassed nature of ambient seismic noise and reduces the effects of persistent monochromatic noise sources, such as the 26 s peak associated with a narrow band noise source in the Gulf of Guinea [Shapiro et al., 2006; Bensen et al., 2007].

For each station pair between a backbone array seismometer and a target sensor, the daily ambient noise recorded on the array seismometer $x$ is cross-correlated with the ambient noise recorded on the target sensor $r$ to produce a noise correlation function. All daily correlation functions computed between each station pair are stacked to construct a single estimate of the Green’s function, $G_H(x, r)$. At positive times we obtain an approximation to $G(x, r)$, the response that would have been recorded at the backbone array seismometer $x$ if an impulsive source had been fired at the location of the target sensor.
r (as in Equation 9). The target sensor r is thus turned into a virtual source. To obtain
the one-sided Green’s function, the acausal part of each stacked correlation function is
time-reversed and added to the causal part to create our final approximation to $G(x, r)$.
Each estimated Green’s function is cut to 8000 s. A second bandpass filter with corner
frequencies at 0.02 Hz and 0.1 Hz is then applied to both the estimated Green’s functions
and the earthquake recordings. This restricts all data to the same length (in time) and to
the same frequency band, within which we expect a significant overlap in signal energy.

Inter-Source Interferometry Step

The earthquake signal recorded at an array seismometer $R(x, s)$ is cross-correlated with
the estimate of $G(x, r)$. According to Curtis et al. [2012] and Equation 12, if we inte-
grate the results over the backbone receiver array for varying x on S, the acausal part
of the constructed seismogram approximates the earthquake response that would have
been recorded at the target sensor r if the sensor had been deployed at the time of the
event. When using correlation-convolution SRI, the earthquake signals recorded at x are
convolved with the estimate of $G(x, r)$ as in Equation 13. Integration over all backbone
array seismometers constructs our estimate of the true response at r from the earthquake
source at s.

The integrand of Equation 12 (or Equation 13) constructed on seismometers located
around the centre of the backbone array contribute most to the final reconstructed seis-
mogram. These seismometers lie within the stationary phase regions of the receiver
boundaries and are highlighted schematically by the thick grey lines in Figures 2a and
2b [Snieder, 2004]. Since the receiver arrays employed at these larger scales are usually irregularly spaced, we found it necessary to perform an interpolated sum over $x$ in this interferometry step. From here onwards we shall thus consider the discretized forms of the inter-source interferometric equations as shown by Equations 14 and 15. The interpolation should be such that it improves the approximation of this summation to the integrations in Equations 12 and 13. Hence, where there is high receiver density, each receiver should individually contribute less to the integration (summation) than when the receivers are sparsely distributed.

To invoke the summations in Equations 14 and 15 more precisely we embed each backbone seismometer array within a rectangular patch of 2D spatial Voronoi cells, such that seismometer coordinates become Voronoi cell centres (for example, see inset in Figure 7). The spatial area of each Voronoi cell is used as the discretization factor $\Delta x_i$ in the summands of Equations 14 and 15 for seismometer $x_i$ at that cell’s centre. In addition the summands are multiplied by a tapered cosine weighting function to remove effects due to the finite boundary length available (see below). Original and adapted codes by Sambridge et al. [1995] and Sambridge [1999a, b] are used to calculate Voronoi cell areas in these computations.

Each rectangular patch (like that in the inset in Figure 7) is divided into a regular grid of square cells of side length approximately 100 km. Each such cell is assigned the value of the summand of Equation 14 or Equation 15 from the seismometer $x_i$ at its Voronoi cell’s centre. Instead of summing over seismometers in the backbone array to approximate the summations in Equations 14 and 15, we sum over this regularly spaced grid of ‘interpolated’ seismometers. First though, a cosine tapered window is applied to
the rectangular grid in the x- and y-directions, such that the first and last 25% of the points in each direction are weighted by half-cosines normalised to lie in the interval [0, 1], and the middle 50% of points are left unchanged. This acts to reduce the edge effects (an effect that would otherwise be imposed by our own rectangular boundary selection) and ensures that contributions from seismometers towards the centre of the array (i.e., within the approximate stationary phase region) provide a stronger contribution to the final summation than seismometers towards the edges. Finally we sum the taper-weighted values of the summands over all cells within the 2D rectangular patch to construct the final estimate of the recording of event $s$ at sensor $r$. This entire 2-step process approximates the application of SRI to the recorded data. Note that this interpolation method uses zeroth-order (constant) Voronoi cell interpolation [Sambridge, 1999a] as represented by the summations in Equation 14 or Equation 15. A more accurate method of interpolation could be obtained by expanding the interpolation within each Voronoi cell to higher orders (e.g., using bilinear interpolation between each cell and neighbouring cells).

Finally, to compare the real and virtual seismograms directly we apply a bandpass filter within the frequency band where both the earthquake and noise spectra overlap with significant energy. Here we present the data within the band 0.04 Hz - 0.06 Hz as these intermediate frequencies performed better on average than higher (0.08 Hz - 0.1 Hz) or lower (0.02 Hz - 0.04 Hz) frequencies when the spatial sampling of the backbone array, and thus the discretization in Equation 14 or 15, varied. Unfortunately in this study, the recovered frequency content of the SRI seismograms is very low which currently prevents the virtual seismograms from being used for high resolution imaging.
Within the 0.04 Hz - 0.06 Hz frequency band, there was only a marginal improvement in results from using 2 lines of seismometers within the backbone array compared to a single line of seismometers. However, if only a single line of seismometers is available and spatial sampling along that line is sparse, higher frequencies (0.08 Hz - 0.1 Hz) were found to perform marginally better than intermediate values. This is surprising as the Nyquist sampling criterion along the boundary would suggest that a denser backbone array is required to capture the characteristics of a higher frequency wavefield. This effect may therefore merit further study. Herein we have consistently used 2 lines of backbone seismometers as there is little variation in the spatial sampling of the arrays chosen for each individual SRI reconstruction in most of our examples. However, when seismometers are missing we observe that Voronoi interpolations help to account for this irregularity by spatially averaging the existing data.

