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Multi-criteria evaluation and GIS

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Multi-criteria evaluation in GIS is concerned with the allocation of land to suit a specific objective on the basis of a variety of attributes that the selected areas should possess. Although commonly undertaken in GIS, it is shown that the approaches commonly used in vector and raster systems typically lead to different solutions. In addition, there are ambiguities in the manner in which criteria should be standardised and aggregated to yield a final decision for the land allocation process. These problems are reviewed and the theoretical structure of fuzzy measures is offered as an approach to the reconciliation and extension of the procedures currently in use. Specifically, by considering criteria as expressions of membership in fuzzy sets (a specific instance of fuzzy measures) the weighted linear combination aggregation process common to raster systems is seen to lie along a continuum of operators mid-way between the hard intersection and union operators typically associated with Boolean overlay in vector systems. A procedure for implementing this continuum is reviewed, along with its implications for varying the degrees of 'ANDORness' and trade-off between criteria. In addition, the theoretical structure of fuzzy measures provides a strong logic for the standardisation of criteria and the evaluation of decision risk (the likelihood that the decision made will be incorrect).

1 INTRODUCTION

One of the most important applications of GIS is the display and analysis of data to support the process of environmental decision-making. A decision can be defined as a choice between alternatives, where the alternatives may be different actions, locations, objects, and the like. For example, one might need to choose which is the best location for a hazardous waste facility, or perhaps identify which areas will be best suited for a new development.

Broadly, decisions can be classified into two extensive categories – policy decisions and resource allocation decisions. Resource allocation decisions, as the name suggests, are concerned with control over the direct use of resources to achieve a particular goal. Ultimately, policy decisions have a similar aim. However, they do so by establishing legislative instruments that are intended to influence the resource allocation decisions of others. Thus, for example, a government body might reduce taxes on land allocated to a particular crop as an incentive to its introduction. This is clearly a policy decision; but it is the farmer who makes the decision about whether to allocate land to that crop or not.

To be rational, decisions will be necessarily based on one or more criteria – measurable attributes of the alternatives being considered, that can be combined and evaluated in the form of a decision rule. In some circumstances, allocation decisions can be made on the basis of a single criterion. However, more frequently, a variety of criteria is required. For example, the choice between a set of waste disposal sites might be based upon criteria such as proximity to access roads, distance from residential and protected lands, current land use, and so on.

This chapter focuses on the very specific problems of spatial resource allocation decisions in the context of multiple criteria – a process most commonly known as *multi-criteria evaluation*

(MCE) (Voogd 1983). In some instances, this term has also been used to subsume the concept of *multi-objective* decision-making (e.g. Carver 1991; Janssen and Rietveld 1990). However, it is used here in a more specific sense. An *objective* is understood here to imply a perspective, philosophy, or motive that guides the construction of a specific multi-criteria decision rule. Thus in siting a hazardous waste facility, the objective of a developer might be profit maximisation while that of a community action group might be environmental protection. The criteria they each consider and the weights they assign to them are likely to be quite different. Each is likely to develop a multi-criteria solution – but a different multi-criteria decision. The resolution of these differing perspectives into a single solution is known as multi-objective decision-making – a topic which will not be covered in this chapter (see Campbell et al 1992 and Eastman et al 1995 for two prominent approaches to this problem in GIS).

Almost all of the case study examples in this chapter are based on an analysis of suitability for industrial development for the region of Nakuru, Kenya. Nakuru is a region of strong agricultural potential that has experienced rapid urban development in recent years. It is also the location of one of the more important wildlife parks in Kenya (the large area of restricted development to the south of Plate 32) – one of Kenya's soda lakes in the Great Rift Valley, it is the home of over two million flamingoes as well as a wide variety of other species.

2 TRADITIONAL APPROACHES TO MCE IN GIS

In GIS, multi-criteria evaluation has most typically been approached in one of two ways. In the first, all criteria are converted to Boolean (i.e. logical true/false) statements of suitability for the decision under consideration. (The term *Boolean* is derived from the name of the English mathematician, George Boole, who first abstracted the basic laws of set theory in the mid 1800s. It is used here to denote any crisp spatial mapping in which areas are designated by a simple binary number system as either belonging or not belonging to the designated set.) In many respects, these Boolean variables can be usefully thought of as constraints, since they serve to delineate areas that are not suitable for consideration. These constraints are then combined by some combination of intersection (logical AND) or union (logical OR) operators. This procedure dominates MCE with vector software systems, but is also commonly used with raster systems. For example, Figure 1 shows how Boolean images, along with their intersection achieved through the characteristic overlay operation of a GIS, may be used here to find all areas suitable for industrial development, subject to the following criteria: suitable areas will be near to a road (within 1 km – upper left), near to a labour force (within 7.5 km of a town – middle left), on low slopes (less than 5 per cent – upper right), and greater that 2.5 km from designated wildlife reserves (middle right). In addition, development is not permitted in wildlife reserves (lower left). These criteria are aggregated by means of an intersection (logical AND) operator, yielding the result on the lower right. Note that the distance to labour force was calculated from a cost distance surface that accounted for road and off-road frictions.

