Allometry in general

Study of variation in size and shape, especially as an organism grows; statistical shape analysis; scaling laws relating to organisms

Early authors were:

- Snell 1917 (brain size and body size)
- D’Arcy Thompson 1917 *On Growth and Form*
- Huxley 1932 (problems of relative growth)
- Schmidt-Nielsen 1984 (metabolic rate and body size)

Example: the bigger the flying object, the larger its wings have to be

\[ y = 0.352x - 0.3804 \]

\[ R^2 = 0.9825 \]

Covering the largest aircraft, goosander and insects (units are kg and m)

It means wing span = constant \( \times \) mass\(^{0.352} \)
Allometry provides a method to estimate carbon stocks of forests

Trees: the taller and heavier they are, the larger their base has to be (or they would fall over)

Sequoiadendron giganteum, Scone Castle, Perthshire
Most of the mass is in the trunk of the tree, and the trunk resembles a cone.

So, we expect tree volume or mass to scale with $r^2h$.

Foresters work with D, diameter, and in a given forest often find this:

- Volume of timber = $aD^b$

  which is the same as

- $\log_e \text{Volume} = \log_e a + b \log_e D$

  (equation for a straight line, $y = c + mx$)

It is an example of an allometric relationship.
Example from Australia. All the Eucalypts fall on one line

Figure 1: Relationship at Howard Springs between Ln(Biomass) and Ln(DBH) for five species
(n = 50). The line plotted accurately represents the relationships for all species, since these did not differ in slope or intercept.

Or a similar relation that includes height $H$

Volume $= aD^bH^c$

Recall the volume of a cylinder is $\pi r^2 H$

a cone is $\frac{1}{3} \pi r^2 H$

Other solids of revolution are possible
Foresters talk of ‘form factors’
Trees not exact cones, so expect departure from $D^2$:

Each has a different relation between height and diameter, and therefore between volume and diameter. See Philip 2002 if you are interested in calculus solutions.

Researcher’s choice

Either
Make your own allometrics, excavate a lot of trees, dry and weigh them

Or
Use published ones
In practice, every species has a characteristic shape and therefore $a$ and $b$ may be expected to be species-dependent. For reference to the range of variability in some common species from 279 cases, look at Zianis & Mencuccini (2004).

Empirical equations can be of any form....

For Sitka spruce we are fortunate to have a set of unpublished empirical equations from Forest Research:

Branches: \(\text{Biomass} = 4.478 \times 10^{-5} \times (6.2396 + (6.896 \times 0.8569D) + 1.43D^2)\)

Stem: \(\text{Biomass} = 0.33(-1.219 \times 10^{-2} + (6.725 \times 10^{-5})(H^{0.8034})\)

Coarse roots: \(\text{Biomass} = 0.33((-2.396 + (6.896 \times 0.8569D) + 1.43D^2)^{2.3302})\)

Where Biomass is in tons, $D$ is in cm, $H$ is height in m.

These equations have a rather unusual form, but they are based upon thousands of trees and should work very well; but most allometric relations are based on small numbers (about 20) and often mixed species.
Typical distribution of biomass according to Philip (1994)

<table>
<thead>
<tr>
<th></th>
<th>Forest trees (%)</th>
<th>Savanna/woodland trees (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Twigs &amp; leaves</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Branches</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td>Bole</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Roots &gt;5 cm diameter</td>
<td>45</td>
<td>30</td>
</tr>
</tbody>
</table>

Bear in mind…

- Mass = volume x density
- When we say ‘mass’ or ‘biomass’ we mean ‘dry mass’
- Wood density varies hugely according to species (need to determine)
- Biomass is about 50% carbon (range is so small that it’s OK to assume 50%)
- Are we including below-ground biomass? This is important for carbon stocks
- Always be clear about your units, as equations are otherwise meaningless.
- See IPCC (2003) Good Practice Guidance for Land Use, Land-Use Change and Forestry for some allometric relationships for boreal, temperate and tropical forests
Observer error, Firbush 2005

<table>
<thead>
<tr>
<th></th>
<th>Alder 1st attempt</th>
<th>Alder 2nd attempt</th>
<th>Oak 1st attempt</th>
<th>Oak 2nd attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean $H$ (m)</td>
<td>14.7±2.4</td>
<td>14.9±0.72</td>
<td>7.2±1.1</td>
<td>6.9±0.39</td>
</tr>
<tr>
<td>Mean $D$ (cm)</td>
<td>19.7±0.6</td>
<td>19.4±0.30</td>
<td>24.5±0.5</td>
<td>24.4±0.5</td>
</tr>
</tbody>
</table>

Permament sample plots (PSPs)

