Title: Anisotropic seismic noise gradiometry by elliptically-anisotropic wave equation inversion: an example at Ekofisk

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Summary

We propose an anisotropic wavefield gradiometry technique to extract azimuthally anisotropic phase velocities from seismic noise that is dominated by a single surface wave mode. The method relies on a two-dimensional elliptical-anisotropic wave equation. This wave equation equates the spatial derivatives of the wavefield amplitudes with the temporal derivatives through the elements of a two-by-two matrix characterizing the medium parameters. The derivatives are evaluated using finite differences, and the system is inverted with a smoothness constraint. We test the procedure on ambient seismic noise recorded in a large and dense array installed over Ekofisk field. Because the station spacing is much larger cross-line than inline, the approximation error of the spatial finite difference results in an apparent anisotropy. From an experiment with synthetic isotropic plane waves, we define a Jacobian to correct the finite difference stencils. With the corrected finite difference stencils, we extracted anisotropic phase velocities at Ekofisk from as little as 10 minutes of seismic noise recordings. The azimuthal anisotropy forms a circular geometry around the production induced subsidence bowl. The methodology is a promising technique for studying changes in the subsurface geomechanical stress-state resulting from time-dependent phenomena operating at a short time-scales.
Introduction

We aim to extract medium parameters close to receivers from the gradients of recorded wavefields. Previously Curtis and Robertsson (2002) proposed to directly invert the 3D elastic wave equation using volumetric recording systems to capture all necessary spatial gradients, however volumetric recordings are rarely available. Langston (2007a) devised wavefield gradiometry to extract properties such as ray parameter and wave directionality from non-overlapping plane waves emitted by sources far away, but the plane wave assumption prohibits this method from application to recordings of seismic noise. A direct estimate for the phase velocity can also be found by evaluating an eikonal or Helmholtz equation on the travel-times of virtual seismic sources (Lin and Ritzwoller, 2011; De Ridder et al., 2015). Li and Holt (2015) described a link between Helmholtz tomography and wavefield gradiometry, as applied to plane waves from large earthquakes. De Ridder and Biondi (2015) introduce a method for surface-wave seismic noise that harnesses the directional invariance of the Laplacian in a two-dimensional wave equation. However, De Ridder and Biondi (2015) found it challenging to evaluate the spatial finite differences, because the velocity reduction method of Langston (2007b) cannot be applied on seismic noise. Here, we propose seismic noise gradiometry for azimuthally elliptically-anisotropic surface waves. We show that this naturally leads to a calibration to correct angle-dependent errors in spatial finite difference stencils. A permanent and spatially dense OBC array installed over Ekofisk oilfield with previously detected surface-wave azimuthal anisotropy offers an ideal opportunity to demonstrate the technique (Kazinnik et al. 2014, De Ridder et al., 2015).

Elliptical-anisotropic wave-equation inversion

Surface waves propagate predominantly in two dimensions and exhibits azimuthal anisotropy, so we construct a time-domain two-dimensional wave equation that exhibits elliptically anisotropic wave propagation. Generally surface waves are dispersive, however after applying a sufficiently narrow bandpass we can neglect the dispersion. A dispersion relationship for anisotropic, monochromatic waves can be expressed as \( c |k| = \omega \), where \( k = k(\phi) = \left[ k_x(\phi), k_y(\phi) \right] \) is the wave vector, in which \( \phi \) is the direction of wave propagation. We define the phase-velocity, \( c = c(\phi) \), to exhibit elliptical anisotropy: \( c^2(\phi) = c^2_s(\phi - \alpha) + c^2_t \sin^2(\phi - \alpha) \), with fast and slow phase velocities \( c_s \) and \( c_t \), and fast azimuth \( \alpha \). Inserting this phase velocity into the dispersion relationship, substituting \( k_x = |k| \cos(\phi) \) and \( k_y = |k| \sin(\phi) \), and applying a spatial and temporal inverse Fourier transformation to find a wave-equation operating on an arbitrary scalar wavefield, \( U = U(x,y,t) \):

\[
\begin{bmatrix}
\partial_x \\
\partial_y
\end{bmatrix}
\begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
\partial_x \\
\partial_y
\end{bmatrix}
U = \partial_x \partial_y U,
\]

where the parameters \( M_{11}, M_{12}, M_{21} \) and \( M_{22} \) form a two-by-two matrix \( M \), and are found as

\[
M_{11} = (c_s^2 - c_t^2) \cos^2(\alpha) + c_t^2, \quad M_{12} = (c_s^2 - c_t^2) \sin^2(\alpha) + c_t^2, \quad M_{21} = (c_s^2 - c_t^2) \cos(\alpha) \sin(\alpha).
\]

