Analytical solutions for assessing pressure buildup during CO$_2$ injection in geological reservoirs

Simon A. Mathias
The Department of Earth Sciences
Durham University
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Approximate solution for pressure-buildup
during CO2 geo-sequestration

- We consider a fluid pressure, $p \text{[ML}^{-1}\text{T}^{-2}]$ that includes an assumption of negligible capillary pressure, and which applies over the entire thickness of a confined porous formation of vertical extent $H \text{[L]}$.

- The CO$_2$ and brine are assumed to be separated by a sharp interface, located at an elevation $h \text{[L]}$ above the base of the formation.

- The CO$_2$ zone is fully saturated with CO$_2$ whilst the brine zone is fully saturated with brine.
What does the relative permeability look like?

![Graph showing relative permeability for CO₂ and brine in Viking #2 and Nisku #2 wells.](image)

- **Viking #2**
- **Nisku #2**

(Benion and Bachu, 2008)
Key parameters

Dimensional analysis reveals that there are three important dimensionless parameter groups:

\[\alpha = \frac{M_0 \mu_o (c_r + c_w)}{2\pi (1 - S_r) H \rho_o k_r k}\]
\[\beta = \frac{M_0 k_r k b}{2\pi H r_w \mu_o}\]
\[\gamma = \frac{\mu_o}{k_r \mu_w}\]

- \(\alpha\): compressibility parameter
- \(\beta\): inertial parameter
- \(\gamma\): viscosity ratio

\(b = \) Forchheimer parameter \([L^{-1}]\)
\(c_r = \) compressibility of formation \([M^{-1}L^2T^2]\)
\(c_w = \) compressibility of brine \([M^{-1}L^2T^2]\)
\(H = \) formation thickness \([L]\)
\(k = \) permeability \([L^2]\)
\(S_r = \) residual brine saturation \([-]\)
\(M_0 = \) mass injection rate \([MT^{-1}]\)
\(\mu_o = \) viscosity of CO\(_2\) \([ML^{-1}T^{-1}]\)
\(\mu_w = \) viscosity of brine \([ML^{-1}T^{-1}]\)
\(\rho_o = \) density of CO\(_2\) \([ML^{-3}]\)
\(\rho_w = \) density of brine \([ML^{-3}]\)
\(k_r = \) relative permeability \([-]\)
Large time solution

\[ p_D \approx -\frac{1}{2} \ln \left( \frac{x}{2\gamma} \right) - 1 + \frac{1}{\gamma} \ln \left( \frac{\alpha}{2\gamma^2} \right) + 0.5772 + \frac{\beta}{r_D} \]

\[ h_D = \frac{h}{H} \]
\[ r_D = \frac{r}{r_w} \]
\[ t_D = \frac{M_0 t}{2\pi(1-S_r)\phi H r_w^2 \rho_o} \]
\[ x = \frac{r_D^2}{t_D} \]
\[ p_D = \frac{2\pi H \rho_o k_p}{M_0 \mu_o} \]
\[ \alpha = \frac{M_0 \mu_o (c_r + c_w)}{2\pi(1-S_r)H \rho_o k_r k} \]
\[ \beta = \frac{M_0 k_r k_b}{2\pi H r_w \mu_o} \]
\[ \gamma = \frac{\mu_o}{k_r \mu_w} \]
Pressure buildup

$$\gamma^{\beta^2_{\gamma}} = c_d t$$

Deep and warm sediments with $\alpha = 10^{-3}$

Dimensionless time, $t_D/\alpha r_B^2$

Finite difference ($\beta = 10$)
Finite difference ($\beta = 7.5$)
Finite difference ($\beta = 5$)
Finite difference ($\beta = 2.5$)
Large time approximation

$10^5$

$10^0$

$$t_{\text{cD}} = \frac{100 \beta^2}{2 \gamma}$$

Dimensionless injection well pressure, $p_d^d \bar{r} = 1$