Title:  Finger-Printing Diffractors Encountered by Multiply-Scattered Waves

Authors:  Giovanni Angelo Meles & Andrew Curtis

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Summary:  We present a model-independent method to ‘finger-print’ or tag individual scatterers (diffractors) inside an acoustic medium using move-outs of interrogative wave energy across source and receiver arrays. For any observed multiply-scattered wave the set of such finger-prints is sufficient information to identify the ordered diffraction path of any recorded wave arrival (i.e., including the sequential order in which each scatterer was encountered). A synthetic example shows that the method is capable of disentangling up to 4th order scattering events into their irreducible single-scattering components. This contributes to proper interpretation of (multiply) diffracted energy.
Introduction
Scattered waves recorded in seismic experiments provide both the interpreted signal (primary reflections), and additionally a source of noise for most migration algorithms (multiply reflected and diffracted waves). Identifying and removing multiples is therefore a major concern (Berkhout and Verschuur, 1997), but another approach is to use multiply scattered wave energy rather than discarding it. We show how (multiply) diffracted waves can be identified and interpreted in recorded data. Specifically, we identify the ordered diffraction path of any recorded (multiply) scattered wave.

Method
A basic geometrical property of diffracted waves is that their move-out with respect to source (receiver) position is invariant with respect to receiver (source) position (Khaidukov et al. 2004). In other words, moving the source only affects the constant component of the diffracted arrival times observed across an array of receivers, but not their variation or move-out across the array (Figure 1). By reciprocity, diffraction move-out across a source array is invariant with respect to receiver position. Each scatterer can then be ‘finger-printed’ or tagged uniquely (except for pathological cases) by their source/receiver move-outs.

Figure 1 Scattered wave ray paths. Red stars and blue triangles are sources and receivers, respectively, and red circles are scatterers. The receiver-array move-out associated with scattered waves does not depend on the position of the source since the relative travel time differences are controlled by the distance between the final scatterer and the receivers (blue rays). The travel time between the source and the scatterer (dashed red rays) only changes the constant component of all travel times (the $x_1$, $x_2$ or $x_3$ lag due to different source positions or scattering paths).

When multiple diffractors are embedded within an otherwise homogeneous or smoothly varying medium, measured seismograms record direct, singly- and multiply-scattered waves. Despite their complexity, multiply diffracted wavefields can always be decomposed into 3 paths: a first connecting the source to the first scatterer, a second connecting all of the scatterers involved, and a third, connecting the last scatterer to the receiver. All waves scattered last at any particular scatterer will share the same receiver move-out (Figure 1), and can thus be identified and grouped. A similar argument holds for move-out across the source array, so all waves scattered first at each scatterer can be identified and grouped. For singly-scattered waves, the first and the last scatterer coincide and the corresponding common-receiver/common-source move-out pairs fully fingerprint the scatterer involved. At no point above or below do we need to identify where the scatterers are located or any ray paths; we only identify scatterers by their move-out based ‘finger-prints’.

Now, let $\alpha$ represent an arbitrary scattering path connecting an arbitrarily located source at $x_1$ to a receiver at $x_3$, and let $x_2$ be part of the set of detectors placed along boundary S’ (Figure 2(a)). The last scatterer of $\alpha$, ‘L’ in Figure 2(a), is identified as above by its unique move-out or fingerprint with respect to the set of receivers. We explain in the following how the algorithm depicted in Figures 2(b) to 2(d) allows one to identify the penultimate scatterer, ‘L-1’, by means of cross-convolution and the comparison of wavefields. By induction, this procedure identifies scatterers L-2, L-3, etc., and hence the full scattering path $\alpha$.

Let $x$ be an arbitrary source on surface S (Figure 2(b)). The first part of the algorithm involves cross-correlating the wavefields corresponding to path [x-to-(L-1)-to-L-to-$x_3$] (identified below) and path $\alpha$. In this and subsequent figures, recordings of waves propagating along dashed rays are cross-correlated with those along the solid ray paths; hence (since correlation subtracts the respective phases) this cancels out the travel time along common solid and dashed rays segments.

Assuming for the moment that we know which scatterer is L-1 in path $\alpha$, the identification of the scattering path associated with [x-to-(L-1)-to-L-to-$x_3$] is theoretically straightforward in recorded data.
sets. From geometrical arguments, this event is the earliest arrival with ‘L-1’ as the first, and ‘L’ as the last scatterer; it can therefore be identified on the basis of its move-out with respect to sources on S and receivers on S’ (this will be examined in more detail using the synthetic example presented in Figure 4). There remains the question of how to identify scatterer L-1, which we address below.