### 4.2.1. Quality Control

Two quality control checks are carried out following the cross-correlation of ambient noise in the inter-receiver step above, based on the signal-to-noise ratio (SNR) of each result. We define the SNR as the ratio of the absolute maximum amplitude within a signal window to the root-mean-square (rms) value of a noise window. The signal window spans a time period within which we expect a surface wave arrival, and the noise window is defined such that it is temporally later than this signal window.

After the daily noise correlation functions have been produced for each $x$ in Equation 9, an approximate surface wave arrival time $\tau$ is calculated for each station pair using the inter-station distance and a mean group velocity of 3.25 km/s. This group velocity was chosen using the group velocity curves provided by Bensen et al. [2008], in which the
authors studied similar inter-station USArray paths. The signal window is chosen to span 
the range \((\tau - 250 \text{ s})\) to \((\tau + 250 \text{ s})\), and a 500 s noise window is defined to follow this 
signal window. The SNR of each daily correlation function is then calculated. Those with 
a SNR value \(\geq 4.8\) are normalised (divided) by the rms-value of their signal window, whilst 
those with a SNR value between 2 and 4.8 are normalised by their absolute maximum 
amplitude as we found this better removed any spurious peaks of energy in lower SNR 
signals. Any daily correlation functions with SNR values \(\leq 2\) are removed from the study 
altogether.

A second SNR check is carried out after the daily correlation functions have been stacked 
and the inter-receiver Green’s functions have been estimated in Equation 9. Those with a 
SNR value \(\geq 15\) are normalised by the rms-value of their signal window, whilst those with 
a SNR value between 3 and 15 are normalised by their absolute maximum amplitude. If 
any have a SNR value \(\leq 3\) at this stage they are removed.

The above set of SNR thresholds were defined following a series of trial and error tests on 
many data. For example, trial and error tests on the daily correlation functions identified 
a possible top SNR limit of between 4.5 and 5, with the majority of “good” signals having 
a SNR value greater than 5. However, to encompass those correlation functions whose 
signals were still strong in comparison to the background noise level (SNR values between 
4.5 and 5), an intermediate value of 4.8 was chosen. For the majority of daily files with 
SNR values less than 4.8 the signal was not necessarily clearly discernible from the noise 
by eye.

4.3. Example 2 at an Intermediate Scale
We first reconstructed the seismograms from a magnitude 5.8 earthquake on 8 target
sensors using a backbone array comprising 93 seismometers that satisfied the spatial sam-
pling criteria outlined above, and correlation-correlation SRI (Equations 9 and 14, Figure
2a). The backbone array and the 8 target sensors at which seismograms were reconstructed
are shown in Figure 7. Earthquake signals were reconstructed at each of the 8 sensors
using combinations of seismometers on 2 lines located on average between approximately
210 km and 540 km from the sensors: the caption in Figure 7 states which lines were used
for which target sensor. On average 22 backbone array seismometers were used for each
reconstruction. Figure 7 (inset) shows the locations of the array seismometers within their
2D spatial Voronoi cells. The number of seismometers available for each reconstruction
varies depending on the deployment history of the seismometers and the outcome of the
quality control checks described above. Each remaining backbone seismometer became
a Voronoi cell centre, and 2-line combinations of Voronoi cells were used to interpolate
across grids of square cells, the grid having dimensions between 1.5° - 1.8° in latitude and
7.7° - 9.9° in longitude. We assume that the backbone array seismometers chosen include
the stationary phase points of the receiver boundary S.

The final SRI seismograms were then compared with the real recordings of the event
that existed on 7 of the target sensors. SRI reconstructions and real earthquake recordings
at all target sensors are plotted within the frequency band 0.04 Hz - 0.06 Hz as a function
of epicentral distance in Figure 8a. All seismograms are normalised to their absolute
maximum amplitude and the quality of the match between the real and SRI seismograms
in the windows shown is quantified by the correlation coefficient as stated on the right of
the figure, above each trace. The moveout of the main surface wave arrival with distance
is clearly visible. Also, the phase of the main arrivals estimated using SRI are in good agreement with the actual recorded seismograms at the same sensor (where the latter exists) and correlation coefficients reach values up to 0.93. Sensor 226A (unfilled triangle in Figure 7), was active prior to the earthquake but was removed from its site before the earthquake occurred. The reconstruction at 226A is thus a new virtual seismogram constructed at a seismometer location selected retrospectively. This also demonstrates that even after sensors have been removed, virtual seismograms can be obtained at their previous locations provided that the backbone array remains intact and the properties of the medium have not changed significantly in the elapsed time between the deployment of the seismometers and the occurrence of the event [Curtis et al., 2012].

Figure 8b shows the results if the integrand in Equation 12 is approximated by a simple summation of the integrands calculated at each station \( x_i \) on \( S \), that is if the Voronoi cell area \( \Delta x_i \) in Equation 14 is omitted. The quality of seismogram reconstructions deteriorates significantly, discussed further below.