In the second most common procedure for MCE, quantitative criteria are evaluated as fully continuous variables rather than collapsing them to

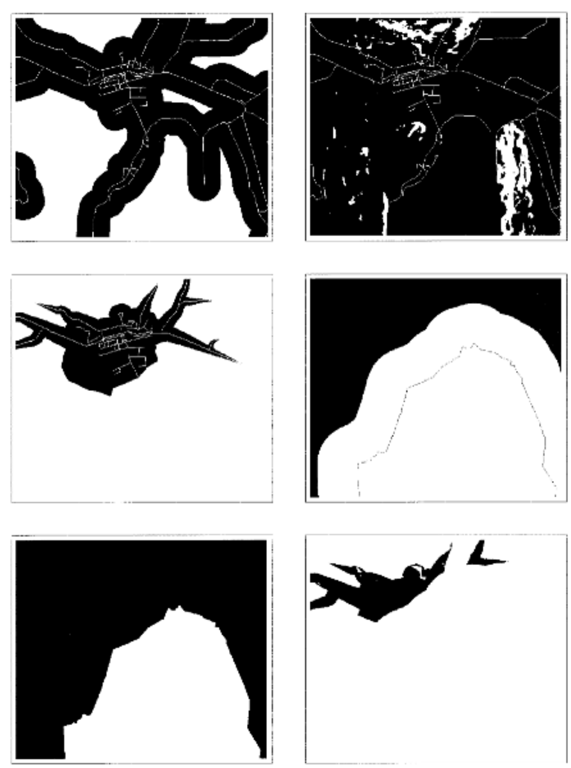

Fig 1. An example of multi-criteria evaluation using Boolean analysis.

Boolean constraints. Such criteria are typically called *factors*, and express varying degrees of suitability for the decision under consideration. Thus, for example, proximity to roads would be treated not as an all-or-none buffer zone of suitable locations, but rather, as a continuous expression of suitability according to a special numeric scale (e.g. 0–1, 0–100, 0–255, etc.). The process of converting data to such numeric scales is most commonly called standardisation (Voogd 1983).

Traditionally, standardised factors are combined by means of weighted linear combination – that is, each factor is multiplied by a weight, with results being summed to arrive at a multi-criteria solution. In addition, the result may be multiplied (i.e. intersected) by the product of any Boolean constraints that may apply (Eastman et al 1995). For example:

$$\text{suitability} = \sum w_i X_i * \prod C_j$$

where w_i = weight assigned to factor i
X_i = criterion score of factor i
C_j = constraint j

Figure 2 illustrates this approach where a comparable example is developed to that in Figure 1. Again, the intention is to find areas suitable for industrial development, subject to the following criteria: suitable areas will be near to a road (as near as possible – upper left), near to a labour force (as near as possible – middle left), on low slopes (as low as possible – upper right) and as far from the wildlife reserve as possible (middle right). As in Figure 1, development is not permitted in wildlife reserves (lower left) through use of a Boolean constraint. These criteria are aggregated by means of a weighted average of the criterion scores. In this case, all criteria were standardised before weighting to a common numeric range using the most commonly used (but not necessarily recommended) technique – linear scaling between the minimum and maximum values of that criterion. The linear rescaling is to a consistent range (0–255) as follows:

$$X_i = (x_i - \min_i) / (\max_i - \min_i)$$

where X_i = criterion score of factor i
x_i = original value of factor i
$\min_i$ = minimum of factor i
$\max_i$ = maximum of factor i

In addition, to provide the most direct comparison to the results of Figure 1, equal weight (0.25) was assigned to each criterion with the wildlife reserve constraint acting as an absolute barrier to development. The result of the averaging process is shown on the lower left. The image on the lower right shows the result of selecting the best areas from this suitability map in order to match the total area of that selected by Boolean analysis in Figure 1. Note that as in Figure 1, the distance to labour force was calculated from a cost distance surface that accounted for road and off-road frictions.