The fast and slow phase velocities and their directions can be found by computing the eigenvalues and eigenvectors of \( M \). (De Ridder et al., 2005) This form of the wave equation can also be found by starting with the isotropic wave-equation and performing a coordinate rotation and scaling. We will invert for a spatial varying anisotropic phase-velocity field by inverting Equation (1) locally. To this end, we evaluate the spatial and temporal derivatives of the wavefield by finite differences (Huiskamp,1991). As a consequence we cannot resolve anomalies on a scale smaller then the spatial stencil size.

In order to invert the wave equation (1), we form a linear system of the form \( \mathbf{F} \mathbf{m} = \mathbf{b} \), where \( \mathbf{F} = \left[ \begin{array}{ccc}
\partial_x \partial_x U & \partial_x \partial_y U & \partial_y \partial_y U \\
\partial_y \partial_x U & \partial_y \partial_y U & \partial_y \partial_y U \\
\partial_x \partial_x U & \partial_x \partial_y U & \partial_y \partial_y U
\end{array} \right], \quad \mathbf{b} = \partial_x \partial_y U - M_0 \left[ \partial_x \partial_x + \partial_y \partial_y \right] U, \quad \text{and} \quad \mathbf{m} = \left[ \begin{array}{c}
\Delta M_{11} \\
\Delta M_{12} \\
\Delta M_{22}
\end{array} \right]^T.
\]

We posed the medium parameters as a perturbation on a homogenous average velocity \( M_0 = c_t^2 \). This
system can, for example, be solved by LU decomposition of the least-squares form constrained by 1st and 2nd order Tikhonov regularization, 

$$\sum F_i^T F_i + \varepsilon_1^2 L^T L + \varepsilon_2^2 I \right) m = \sum F_i^T b_i,$$

where I is a three-by-three identity matrix, and 

$$L = \text{diag}\{\nabla^2, \nabla^2, \nabla^2\}.$$

Calibration of the finite difference stencils

Although the time-sampling of a wavefield is generally sufficiently high to avoid large errors in the finite difference approximation of derivatives, spatial sampling is often sparse compared to minimum wavelengths. For the permanently installed seismic array, we find that surface waves travelling with about 500 m/s are aliased in neither inline nor cross-line directions below 0.8 Hz. However, the cross-line spatial sampling is close to Nyquist levels, and finite difference errors for plane-waves therefore depend on the propagation direction. This is apparent if we perform a synthetic-data experiment, using the geometry of the Ekofisk array and simulating plane waves at 0.8 Hz travelling isotropically at 500 m/s. For each station in the array, we used all other stations within 400m radius to compute the spatial derivatives of the recorded by inverting a set of Taylor series expansion (Huiskamp, 1991). Figure 1 shows the apparent anisotropic velocity, $M_b$, resulting from the finite difference stencil error. The dashes indicate the direction of fast velocity, their length the magnitude of anisotropy as a percentage of the isotropic component, $(c_f - c_i)/c_0$.

We use this measurement to calibrate our finite difference stencils. We factor $M_b$ into a diagonal form using the eigenvectors and eigenvalues to rescale the diagonals to match the velocity of the synthetic data: 

$$M_b = P S^T P^T \left[ \begin{array}{ccc} c_0^2 & 0 & 0 \\ 0 & c_1^2 & 0 \\ 0 & 0 & c_2^2 \end{array} \right] P S P^T,$$

where 

$$S = \left[ \begin{array}{cc} \sqrt{\lambda_1} / c_0 & 0 \\ 0 & \sqrt{\lambda_2} / c_0 \end{array} \right]$$

contains the two scaled eigenvalues, and 

$$P = \left[ \begin{array}{c} p_1 \\ p_2 \end{array} \right],$$

contains the two eigenvectors in the columns.