Next we used correlation-convolution SRI (Equations 9 and 15, Figure 2b) to reconstruct earthquake seismograms at target sensors Z27A, 127A, 227A and 226A using the backbone array shown in Figure 9 and 2-line combinations of the Voronoi cells shown in the inset to Figure 9 and described in the caption to that figure. The SRI seismograms are compared with the real recordings of the earthquake at the target sensors in Figure 10. We again observe clear surface wave arrivals in the SRI seismograms that are in phase with the real recordings at the target sensors (where the latter exists) and which agree well with the top four traces in Figure 8a which are constructed at the same target sensors using correlation-correlation SRI. Thus we verify that both correlation-correlation
and correlation-convolution SRI can be used for this form of retrospective seismology, and the method used depends on the backbone array geometry available. Furthermore, the spurious events in the correlation-correlation SRI seismograms that occur prior to the main surface wave arrivals disappear when one uses the correlation-convolution approach. For example, compare the SRI reconstructions at target sensor Z27A in Figure 8a to the same sensor in Figure 10 (top trace). When using correlation-correlation SRI, artefacts are introduced between approximately 300 s and 550 s (see grey ellipse in Figure 8a) and this lowers the correlation coefficient of the SRI seismogram (a value of 0.42 is obtained). These non-physical arrivals are not constructed when using the correlation-convolution approach (see grey ellipse in Figure 10) and the correlation coefficient subsequently increases to a value of 0.68.

4.4. Example 3 at the Largest Scale

Finally we created a large scale example designed to challenge the method. Correlational interferometry as described herein contains an underlying assumption of zero attenuation, and as propagation distance increases this assumption becomes increasingly questionable. We also wanted to test the method when using target sensors that lie relatively close to the earthquake source compared to the locations of the backbone array seismometers. We first reconstructed earthquake seismograms at target sensors WDC, BMN, DUG and P21A using correlation-correlation SRI (Equations 9 and 14, Figure 2a) and large arrays of backbone seismometers that spanned almost 20° in latitude (see Figure 11 and the inset to that figure for the locations of the seismometers within 2D spatial Voronoi cells). The seismometers were located along lines 24A and 25A of the USAArray Transportable Array network and a total of 54, 55, 56 and 51 array seismometers were used as the backbone
seismometers to reconstruct the earthquake seismograms at target sensors WDC, BMN, DUG and P21A, respectively.

These virtual seismograms are shown in Figure 12a. SRI results are plotted against epicentral distance and overlain by the real earthquake recordings at these locations (where the latter exists). As for the previous example, a bandpass filter with corner frequencies at 0.04 Hz and 0.06 Hz has been applied to both the SRI reconstructions and the real recordings and all seismograms have been normalised to their absolute maximum amplitude. The main surface wave arrivals are reconstructed by SRI but there are differences in the phase of these virtual arrivals compared to the recorded data. Artefacts are present both before and after the main surface wave arrivals at sensors BMN and DUG, but we note that the best reconstruction is at target sensor WDC, which is located over 1900 km from the backbone array and within 200 km of the earthquake epicentre. The quality of the match between the real and SRI seismograms is quantified by the correlation coefficient and we observe a maximum value of 0.71 for the reconstruction at WDC.

Target sensor P21A (unfilled triangle in Figure 11) was previously active but was removed from its site before the earthquake occurred. This seismogram is thus constructed at a truly retrospective location - a location chosen from previous seismometer locations after the event occurred. The result is shown at the top of Figure 12a and constitutes a new virtual seismogram for that receiver, from an earthquake that occurred after the receiver had been removed. The main surface wave appears to be constructed but large amplitude non-physical arrivals are also constructed at earlier travel times (see grey ellipses), indicating that this reconstruction is unreliable.
We propose that the largely inaccurate SRI reconstructions at target sensors BMN, DUG and P21A are a consequence of incorrectly approximating the stationary phase region of the receiver boundary $S$, i.e., fewer backbone seismometers are required on $S$ when the backbone seismometer-target sensor distance is short [Snieder et al., 2006b]. Thus, we perform correlation-correlation SRI using spatially restricted backbone arrays that consist of only those seismometers whose locations approximate the stationary phase points of the receiver boundary $S$ in Equation 12. The arrays used are shown in Figure 12c, embedded within Voronoi cells, and are used to reconstruct the SRI seismograms at target sensors BMN, DUG and P21A as shown in Figure 12b. All backbone seismometers as shown in Figure 11 are used to construct the SRI seismogram at target sensor WDC. Note that the SRI reconstruction at P21A requires the fewest backbone seismometers. This is a consequence of the close proximity of target sensor P21A to the backbone array in Figure 11 and thus a narrower stationary phase region around the receiver boundary $S$. We observe a significant improvement in the results of correlation-correlation SRI when using the shorter backbone seismometer arrays that more accurately approximate the stationary phase regions of $S$ for each target sensor at $r$ in Equation 12. For example, compare the SRI seismograms constructed at target sensor BMN in Figures 12a and 12b. When using all 54 seismometers in (a) the match between the real and SRI seismograms is poor as the correlation coefficient is just 0.51. This correlation coefficient increases to 0.71 in (b) when using the restricted array of only 40 seismometers to construct the SRI seismogram. Similarly, the amplitudes of the non-physical arrivals constructed between $\sim 300$ s and $450$ s in the SRI seismogram at target sensor P21A in (a) (grey ellipses) are significantly reduced when using the shorter array. We found that to construct the SRI
seismogram at a given target sensor, only those backbone array seismometers located at
distances within $\sim 250$ km of the shortest backbone seismometer-to-target sensor distance
should be used.