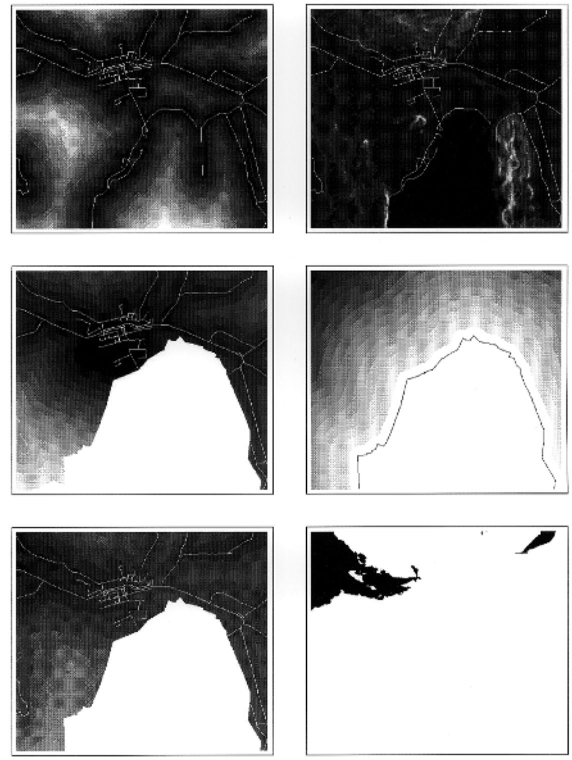

Fig 2. An example of multi-criteria evaluation using weighted linear combination.

The continuous suitability map shown in Figure 2 has the same numeric range as the standardised factors if the weights that are applied sum to 1.0. A specific decision can then be reached by rank ordering the alternatives (in this case, pixels) and selecting as many of the best ranked areas as is required to meet the objective of the analysis in question. In Figure 2, this has been done in order to select as many of the best areas as were selected by the Boolean analysis in Figure 1.

This procedure of weighted linear combination dominates multi-criteria approaches with raster-based GIS software systems. However, there are a number of problems with both approaches to multi-criteria evaluation.

First, despite a casual expectation that the two procedures should yield similar results, they very often do not. For example, the results of the decision portrayed in the lower right of Figures 1 and 2 are in agreement only by 53 per cent. The reason clearly has to do with the logic of the aggregation operation. For example, Boolean intersection results in a very hard AND – a region will be excluded from the result if any single criterion fails to exceed its threshold. Conversely, the Boolean union operator implements a very liberal mode of aggregation – a region will be included in the result even if only a single criterion meets its threshold. Weighted linear combination is quite unlike these options. Here a low score on one criterion can be compensated by a high score on another – a feature known as *trade-off* or *substitutability*. While human experience is replete with examples of both trade-off and non-substitutability in decision making, the tools for flexibly incorporating this concept are poorly developed in GIS. Furthermore, a theoretical framework that can link the aggregation operators of Boolean overlay and weighted linear combination has, until recently (Eastman and Jiang 1996), been lacking.

The second problem with MCE has to do with the standardisation of factors in weighted linear combination. The most common approach is to rescale the range to a common numerical basis by simple linear transformation (Voogd 1983), as was applied in Figure 2. However, the rationale for doing so is unclear. Indeed, there are many instances where it would seem logical to rescale values within a more limited range. Furthermore, there are cases where a non-linear scaling may seem appropriate. Since the recast criteria really express suitability, there are many cases where it would seem appropriate that criterion scores asymptotically approach the maximum or minimum suitability level.

The third issue concerns the weights that are applied. Clearly they can have a strong effect on the outcome produced. However, not much attention has been focused in GIS on how they should be developed. Commonly they represent the subjective (but no less valid) opinions of one or more experts or local informants. How can consistency and overt validity be established for these weights? Furthermore, how should they be applied in the context of varying trade-off between factors?

A fourth problem concerns decision risk. Decision risk may be considered as the likelihood that the decision made will be wrong. For both procedures (Boolean analysis and weighted linear combination) it is a fairly simple matter to propagate measurement error through the decision rule and subsequently to determine the risk that a given location will be assigned to the wrong set (i.e. the set of selected alternatives or the set of those not to be included). However, the continuous criteria of weighted linear combination would appear to express a further uncertainty that is not so easily accommodated (see Fisher, Chapter 13). The standardised factors of weighted linear combination each express a perspective of suitability – the higher the score, the more suitable. However, there is no real threshold that can definitively allocate locations to one of the two sets involved (areas to be chosen and areas to be excluded). How are these uncertainties to be accommodated in expressions of decision risk? If these criteria really express uncertainties, why are they combined through an averaging process?