Next, we convolve the result of the previous operation with the trace corresponding to the primary scattering event [x-to-(L-1)-to-x’], x’ being the location of an arbitrary receiver on surface S’ (Figure 2(c)). In the synthetic experiments below, we will use x’=x, but x’ this is not necessary. If and only if ‘L-1’ is the penultimate scatterer of α, then the above operations reproduce a wave arrival with the travel time of [x-to-(L-1)-to-x’] that is observed in the recorded data at receiver x’ (Figure 2(d)). This is because convolution results in the addition of the phases (travel times) of the two convolved waves; hence the travel time of the path [x-to-(L-1)] which was subtracted earlier is cancelled by this convolution. More precisely, the presence of energy on the recorded seismograms corresponding to source x1 matching the travel time and move-out (scatterer) L-1 means that the penultimate scatterer ‘L-1’ has been identified. By induction any scatterer in ‘α’ may be identified similarly: the algorithm is simply iterated to identify the penultimate scatterer of this newly identified energy observed in the recordings at x’.

Figure 2. Graphical representation of algorithm discussed in the text. Key as in Figure 1.

Numerical example
We now show how to apply the above algorithm using 2-D acoustic synthetic data computed with a numerical implementation of Foldy’s method (Foldy, 1945; Galetti et al. 2012). The medium has 1 km/s velocity, unit density, and three isotropic scatterers, with 160 sources and 400 receivers (Figure 3). Corresponding gathers are shown in Figure 4.

Actual implementation of the algorithm to decompose the scattering path of events in Figure 4(a) requires the identification in Figure 4(a) of the same travel time differences and move-out (scatterer)
pairs as are required for the combination of primaries and secondaries in Figure 2(a)-2(d). To practically identify the key events depicted in Figure 2 from the data, we proceed as follows:

1) Primaries are the events with the shortest travel time for any given common-source/common-receiver move-out. By comparing primaries at the same source-receiver pair (black lines in Figure 4(b) and 4(c)), link common-source and common-receiver move-outs for any scatterer. Thus we determine the full fingerprint of the scatterer.

2) Secondaries are the events with shortest travel time for a pair of different last- and first-scatterers. Identify these by finding pairs of common-source and common-receiver move-outs that are not both in the fingerprint of a single scatterer (see Figure 4(b) and 4(c), respectively).

The travel time differences between secondaries and primaries for the first scatterer (black braces in Figures 4(a) and (b)) are first found by comparing Figures 4(b) and 4(c), then are identified in 4(a).

Figure 5 shows the seismogram along the bold line in Figure 4(a) along with the ordered scattering paths associated with most of its multiply-scattered events, as identified by the algorithm above. A total of 5 second-order, 5 third-order, and 1 fourth-order events were identified. Thus we prove that the new algorithm reduces recorded multiply-scattered waves into known individual scattering events.

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**Figure 3** Geometry of the model used in numerical examples. Key as in Figure 1. A total of 160 sources and 400 receivers are employed in horizontal arrays (only every twentieth source and fiftieth receiver is plotted for clarity). The 60th source and the 190th receiver (used in Fig. 4) are highlighted by black circles.

**Figure 4** (a) Common-source gather for source $x_1$ of Fig. 3 and, (b) similarly for source number 60. (c) Common-receiver gather for receiver number 190 in Fig. 3. The vertical black line indicates the trace corresponding to source number 60 and receiver 190 where panels (b) and (c) must coincide by reciprocity, thus identifying common-source and common-receiver move-out pairs for the same scatterer. A gain has been applied to the traces at later times.
Figure 5 For the trace along the vertical bold line in Figure 4(a), the scattering path corresponding to the different recorded events are identified by the algorithm (each panel shows a different portion of the waveform). The sequence of scatterers involved in each arrival is shown in the order in which scatterers were visited. Mutually overlapping events were not recovered by the algorithm due to difficulties in move-out discrimination. A gain has been applied to the lower plot (later times).

Conclusions
We have presented the first algorithm to identify full scattering paths of multiply-scattered waves without requiring any model of the interior of the medium of propagation. The algorithm uses move-out based ‘finger-printing’ of individual scatterers. While its efficacy will of course be affected by signal-to-noise issues, in synthetic tests it allowed up to 4th order scattering events to be identified.

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References