Next we used correlation-convolution SRI (Equations 9 and 15, Figure 2b) to recon-
struct earthquake seismograms at 4 additional target sensors (sensors R29A, R30A, R31A
and GOGA). Again, we begin by using large backbone arrays (as shown in Figure 13)
to construct the SRI seismograms. We extend the source-receiver geometry to target
sensor GOGA located up to 2420 km from the backbone seismometer array and over
3700 km from the earthquake epicentre. Note that since processes of convolution do not
involve time reversal (complex conjugation), convolutional interferometry does not have
an implicit assumption of zero attenuation [Halliday and Curtis, 2009]. In this example
however, noise-based correlational interferometry in Equation 9 is still used to construct
the SRI propagators $G(x, r)$, hence this correlation-convolution approach to SRI still has
an implicit acoustic assumption, which is questionable at these propagation distances.

The SRI seismograms are plotted in Figure 14a and compared with the real recordings
of the earthquake at those same sensors. Despite the assumption made herein for lossless
media, the main surface wave arrivals can be traced across all target sensors, and the SRI
seismograms are in reasonable agreement with the real recordings at target sensors R29A,
R30A and R31A. The quality of these matches is quantified by the correlation coefficients
which have intermediate values of 0.57, 0.51 and 0.70, respectively. However we note that
there is a time shift in the reconstructions of these main arrivals compared to the real
recordings.
Again, we propose that these inaccurate reconstructions are a consequence of the large backbone arrays used and thus an incorrect destructive/constructive interference of the data in Equation 15. In Figure 14b we reconstruct the SRI seismograms at R29A, R30A and R31A by invoking the criteria described above for an array restricted to backbone seismometers located within 250 km of the shortest backbone seismometer-target sensor distance. Thus, all backbone seismometers are located within 450 km of target sensors R29A, R30A and R31A, in comparison to backbone seismometer-target sensor distances up to 1200 km in (a). For the reconstruction at target sensor R31A we used seismometers along lines 26A and 27A in this example as Line 28A is largely incomplete within the central region of the array that approximates the stationary phase region (see Figure 13). Invoking the criteria for a restricted backbone array increases the correlation coefficients of the SRI seismograms as the main surface wave arrivals constructed by SRI are now in phase with the real earthquake recordings and the time shifts observed in Figure 14a have been corrected.

At the largest length scale at target sensor GOGA, all backbone array seismometers along lines 24A and 25A of the array shown in Figure 13a are used to construct the SRI seismogram in Figure 14a, as little increase in quality was observed by further limiting the array. The main surface wave is reconstructed but is of a similar amplitude to earlier spurious arrivals at $\sim 800$ s (see grey shaded ellipse in Figure 14a). This makes it difficult to distinguish and identify the main arrivals without the real recording of the event for reference. Furthermore, the correlation coefficient is low at 0.15. SRI seismograms constructed at this largest length scale are thus less accurate than those constructed over shorter backbone seismometer-target sensor distances.
As shown for correlation-correlation SRI it appears that it is better to use only the backbone seismometers that are necessary to construct the arrivals of interest (e.g. those within the region in which the phase of the integrand of SRI integrals is approximately stationary - Snieder et al. [2006b]), rather than including all possible backbone sensors. In the latter case one would hope that contributions to the SRI integrands from those backbone seismometers that do not construct arrivals of interest will cancel out to zero. We suspect that such cancellation imposes more stringent requirements on the backbone array geometry than can generally be accommodated using real arrays.

5. Discussion

By using 3 separate example applications we are able to assess the ability of source-receiver interferometry (SRI) to reconstruct seismograms on target seismometers across different spatial scales. On the two smaller length scales (up to ~ 540 km), the main surface wave arrivals of the SRI seismograms were in agreement with the real recordings, and some of the virtual reconstructions matched the recorded data some way into the coda (as found at a single length scale by Curtis et al. [2012]). However, at the largest length scale the SRI seismograms were poorly constructed as large spurious arrivals and significant time shifts were present in the virtual reconstructions compared to the real seismograms. These inaccurate reconstructions were a consequence of using all available backbone array seismometers, which led to incorrect constructive/destructive interference of the data in SRI integrals. We instead found that using spatially restricted backbone arrays that were scaled by the minimum backbone seismometer-target sensor distance constructed more reliable SRI seismograms.
5.1. Comparison of correlation-correlation SRI and correlation-convolution SRI

At the intermediate length scale we were able to compare directly the SRI seismograms constructed using two correlation integrals within SRI (Figure 8a, top 4 traces) to those seismograms constructed using the joint correlation-convolution approach (Figure 10). In the correlation-correlation reconstructions we identify spurious events that appear prior to the first surface wave arrivals (see the grey shaded ellipse in 8a). These spurious arrivals are associated with non-physical energy that should cancel in the integration but does not when the medium is strongly scattering or when the surrounding source (or receiver) boundary is incomplete. Such non-physical arrivals are constructed by cross-correlating direct wavefields with scattered wavefields [Halliday and Curtis, 2009]. Under ideal conditions these non-physical arrivals mutually cancel with non-physical arrivals constructed by the cross-correlation of purely scattered wavefields. However, when the medium is strongly attenuating energy is lost during the propagation of the wavefields from the boundary sources (in this case noise sources) to the receiver locations. This introduces amplitude imbalances into the non-physical arrivals and they no longer mutually cancel, introducing artefacts into the interferometric estimations. Furthermore, Halliday and Curtis [2009] find that non-physical arrivals are also enhanced when the source aperture is limited i.e., when sources are not present at all required stationary phase points. The same argument can also be made for a limited receiver aperture in inter-source interferometry, i.e., when spatial irregularities in the backbone array result in unoccupied stationary phase points on the receiver boundary.