The surprising feature of multi-criteria evaluation is that, despite its ubiquity in environmental management, so little is understood of its character in GIS. In the following sections we survey the issues involved, and offer a perspective on a resolution through the concept of *fuzzy measures*.

3 FUZZY MEASURES

This discussion of fuzzy measures is adapted from Eastman and Jiang (1996). The term fuzzy measure refers to any set function which is monotonic with respect to set membership (Dubois and Prade 1982; see also Fisher, Chapter 13). Notable examples of fuzzy measures include *probabilities*, the *beliefs*, and *plausibilities* of Dempster-Shafer theory, and the *possibilities* of fuzzy sets. Interestingly, if we consider the process of standardisation in MCE to be one of transforming criterion scores into set membership statements (i.e. the set of suitable choices), then standardised criteria *are* fuzzy measures.

A common trait of fuzzy measures is that they follow DeMorgan's Law in the construction of the intersection and union operators (Bonissone and Decker 1986). DeMorgan's Law establishes a triangular relationship between the intersection, union, and negation operators such that:

$T(a,b) = \sim S(\sim a, \sim b)$

where T = intersection (AND) = T-Norm
and S = union (OR) = T-CoNorm
and $\sim$ = negation (NOT)

The intersection operators in this context are known as *triangular norms*, or simply T-*Norms*, while the union operators are known as *triangular co-norms*, or T-*CoNorms*.

4 FUZZY MEASURES AND AGGREGATION OPERATORS

A T-Norm can be defined as (Yager 1988):

a mapping T: [0,1] * [0,1] → [0,1] such that:

$T(a,b) = T(b,a)$ (commutative)
$T(a,b) \geq T(c,d)$ if $a \geq c$ and $b \geq d$ (monotonic)
$T(a,T(b,c)) = T(T(a,b),c)$ (associative)
$T(1,a) = a$

Some examples of T-norms include:

$\min(a,b)$ (the intersection operator of fuzzy sets)
$a * b$ (the intersection operator of classical sets)
$1 - \min(1,((1-a)^p + (1-b)^p)^{(1/p)})$ (for $p \geq 1$)
$\max(0,a + b - 1)$

Conversely, a T-CoNorm is defined as:

a mapping S: [0,1] * [0,1] → [0,1] such that:

$S(a,b) = S(b,a)$ (commutative)
$S(a,b) >= S(c,d)$ if $a \geq c$ and $b \geq d$ (monotonic)
$S(a,S(b,c)) = S(S(a,b),c)$ (associative)
$S(0,a) = a$

Some examples of T-CoNorms include:

$\max(a,b)$ (the union operator of fuzzy sets)
$a + b - a*b$ (the union operator of classical sets)
$\min(1,(a^p + b^p)^{1/p})$ (for $p \geq 1$)
$\min(1,a + b)$

Interestingly, while the intersection ($a*b$) and union ($(a+b) - (a*b)$) operators of Boolean overlay represent a T-Norm/T-CoNorm pair, the averaging operator of weighted linear combination is neither, because it lacks the property of associativity (Bonissone and Decker 1986). Rather, it has been determined (Bonissone and Decker 1986) that the averaging operator falls midway between the extreme cases of the T-Norm (AND) of fuzzy sets (the minimum operator) and its corresponding T-CoNorm (OR – the maximum operator) – in essence, a perfect ANDOR operator. In fact, Yager (1988) has proposed that weighted linear combination is one of a continuum of aggregation operators that lies between these two extremes. Further, he has proposed the concept of an *ordered weighted average* that can produce the entire continuum. Recently, Eastman and Jiang (1996) have implemented this operator, with modifications, in a raster GIS. In doing so, the traditional aggregation operators of vector and raster GIS have been united into a single theoretical framework.