We recognize that if we define $J = PSP^T$, and insert $M = J^T M^T J$ into Equation (1), we have effectively used a Jacobian to calibrate the finite difference stencils and find a calibrated $M'$.

We test our procedure by performing another five synthetic tests, one with isotropic plane-waves, and four with anisotropic plane-waves with different principle directions, see Figure 2. We retrieve approximately the correct direction of anisotropy with a significant underestimation.

![Figure 1](image1.png)

**Figure 1** The apparent anisotropic velocity resulting from wavefield gradiometry on synthetic isotropic plane waves in the Ekofisk array. The dash in the upper-right corner indicates an anisotropy magnitude of 5%.

![Figure 2](image2.png)

**Figure 2** Retrieved velocity and anisotropy in 5 synthetic tests with an isotropic velocity of 500 m/s (in a-e) and 0 % anisotropy (in a), and 10 % anisotropy (in b-e). The dash in the upper-right corner indicates an anisotropy magnitude of 10 % and the true direction.
Field data application using Ekofisk’s LoFS array recordings

In order to substantiate this technique proposed in the first two sections, we carried out a field data study using ambient seismic noise recordings made in a large and dense array in the Norwegian North Sea. We use ambient seismic recordings from a large and dense array installed over Ekofisk field (Eriksrud, 2010) to verify the method with a field data experiment. The array has dense in-line and sparse cross-line station spacing, respectively 50 m and 300 m, see station map in Figure 3a.

In the absence of seismic shooting, the pressure sensors record strong seismic energy at frequencies between 0.35Hz and 1.35Hz known as microseism noise, which is dominated by fundamental-mode Scholte waves propagating along the seafloor (De Ridder and Biondi, 2015). Below 0.8 Hz, these waves are recorded unalised in both the in-line and cross-line directions. The cables are buried in mud on the seafloor and the stations generally exhibit similar coupling. No strong sources of seismic energy in the microseism frequency range where found within the array.

We bandpass filtered 10 minutes of recording for frequencies between 0.6 Hz and 0.8 Hz. We solved the proposed inverse system without (Figure 3b) and with (Figure 3c) the stencil calibration proposed in the previous section. The resolution of our technique is limited to 800 m by the stencil span.

The fast directions of anisotropy form a subsidence-induced circle tracing the rim of the subsidence bowl. The anisotropy is greater where the near-surface is stretched due to seafloor subsidence (Figure 3a). These results match qualitatively very well with those found using eikonal tomography on the travel-time surfaces extracted from noise correlations (De Ridder et al., 2015) and refracted and surface wave analysis of controlled source seismic (Kazinnik et al. 2014). This anisotropy can be used to infer geomechanical changes in the reservoir and overburden (Herwanger and Horne, 2009).

We significantly underestimate the magnitude of the velocity anomalies, and we under-estimate the magnitude of anisotropy. The latter, is consistent with the observation of the synthetic tests of the previous section. This is likely to be an effect of optimizing the stencils for one particular isotropic phase velocity: because the error in finite difference stencils increases with shorter wavelengths, we underestimate the velocity for higher velocities, and over estimate the velocity for lower frequencies. Iterating the stencil calibration using the retrieved phase velocities may improve the magnitude of the anomalies and anisotropy.

Figure 3 (a) Ekofisk LoFS array stations (white dots), overlain on a map of the Laplacian of the bathymetry, showing the area of extensional stress due to seafloor subsidence in dark blue. (b–c) The anisotropic velocities at Ekofisk, from 10 minutes of seismic noise recordings, (b) without stencil calibration, and (c) with stencil calibration. The dash in the upper-right corner of (b–c) indicates an anisotropy magnitude of 5%.
Conclusions

We propose an anisotropic wavefield gradiometry technique for surface-wave ambient seismic noise based on elliptical-anisotropic wave-equation inversion. A major challenge is evaluating spatial derivatives by finite differences. We overcome this difficulty by a stencil calibration procedure that finds a Jacobian to correct the finite difference stencils. In a field data test application we extracted anisotropic phase velocities from just 10 minutes of seismic noise recordings. The method is a promising technique for studying changes in the subsurface geomechanical strain resulting from time-dependent phenomena operating at a short time-scales – herein, likely subsidence-related extension.

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References