Since correlation refers to processes of complex conjugation (time-reversal), and wavefields can not theoretically be time-reversed in an attenuative medium without the re-
injection of all of the lost (attenuated) energy, Halliday and Curtis [2009] propose that
the method of convolution may be performed over correlation when attenuation is strong.
When following a convolution approach, all non-physical arrivals cancel to zero [Halliday
and Curtis, 2009] and no artefacts are introduced into the interferometric result due to
amplitude imbalances. Similarly, limited apertures do not introduce non-physical arrivals
when using a convolution approach. We conclude that when the target sensors are located
at intermediate distances from the backbone seismometer array (here, up to 540 km), and
when the backbone seismometer geometry allows, correlation-convolution SRI is advan-
tageous as the SRI propagators are constructed well via processes of correlation, and any
non-physical arrivals that do exist are suppressed in the inter-source interferometry step
by processes of convolution.

5.2. Observing the effect of spatial irregularities within the backbone arrays

To compensate for spatial irregularities within the backbone arrays, and to thus dis-
cretize the inter-source interferometric integrals in Equations 12 and 13 more precisely,
the backbone seismometer arrays used in Sections 4.3 and 4.4 were embedded within 2D
spatial Voronoi cells, the area of which provide $\Delta x_i$ in Equations 14 and 15. Figure 8b
shows the SRI seismograms of the M 5.8 Mexico earthquake studied in Section 4.3 con-
structed without Voronoi interpolation of the summand in Equation 14. Instead, a simple
summation was performed over the original (un-weighted) values of the summand (i.e.,
without term $\Delta x_i$) for each backbone seismometer. Comparing Figure 8b with Figure 8a
we observe that all matches between the real recordings and the SRI reconstructions are
of a much poorer quality when the integration is performed without Voronoi interpola-
tion, and include large amplitude artefacts prior to the main surface waves. Since we use
Voronoi interpolation to approximate the second step of SRI above, and since that step is simply the reciprocal of inter-receiver interferometry, it is likely that more generally Voronoi interpolation would increase the accuracy of most applications of interferometry where boundary source or receiver locations are known.

When reconstructing SRI seismograms at the intermediate length scale (Section 4.3), only a small subset of the array seismometers were actually used in any one reconstruction, despite over 90 seismometers fitting the criteria outlined in Section 4.1, and in both earthquake seismology examples (Sections 4.3 and 4.4) only 2 lines of seismometers were used to create the SRI reconstructions. Surprisingly, preliminary studies on many data showed that a thicker band of seismometers did not improve the SRI results. This contradicts what might be expected given the results of Draganov et al. [2004], Halliday and Curtis [2008] and Kimman and Trampert [2010] whose work suggests that thicker boundaries should provide better results if the boundary is not in the very far field. For a target sensor located within or close to the backbone array, the final geometrical criteria that were highlighted by the preliminary studies and found to work best are described in Section 4.1 and illustrated schematically in Figure 6.

We observed that even a slight deviation from these criteria can result in significantly poorer SRI reconstructions. For example, consider the first criterion in Section 4.1, that the backbone array should comprise 2 parallel lines of approximately regularly spaced seismometers. This condition aims to fulfil the requirement that the backbone array seismometers occupy the stationary phase points of the receiver boundaries in Equations 14 and 15 [Halliday and Curtis, 2009]. In the backbone array shown in Figure 7 we observe that lines V and 3 are the most spatially irregular, comprising far fewer seismometers
than the other lines. Seismometers along lines V and 3 were used to reconstruct the SRI
seismograms at target sensors Z27A and 627A, respectively, and in Figure 8a we observe
that the SRI seismograms constructed at these target sensors exhibit the lowest correlation
coefficients. We thus conclude that these poorer reconstructions are partly a consequence
of spatial irregularities within the 2 parallel lines of backbone array seismometers. This
breaks the condition that receivers are to be located around the stationary phase points
of the receiver boundary, and consequentially introduces non-physical arrivals into the
interferometric reconstructions as described above. Nevertheless, a comparison of Figure
8a with Figure 8b shows that Voronoi interpolation does contribute to resolve the issue of
spatial irregularity as data are effectively interpolated into areas of the receiver boundary
where seismometers are missing. It may be that in future a more accurate (higher order)
method of interpolation within and across Voronoi cells may further diminish such issues.

5.3. Performing SRI at the largest length scale

The large scale source and receiver geometries outlined in Section 4.4 were chosen specif-
ically to challenge the method: the limitations of SRI were identified and new criteria were
established to correct for inaccurate data. We demonstrated the importance of correctly
approximating the stationary phase region of the receiver boundary $S$ in Equation 12 (or
Equation 13) by comparing SRI seismograms constructed using all available backbone
seismometers to those constructed using spatially restricted backbone arrays. For a given
target sensor the restricted array consisted of seismometers located within 250 km of the
shortest backbone seismometer-target sensor distance. Invoking this criteria for restricted
arrays constructed SRI seismograms that were a significant improvement on SRI seismo-
grams constructed using all available backbone seismometers (see Figures 12a and 12b, and Figures 14a and 14b for comparisons of SRI seismograms).

We note however that at the largest length scale presented here at target sensor GOGA, the SRI seismogram of the M 6.5 earthquake was constructed poorly. Here we propose that this is a consequence of poorly constructed SRI propagators that are thus unable to redatum the earthquake energy from the backbone array seismometers to the locations of the most distant target sensors (e.g. due to a breakdown in the underlying assumption of zero attenuation for correlational interferometry).