4.1 The ordered weighted average

With the ordered weighted average, criteria are weighted on the basis of their rank order rather than their inherent qualities. Thus, for example, we might decide to apply order weights of 0.5, 0.3, 0.2 to weight a set of factors *A*, *B,* and *C* based on their rank order. Thus if at one location the criteria are ranked *BAC* (from lowest to highest), the weighted combination would be $0.5*B + 0.3*A + 0.2*C$. However, if at another location the factors are ranked *CBA*, the weighted combination would be $0.5*C + 0.3*B + 0.2*A$. In the implementation of Yager's concept by Eastman and Jiang (1996), the concept of weights that apply to specific factors has also been incorporated, yielding two sets of weights – *criterion weights* that apply to specific criteria and *order weights* that apply to the ranked criteria, after application of the criterion weights.

The interesting feature of the ordered weighted average is that it is possible to control the degree of ANDORness and trade-off between factors within limits. For example, using order weights of [1 0 0] yields the minimum operator of fuzzy sets, with full ANDness and no trade-off. Using order weights of [0 0 1] yields the maximum operator of fuzzy sets with full ORness and no trade-off. Using weights of [0.33 0.33 0.33] yields the traditional averaging operator of MCE with intermediate ANDness and ORness, and full trade-off. Trade-off is thus controlled by the degree of dispersion in the weights while ANDness or ORness is governed by the amount of skew. For example, order weights of [0 1 0] would yield an operator with intermediate ANDness and ORness, but no trade-off, while the original example with order weights of [0.5 0.3 0.2] would yield an operator with substantial trade-off and a moderate degree of ANDness.

This quality of variable ANDORness has interested some in the decision science field (e.g. Yager 1988) because of the recognition that in human perception of decision logics, it is not uncommon to wish to combine criteria with something less extreme than the hard operations of union or intersection. In the context of GIS, however, it is the property of trade-off that is of special interest. The minimum operator is occasionally used in GIS applications, and represents a form of limiting factor analysis. Here the intent is one of risk aversion, by characterising the suitability of a location in terms of its worst quality. The maximum operator is the opposite, and can thus be thought of as a very optimistic aggregation operator – an area will be suitable to the extent of its best quality. Both of these operations permit no trade-off between the qualities of the criteria considered. Furthermore, in cases where set membership approaches certainty, the results from fuzzy sets will be identical to those of Boolean overlay. However, with weighted linear combination, trade-off is clearly and fully present.

Figure 3 illustrates various degrees of trade-off (and ANDORness) between the same four factors considered in Figures 1 and 2. The same criteria were used as for Figure 2, except that the scaling was changed to facilitate comparison to the results in Figure 1 – a sigmoidal fuzzy membership function was used such that the thresholds used to create the Boolean images in Figure 1 correspond to membership values of 0.5 for the fuzzy criteria in this example (i.e. scaling was asymptotic to membership values of 1.0 and 0.0 at values of 0–2 km for proximity to roads, 0–15 km for proximity to the labour force, 0–10 per cent for slope gradients, and 5–0 km for distance from designated wildlife reserves). In addition, to facilitate comparison, equal criterion weights were applied as in Figure 2. Thus the differences between these aggregations arise solely from the effects of different order weights. The v-shaped sequence, from top to bottom, used order weights of [1 0 0 0], [.60 .20 .15 .05], [.4 .3 .2 .1], [.25 .25 .25 .25], [.1 .2 .3 .4], [.05 .15 .20 .60], and [0 0 0 1]. This sequence progresses from full ANDness and no trade-off for the first (the minimum function), to intermediate ANDORness and full trade-off for the middlemost (equivalent to a standard weighted linear combination), to full ORness and no trade-off with the last (corresponding to the maximum operator). The image on the middle left is a median operator produced with order weights of [0 .5 .5 0], producing an aggregation with intermediate ANDORness (like weighted linear combination) but almost no trade-off.

The similarity of the result with full ANDness (and thus no trade-off) to the Boolean result in Figure 1 is striking. In fact, when these suitability values are rank ordered and enough of the best pixels are selected to equal the area of the Boolean result, the solution is identical. Thus the reason for the difference in the Boolean and weighted linear combination results is clear – the characteristic Boolean overlay operation of vector GIS produces an aggregation of criteria with full ANDness and no trade-off while the typical weighted linear combination operation of raster GIS produces intermediate ANDness and full trade-off. The results are different because the aggregation operators are different.

Recognising that a full spectrum of aggregation operators exists opens up a much richer set of possibilities for implementing decision rules in GIS.