- Geo-referenced and marked, so can be revisited
- Often 1 ha, square, divided into 20 m parts
- Trees over 10 cm diameter at 1.3 m measured
- Tag trees with nailed metal labels, often paint the mark at 1.3 m
- For butressed trees, climb above the butress.
Using allometrics to infer the forest carbon sink

Changes in the Carbon Balance of Tropical Forests: Evidence from Long-Term Plots
Oliver L. Phillips,* Yadvinder Malhi,* Níro Higuchi,
William F. Laurance, Percy V. Núñez, Rodolfo M. Vásquez,
Susan G. Laurance, Leandro V. Ferreira, Margaret Stern,
Sandra Brown, John Grace

Science 282, 439-442 1998

The role of the world's forests as a "sink" for atmospheric carbon dioxide is the subject of active debate. Long-term monitoring of plots in mature tropical forests in South America revealed that biomass gain by tree growth exceeded losses from tree death in all plot Neotropical sites. These forest plots have accumulated 0.71 t C ha⁻¹ yr⁻¹ on average, 0.54 t C ha⁻¹ yr⁻¹ of carbon per hectare per year in recent decades. The data suggest that Neotropical forests may be an important carbon sink, reducing the rate of increase in atmospheric carbon dioxide.

Phillips O. L., Malhi Y., Vinceti B., Baker T., Lewis S. L.,
Núñez N., Laurance W. F., Vásquez P. N., Martínez V. N.,

references

  http://www.sciencemag.org/cgi/content/full/282/5388/439
  http://www.nature.com/cgi-taf/DynaPage.taf?file=/nature/journal/v400/n6745/full/400664a0_fs.html&content_filetype=pdf
Propagation of errors

Result often calculated from several measured variables,

Eg \( Z = a + b \)

Suppose \( a \) and \( b \) are uncertain, what is the error in the derived value of \( Z \)?

Errors are not additive (do you see why?)

Instead \( \Delta Z = \sqrt{(\Delta a)^2 + (\Delta b)^2} \)

Pythagoras theorem
1.3.1 Log Derivative Method

The errors in each of the measured variables can be combined to yield an estimate of total measurement error for a multiple variable problem. The log derivative approach can separate the contributions of each variable. Assume that $A, B, C, D, F,$ and $G$ are measured quantities, $H$ is a constant, and $E$, the combined value of these quantities, is given by

$$E = \frac{H A B P C}{D(F + G)}.$$  \hfill (1.1)

To apply the log derivative method, first compute the unit error of each quantity, i.e., $\pm \Delta A/A, \pm \Delta B/B, \text{etc.}$, from supplementary information provided with the instruments or the components in the system. Next, take the logarithm of the equation (1.1)

$$\log E = \log H + \log A + 4 \log B + \log C - \log D - \log(F + G).$$  \hfill (1.2)

Then differentiate the equation, recalling that the derivative of $\log y/dy = 1/y$ and the derivative of a constant is zero,

$$\frac{dE}{E} = \frac{dA}{A} + \frac{dA}{A} + \frac{dC}{C} - \frac{dD}{D} - \frac{dF}{F} - \frac{dG}{G}.$$  \hfill (1.3)

Rearranging and replacing the derivatives with finite differences (say $dE/E = \Delta E/E$) yields

$$\frac{\Delta E}{E} = \frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C} + \frac{\Delta D}{D} - \frac{\Delta F}{F} - \frac{\Delta G}{G}.$$  \hfill (1.4)

We can now substitute the unit errors calculated from supplementary information in the first step. In order for this method to apply, the unit errors should be less than 0.1. Since a unit error may be either positive or negative, we shall sum only absolute values to insure that the errors combine in the most unfavorable way.

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<table>
<thead>
<tr>
<th>case</th>
<th>How to work out the combined error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z=a+b$</td>
<td>$(\Delta Z)^2 = (\Delta a)^2 + (\Delta b)^2$</td>
</tr>
<tr>
<td>$Z=a-b$</td>
<td>$(\Delta Z)^2 = (\Delta a)^2 + (\Delta b)^2$</td>
</tr>
<tr>
<td>$Z=ab$</td>
<td>$(\Delta Z/Z)^2 = (\Delta a/a)^2 + (\Delta b/b)^2$</td>
</tr>
<tr>
<td>$Z=a/b$</td>
<td>$(\Delta Z/Z)^2 = (\Delta a/a)^2 + (\Delta b/b)^2$</td>
</tr>
<tr>
<td>$Z = \ln a$</td>
<td>$\Delta Z = (\Delta a/a)$</td>
</tr>
<tr>
<td>$Z = e^a$</td>
<td>$(\Delta Z/Z) = 2\Delta a/a$</td>
</tr>
</tbody>
</table>

http://teacher.nsrl.rochester.edu/phy Labs/AppendixB/AppendixB.html