5.4. The role of the SRI propagators

The inter-receiver Green’s function estimates (the SRI propagators) are clearly key in SRI. They were constructed using noise interferometry in Equation 9 and were selected to have a high signal-to-noise ratio (SNR) and hence (by our definition of SNR) are usually dominated by a main surface wave arrival. During the second correlational step of SRI, the phase of the propagators is subtracted from the phase of the backbone recordings through processes of time-reversal. This has the effect of (computationally) redatuming the recorded energy onto the target sensor location, but dominantly this focusses the main surface wave arrival since that energy dominates the SRI propagators. To get the one-sided Green’s function $G(x, r)$ for this redatuming step in Equations 14 and 15, the Green’s function constructed at negative time in $G_H$ was time-reversed and added to the Green’s function constructed at positive time. Since the results of noise interferometry are dependent on the directivity of the ambient wavefield, this would not be necessary if one side of the estimated Green’s function was predominantly noise; instead one could simply take the side within which the signal is constructed. Here we did not notice
any particular consistent increase or decrease in the quality (signal-to-noise ratio) of the
estimated Green's function when using a stacked summation of both sides of the Green's
function, so we decided to continue with that approach to be consistent with previous
studies.

Since attenuation may be an issue over long inter-receiver distances (e.g. from the
backbone array to target sensor GOGA), another approach to estimate these propagators
might be to use deconvolutional interferometry in place of correlational interferometry
[Snieder and Safak, 2006]. The use of multidimensional deconvolution [Wapenaar et al.,
2008] has been shown to be able to compensate for various deficiencies in correlational
interferometry, so there may be merit in future testing of deconvolutional forms of SRI.

5.5. Future applications of SRI

In this study, target sensor WDC was located less than 200 km from the epicentre of
the M 6.5 earthquake that occurred off the coast of California. Thus, it approaches the
proximity of the location of rapid response local temporary seismometer arrays that are
often deployed around large earthquake epicentres after the event has occurred, in order
to continuously monitor the area for subsequent aftershocks. If such temporary seismome-
ters are deployed for 6 months and ambient noise data is collected, this example shows
that SRI might then be applied to construct local virtual seismograms of the earthquake
retrospectively, even when the event occurs far away from the permanent backbone array.
These virtual earthquake seismograms could then also be compared with any subsequent
aftershocks recorded on the temporary seismometer array during its deployment. Alter-
natively, one can construct virtual SRI seismograms of an earthquake in almost real time
using seismometers that used to be deployed close to where an earthquake has occurred,
but which have since been removed. All one requires is a backbone array of seismometers whose deployment spans both the time at which the earthquake occurred and the time during which the target sensors were previously deployed. In addition the C3 (correlation of coda of correlation) method described by [Ma and Beroza, 2012] allows for the retrieval of Green’s functions between pairs of seismometers whose times of deployment do not coincide. SRI propagators constructed from ambient noise observations recorded asynchronously could significantly increase the number of practical applications of SRI as all target sensor-backbone seismometer pairs would not need to be deployed coincidentally in time.

Furthermore, we see the benefit of seismometer arrays with deployment strategies similar to the USArray Transportable Array which routinely deploys approximately regularly spaced seismometers for up to 2 years before they are moved to a set of new locations. During their deployment, recordings of ambient seismic noise can be made on all seismometers and cross-correlated to construct all possible inter-receiver Green’s functions. This creates an archive of the propagators needed for SRI which can subsequently be inserted into Equation 14 or Equation 15 when an earthquake occurs. Virtual earthquake seismograms can then be reconstructed on any seismometer deployed before, during or after the event, provided that a suitably dense geometry of other seismometers recorded the earthquake. We note though that the ideal array deployment strategy for SRI would be for a dense backbone array to remain permanently in place while other roving sensors occupy temporary recording locations, as outlined in Curtis et al. [2012], which unfortunately was not the design of the deployment strategy used for USArray.
Finally, we comment on the results of Curtis et al. [2012] in which the SRI reconstructions were used to determine independent information about the source phase. In order to obtain this information correctly one would require the source-receiver geometry outlined in Figure 2a in which backbone seismometers occupy both stationary phase regions of the boundary $S$ surrounding the earthquake source at $s$ and the target sensor at $r$ (see the grey shaded regions in Figure 2a for a schematic representation of these stationary phase regions). Using both stationary phase regions of the receiver boundary, one would correctly construct both the causal ($C = T^*G$) and acausal ($A = T^*G^*$) sides of $T^*G_H(r, s)$ in Equation 14. Curtis et al. [2012] then theoretically show that by calculating the ratio $C/A$, the phase of the source time function can be estimated independently from the phase of the Green’s function, without using inverse theory. Such independent phase information can then be used to obtain surface wave phase velocity estimates along event-to-seismometer paths that are an improvement on the estimates obtained from real recorded data alone. This application of SRI could thus improve the results of surface wave tomography studies. Unfortunately in the current study we did not have backbone array seismometers in both stationary phase regions, hence we could not test this method here.

6. Conclusion

We demonstrate the construction of band-limited virtual seismograms of waves from seismic sources on sensors that were not necessarily deployed when the source occurred. We do this using both correlation-correlation and correlation-convolution source-receiver interferometry (SRI): cross-correlations of seismic energy recorded over one period of time are used as propagators to redatum the source signals currently recorded at one
set of locations onto new sensor locations, at any time after the source has occurred, using processes of correlation or convolution. The source signals need never be physically recorded at the new sensor locations.