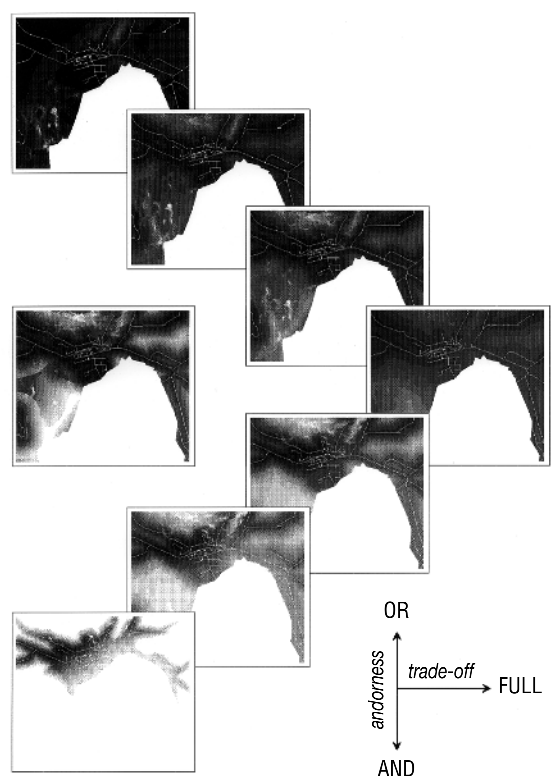

Fig 3. A spectrum of aggregations using the ordered weighted average procedure.

For example, Plate 32 illustrates the effects of combining three of the factors with trade-off and one without. In this case, proximity to roads is given a criterion weight of 0.45, proximity to the labour force is given a weight of 0.12, and the slope factor is given a weight of 0.43. These are combined using a standard weighted linear combination. This result is then combined with the distance from wildlife reserve factor using a minimum operator. The absence of trade-off in this last step is clear – the distance from wildlife reserve factor dominates the result until it no longer represents the limiting factor. The effect is clearly similar to that of a constraint, but lacks the crispness of a traditional constraint. In effect, the minimum operator with a fuzzy measure represents a form of soft constraint. Soft constraints are particularly useful where a specific boundary cannot be reasonably established. Indeed, it might be argued that this is more commonly appropriate than the artificial boundaries of traditional constraints.

5 FUZZY MEASURES AND STANDARDISATION

Clearly, this consideration of fuzzy measures has implications beyond those of the aggregation process alone. It also provides a very strong logic for the process of standardisation. In this context, the process of standardising a criterion can be seen as one of recasting values into a statement of set membership – the degree of membership in the final decision set. Indeed, Eastman and Jiang (1996) argue that such statements of set membership in fact constitute fuzzy sets (a particular form of fuzzy measure), while those of Boolean constraints represent classical sets. This clearly opens the way for a broader family of set membership functions than that of linear rescaling alone. For example, the commonly used sigmoidal (s-shaped) function of fuzzy sets provides a simple logic for cases where a function is required that is asymptotic to 0 and 1. It also suggests that the minimum and maximum raw factor values should not blindly be used as the anchor points for such a function. Rather, anchor points that are consistent with the logic of set membership are clearly superior. For example, in Figure 4, sigmoidal membership functions were created for each factor, with anchor points set at the points where the factor begins to have an effect and where the effect is no longer relevant. The distance to wildlife reserve factor, for instance, starts to rise above 0.0 immediately at the park boundary, but approaches 1.0 at a distance of 5 kilometres. Further distance does not lead to an increase in the factor score since the distance is far enough.

6 DETERMINATION OF WEIGHTS

Given the consideration of factors as fuzzy sets and the nature of the aggregation process, the criterion weights of weighted linear combination clearly represent trade-off weights – that is, expressions of the manner in which they will trade with other factors when aggregated in multi-criteria evaluation. Rao et al (1991) have suggested that a logical process for the development of such weights is the procedure of pairwise comparisons developed by Saaty (1977). In this process each factor is rated for its importance relative to every other factor using a 9-point reciprocal scale (i.e. if 7 represents substantially more important, 1/7 would indicate substantially less important). This leads to a *n* x *n* matrix of ratings (where *n* is the number of factors being considered). Saaty (1977) has shown that the principal eigenvector of this matrix represents a best fit set of weights. Figure 5, for example, illustrates this rating scale along with a completed comparison matrix and the best fit weights produced. Eastman et al (1993) have implemented this procedure in a raster GIS with a modification that also allows the degree of consistency to be evaluated as well as the location of inconsistencies to allow for an orderly re-evaluation. The process is thus an iterative one that converges on a consistent set of consensus weights.