This is possible on a variety of length scales and in a variety of settings. A small length scale, industrial seismics example uses correlation-correlation SRI to yield band-limited reconstructions of an active seismic shot on 6 sensors located up to 62 m from a regular line of receivers. An intermediate length scale example in an earthquake seismology setting uses correlation-correlation SRI to reconstruct virtual seismograms of a M 5.8 earthquake on 8 target sensors located approximately between 210 km and 540 km from a backbone array of seismometers. Correlation-convolution SRI is also applied and is found to stabilise the SRI reconstructions, allowing spurious events to be identified that are associated with terms in equations that should (but may not) mutually cancel when using correlation-correlation SRI. The virtual earthquake seismograms constructed using SRI were compared with the real event recordings and the quality of this match, as quantified by the correlation coefficient, was as high as 0.93 on one occasion. Finally, a large length scale example within an earthquake seismology setting was designed to challenge the methodology: this showed the limitations of correlation-correlation SRI and correlation-convolution SRI as virtual seismograms of a M 6.5 earthquake were reconstructed poorly (correlation coefficients as low as 0.15) on target sensors located up to 2420 km from a backbone array of seismometers. However, we showed that more robust SRI seismograms could be constructed by restricting the backbone array to include only those seismometers whose locations better approximated the stationary phase points of the receiver boundary.
We show that discretizing the integrals from interferometric theory using Voronoi cell based interpolation between seismometers on integral boundaries improves interferometric results significantly. We also constructed two completely new virtual seismograms. These are reconstructions of the earthquake seismograms on two sensors that were not deployed when the events occurred. When plotted alongside the virtual seismograms reconstructed on the target sensors at which we have real recordings for comparison, we see that all reconstructions follow the same moveout curves. All virtual earthquake seismograms were however constructed within a very narrow frequency band (0.04 Hz - 0.06 Hz) which currently limits the implementation of the data in future crustal seismology studies.

These multi-length scale applications of SRI in both engineering and earthquake seismology settings begin to pave the way for a new type of seismology: a form of “retrospective seismology” where virtual seismograms can potentially be constructed at new, desired locations - locations determined after the energy from the source has dissipated and where, with hindsight, one would have liked to have had sensors installed.

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Figure 1. Schematic source and receiver geometries for (a and b) inter-receiver interferometry and (c and d) inter-source interferometry to construct the Green’s function $G(r_1, r_2)$ or $G(s_1, s_2)$. Stars are sources, triangles are receivers. In (a) and (b) the responses from each source at locations $x'$ on boundary $S'$ are recorded on receivers $r_1$ and $r_2$ which are then (a) cross-correlated or (b) convolved. (c) and (d) By applying source-receiver reciprocity to (a) and (b), the responses from sources at $s_1$ and $s_2$ are recorded by each receiver at locations $x$ on boundary $S$ and (c) cross-correlated or (d) convolved. In all four cases the results of all cross-correlations or convolutions are summed (integrated over $S'$ or $S$) to obtain an approximation of the Green’s function between $r_1$ and $r_2$, or $s_1$ and $s_2$. 
Figure 2. Three possible geometries for source-receiver interferometry (SRI) pertaining to (a) correlation-correlation SRI, (b) correlation-convolution SRI, and (c) convolution-convolution SRI. Key as in Figure 1. The grey shaded regions in (a) and (b) highlight schematically the stationary phase regions on the receiver boundary which contribute constructively to energy in SRI integrals; in (a), only one or other stationary phase region need be used provided back-scattered energy is weak across the boundary. For full derivations of the SRI integrals used in each example see Curtis and Halliday [2010].
Figure 3. Illustrations of the discretization of Equation 12 or Equation 13 for seismometers (black circles) at positions $i \ldots n$ located: a) along a 1D line such that $\Delta x_i$ is the sum of half-distances to the next seismometer on either side, and b) within a 2D latitude/longitude ($\lambda/\phi$) grid such that $\Delta x_i$ is the surface area of the portion of space closer to that seismometer than neighbouring seismometers. In (b) the cells (polygons) have been shaded according to their size, normalised to the largest cell.
Figure 4. Geometry for the small-scale seismic experiment. (a) Active sources are located along the dashed boundary $S'$ (small/black stars) and at position $s$ (large/red star). Receivers are located along line $S$ (circles - every second receiver is shown here for clarity) and at points $r_i$ (triangles) close to the source at $s$. Only those boundary sources located approximately around the stationary-phase region of $S'$ are used (small/black stars). (b) Magnified view of the active source and the target receivers marked by triangles with receiver numbers shown in the numbering scheme of Duguid et al. [2011]: results for these receivers are shown in Figure 5.
Figure 5. Comparison of surface wave (ground roll) seismograms constructed using SRI (solid/red traces) with the real recordings (dashed/blue traces) at the target sensors \( r_i \) shown in Figure 4b. All seismograms are bandpassed in the frequency range 15.5 Hz - 20 Hz, chosen because that range contained all dominant amplitudes that were common to all recordings at \( x \) and \( r_i \) in Figure 4.