A problem still exists, however, in how these weights should be applied in the context of the ordered weighted average discussed above. It seems clear that these weights will have full effect with the weighted linear combination operator (where full trade-off exists), and that they should have no effect when no trade-off is in effect (i.e. with the minimum and maximum operators). It seems logical, therefore, that their effect should be graded between these extremes as the degree of trade-off is manipulated with the ordered weighted average process. However, the logic for this gradation has not been established. In their implementation of the ordered weighted average for GIS, Eastman and Jiang (1996) have used a measure of relative

1/9	1/7	1/5	1/3	1	3	5	7	9
extremely	very	strongly	moderately	equally	moderately	strongly	very	extremely

less important ⟶ more important

(a)

	Proximity to roads	Proximity to labour force	Slope gradient	Distance from wildlife reserves
Proximity to roads	1			
Proximity to labour force	1/3	1		
Slope gradient	1	4	1	
Distance from wildlife reserves	1/3	2	1/2	1

(b)

Factor	Derived weight
Proximity to roads	0.3770
Proximity to labour force	0.0979
Slope gradient	0.3605
Distance from wildlife reserves	0.1647

(c)

Fig 4. Saaty's pairwise comparison procedure for the derivation of factor weights. Using a 9-point rating scale (a) each factor is compared to each other factor for its relative importance in developing the final solution (b). The principal eigenvector of this matrix is then calculated to derive the best-fit set of weights (c).

dispersion (a measure closely related to the entropy measure of information theory) as the basis for this gradation. However, further research is needed on this important aspect of the ordered weighted average procedure.

7 DECISION RISK

Uncertainty in the decision rule, and in the criteria that are considered, implies some risk that the decision made will be wrong. In the case of measurement error, the effects of uncertainty can fairly easily be propagated to the suitability map that is produced in MCE (see Heuvelink, Chapter 14; Heuvelink 1993). Furthermore, Eastman (1993) has developed a simple operator that can convert such an evaluation into a mapping of the probability

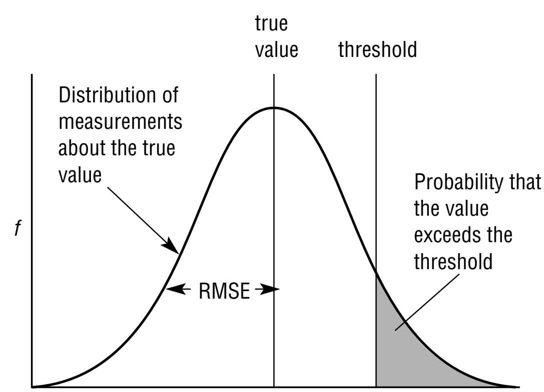

Fig 5. A procedure for calculating decision risk. Assuming a normal distribution of errors, the probability that a data value exceeds or is exceeded by a threshold can be calculated as the area of the normal curve subtended by that threshold. [RMSE = root mean square error]

that locations belong to the decision set (the PCLASS operator of the Idrisi software system). The operator assumes a normal distribution of errors and calculates the area under the normal curve subtended by a threshold that can distinguish the cases that belong in the decision set from those that do not (Figure 5). The result is an expression of decision risk that is directly analogous to the concept of a Type II error in statistical hypothesis testing – that is, the likelihood that the alternative does not belong to the decision set if we assume that it does (Plate 33). The resulting probability map can subsequently be thresholded to see the nature of the decision set at any specified risk level (again see Plate 33).

To the extent that measurement error can be quantified and propagated through an analysis, an expression of decision risk is thus not very difficult to achieve. However, the recognition of factors in MCE as fuzzy sets implies a very different form of uncertainty from that of measurement error. The suitability map that results from weighted linear combination is a clear expression of uncertainty about the suitability of any particular piece of land for the objective under consideration. However, as an expression of uncertainty, it has no relationship to the frequentist notion of probability that underlies the treatment of decision risk in the context of measurement error. Thus a traditional treatment of decision risk as the probability that the decision made will be wrong cannot be developed. Eastman (1996) has therefore suggested that decision risk for such cases be expressed by the concept of *relative risk*.

A mapping of relative risk can be very simply achieved by rank ordering the alternatives and dividing the result by the maximum (i.e. worst) rank that occurs. The outcome is a proportional ranking that can directly be interpreted as relative risk. Then in cases where no specific area requirement for the decision set is being sought (e.g. the best 10 hectares), the final decision set can be established by selecting the alternatives where the relative risk does not exceed a specific threshold (e.g. the best 5 per cent of the areas under consideration). Figure 6 illustrates such a mapping of relative risk for the result of Plate 32 along with a mapping of the best (least risky) 10 per cent of cases outside the wildlife reserve.