Figure 6. Schematic diagram illustrating the spatial criteria specified in Section 4.1 for a source \( s \) (star), target sensor \( r \) (triangle), band of backbone seismometers \( x \) (circles) and great circle path (GCP) passing between \( s \) and \( r \). The criterion used in Section 4.3 that the backbone seismometers should lie between a distance of 210 km and 540 km from the target sensor is also indicated, as is the intention that the intersection angle \( \Theta \) should be between 70° and 110°.
Figure 7. Source and receiver geometries used to reconstruct virtual seismograms of the 27/04/2009 M 5.8 Mexico earthquake (large star with source mechanism) using correlation-correlation SRI (Figure 2a). Ambient noise fluctuations from source locations such as $x'$ (small stars positioned schematically in oceans) are recorded on the backbone seismometer array $x$ (circles) and at the 8 target sensors $r$ (filled and unfilled triangles) located within New Mexico. The backbone array consists of 8 approximately parallel lines of seismometers from Line V in the North of the array, to Line 3 in the South (the letters derive from station notation employed in naming the USAArray Transportable Array). 2-line combinations of the backbone array seismometers are used to reconstruct virtual earthquake seismograms at each target sensor by interpolating interferometric integrands across the Voronoi cells (polygons - shaded according to their area). To reconstruct the virtual earthquake seismogram at sensor Z27A, only those seismometers along lines V and W comprise the backbone array; to reconstruct the virtual earthquake seismogram at sensor 127A, only seismometers along lines W and X comprise the backbone array; and so on, until the virtual earthquake seismogram at sensor 627A is reconstructed using an array comprised of seismometers from lines 2 and 3 only.
Figure 8. Comparison of seismograms for the M 5.8 Mexico earthquake constructed using correlation-correlation SRI (solid/red traces) with the real recordings (dashed/blue traces) at target sensors Z27A to 627A and at 226A in (a). At 226A there is no real recording of the event. The quality of the match between the real and SRI seismograms is quantified by the correlation coefficient, stated on the right above each trace. (a) SRI seismograms are constructed by integrating over 2-line combinations of seismometers within the Voronoi cells shown in Figure 7 as described in the caption to that figure and stated here on the left above each trace. (b) The SRI integral in Equation 12 is approximated by performing a direct summation over all backbone seismometer locations, rather than as an interpolated sum over Voronoi cells. The grey shaded ellipse in (a) highlights the spurious, non-physical arrivals that occur prior to the main surface wave arrivals constructed at target sensor Z27A.
Figure 9. Source and receiver geometries used to reconstruct virtual seismograms of the 27/04/2009 M 5.8 earthquake in Mexico using correlation-convolution SRI (Figure 2b). Key as in Figure 7. 2-line combinations of backbone array seismometers are used to reconstruct virtual earthquake seismograms of the earthquake at sensors Z27A, 127A, 227A and 226A by interpolating the interferometric integrands in equations in the text over the Voronoi cells shown in the inset (shaded polygons). To reconstruct the virtual earthquake seismogram at sensor Z27A, only those seismometers along lines 3 and 4 comprise the backbone array; to reconstruct the virtual earthquake seismogram at sensor 127A, only seismometers along lines 4 and 5 comprise the backbone array, and finally, the virtual earthquake seismograms at sensors 226A and 227A are constructed using an array comprised of seismometers along lines 5 and 6 only.
Figure 10. Comparison of seismograms for the M 5.8 Mexico earthquake constructed using correlation-convolution SRI (solid/red traces) with the real recordings (dashed/blue traces) at target sensors Z27A, 127A, 227A and 226A. Key as in Figure 8. The geometry for this correlation-convolution approach to SRI is shown in Figure 9. The grey shaded ellipse highlights the region in which higher amplitude spurious, non-physical arrivals were reconstructed when using correlation-correlation SRI (see Figure 8, top trace).
Figure 11. Source and receiver geometries used to reconstruct the virtual seismograms of the 10/01/2010 M 6.5 earthquake that occurred off the coast of California using correlation-correlation SRI (Figure 2a). Key as in Figure 7. The backbone array consists of 2 adjacent lines of seismometers with Line 24A on the left and Line 25A on the right. Combinations of the 57 seismometers within the array are used to reconstruct virtual earthquake seismograms at target sensors WDC, BMN, DUG and P21A by interpolating interferometric integrands across the Voronoi cells shown in the inset (shaded polygons).
Figure 12. a) Comparison of seismograms for the M 6.5 earthquake that occurred off the coast of California constructed using correlation-correlation SRI (solid/red traces) with the real recordings (dashed/blue traces) at target sensors WDC, BMN, DUG and P21A. At P21A there is no real recording of the event. Key as in Figure 8. Virtual earthquake seismograms are constructed using the full backbone receiver array shown in Figure 11. b) As (a) but virtual earthquake seismograms at BMN, DUG and P21A are constructed using spatially restricted backbone arrays consisting of seismometers whose locations are expected to better approximate the stationary phase points of the receiver boundary $S$ for the target sensor at $r$ in Equation 12: these restricted arrays within 2D spatial Voronoi cells are shown in (c). The Voronoi cells are shaded according to their area as in Figure 7.
Figure 13. Source and receiver geometries used to reconstruct virtual seismograms of the 10/01/2010 M 6.5 earthquake that occurred off the coast of California using correlation-convolution SRI (Figure 2b). Key as in Figure 7. The backbone array consists of 5 adjacent lines of seismometers from Line 24A on the left of the array, to Line 28A on the right of the array. 2-line combinations of the backbone array seismometers are used to reconstruct virtual earthquake seismograms at target sensors R29A, R30A, R31A and GOGA by interpolating interferometric integrands over the corresponding 2D Voronoi cells. (b) The Voronoi cells within which the array seismometers are embedded.
Figure 14. a) Comparison of seismograms for the M 6.5 earthquake that occurred off the coast of California constructed using correlation-convolution SRI (solid/red traces) with the real recordings (dashed/blue traces) at target sensors R29A, R30A, R31A and GOGA. Key as in Figure 8. Virtual earthquake seismograms are constructed using 2-line combinations of the full backbone receiver array shown in Figure 13. b) As (a) but for target sensors R29A, R30A and R31A only. Virtual earthquake seismograms are constructed using restricted backbone arrays consisting of only 11 or 12 seismometers that are located along the central region of the receiver lines stated on the left above each trace and displayed in Figure 13.