Such an expression of relative risk is quite familiar in human experience. By rank ordering the alternatives (on the basis of suitability) and choosing the best ones, we use a procedure that strives to pick the least risky alternatives (i.e. the ones that are least likely to be poor choices). For example, in screening applicants for employment we may make use of a variety of criteria (e.g. test scores, reference evaluations, years of experience, etc.) that will allow the candidates to be ranked. Then by choosing only the highest ranked candidates we minimise our risk of choosing someone who will perform poorly. However, we do not know the actual degree of risk we are taking; only that the candidates we have chosen are the least risky of the alternatives considered.

From the perspective of considering the criteria of MCE as fuzzy measures, then, it would appear that the expression of decision risk needs to be different from that which arises from a consideration of measurement error. However, in most cases, both

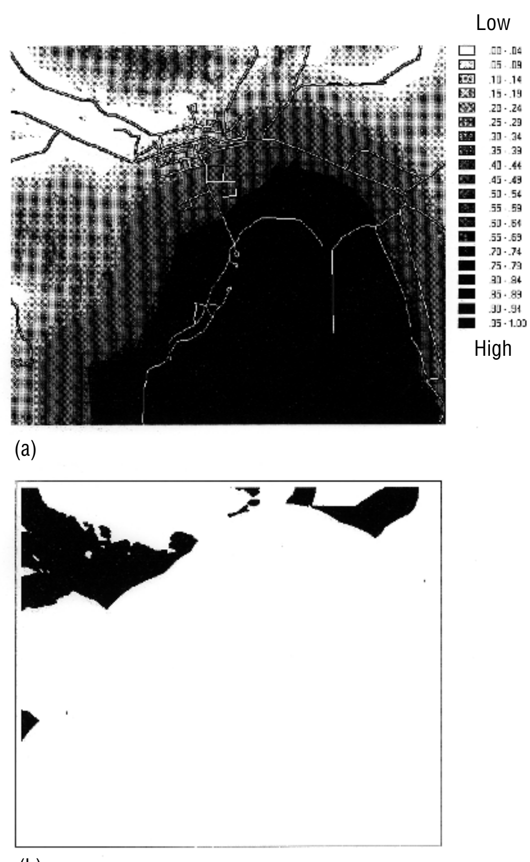

Fig 6. (a) A mapping of relative risk for the result in Plate 32. The procedure involves a simple rank ordering of alternatives followed by division by the maximum rank. (b) Areas outside the wildlife reserve with a relative risk of 10 per cent or less.

forms of uncertainty exist. Thus one might anticipate the problem of having to express both forms of risk. For example, given the presence of measurement error in the development of a multi-criteria suitability mapping, and a propagation of those errors to the final suitability map, one realises that this mapping is only one of a large number of possible outcomes that might be produced by randomly introducing the uncertainties in measurement that exist. Thus by Monte Carlo simulation (a capability that unfortunately exists in only a small number of GIS software programs) one could thus tabulate the proportion of simulations in which each location falls within a specific threshold of relative risk, or a specific areal requirement. This then restores the frequentist notion of probability and the usual expression of decision risk.

8 CONCLUSION

In this chapter an attempt has been made to reconcile the differences between the typical approaches to MCE used in vector and raster GIS. By using the theoretical structure of fuzzy measures, both approaches can be seen as special cases of a single family of aggregation operators. In the case of Boolean overlay as very typically used in vector GIS, the decision problem is treated as one of classical set membership, with the intersection and union operations resulting in strict ANDness or ORness with no trade-off. However, it has been shown here that these hard constraints represent no more than the crisp extremes of an underlying logic of fuzzy sets. By considering the more general class of fuzzy measures (of which fuzzy sets are a member) it has been shown that similar operations exist for the continuous factors more commonly associated with raster systems (the minimum and maximum operators). In addition, it has been shown that the weighted linear combination operator commonly used with such factors lies on a continuum with these operators, where it represents the case of intermediate ANDness and ORness, and full trade-off between the factors considered. Furthermore, it has been shown that a more general operator (the ordered weighted average) can produce all of these results along with a continuum of other operators with varying degrees of trade-off and ANDORness. This not only acts as a strong theoretical framework for consideration of the aggregation operator, but also provides a logic for the standardisation of factors, a rationale for the expression of decision risk, and a high degree of flexibility in the land allocation decision process.

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