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Models of uncertainty in spatial data

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Spatial information is rife with uncertainty for a number of reasons. The correct conceptualisation of that uncertainty is fundamental to the correct use of the information. This chapter attempts to document different types of uncertainty – specifically error, vagueness, and ambiguity. Examples of these three types are used to illustrate the classes of problems which arise, and to identify appropriate strategies for coping with them. The first two categories are well documented and researched within the GIS field, and are now recognised in many varied contexts. The third has not been so widely researched. Cases are also identified where uncertainty is deliberately introduced into geographical information in order to anonymise individuals. Examples are given where both error and vagueness can be applied to the same phenomenon with different understandings and different results. Methods to address the problems are identified and are explored at length.

1 INTRODUCTION

'The universe, they said, depended for its operation on the balance of four forces which they identified as charm, persuasion, uncertainty and bloody-mindedness.'

Terry Pratchett (1986)

acuracy *n.* An absence of erors. 'The computer offers both speed and acuracy, but the greatest of these is acuracy' (*sic*)

Kelly-Bootle (1995)

The handling of large amounts of information about the natural and built environments, as is necessary in any GIS, is prone to uncertainty in a number of forms. Ignoring that uncertainty can, at best, lead to slightly incorrect predictions or advice and at worst can be completely fatal to the use of the GIS and undermine any trust which might have been put in the work of the system or operator. It is therefore of crucial importance to all users of GIS that awareness of uncertainty and error should be as widespread as possible. Fundamental to such understanding is the nature of the uncertainty, in its different guises. This is the subject of this chapter. A minimal response should be that users of the GIS be aware of the possible complications to their analysis caused by uncertainty, and at best present the user of the analysis with a report of the uncertainty in the final results together with a variety of plausible outcomes. A complete response to uncertainty is to present the results of a full modelling exercise which takes into account all types of uncertainty in the different data themes used in the analysis. It seems that neither response is widespread at present, and in any case the tools for doing the latter are currently the preserve only of researchers.

This chapter explores the developing area of the conceptual understanding (modelling) of different types of uncertainty within spatial information. These are illustrated in Figure 1. At the heart of the issue of uncertainty is the problem of defining both the class of object to be examined (e.g. soils) and the individual object (e.g. soil map unit) – the so-called problem of definition (Taylor 1982). Once the conceptual modelling identifies whether the class of objects to be described is well or poorly defined the nature of the uncertainty as follows:

1. If both the class of object and the individual are well defined then the uncertainty is caused by errors and is probabilistic in nature;

2 If the class of object or the individual is poorly defined then additional types of uncertainty may be recognised. Some have been explored by GIS researchers and others have not:
   a If the uncertainty is attributable to poor definition of class of object or individual object, then definition of a class or set within the universe is a matter of *vagueness*, and this can conveniently be treated with fuzzy set theory.
   b Uncertainty may also arise owing to ambiguity (the confusion over the definition of sets within the universe) owing, typically, to differing classification systems. This also takes two forms (Klir and Yuan 1995), namely:
      i Where one object or individual is clearly defined but is shown to be a member of two or more different classes under differing schemes or interpretations of the evidence, then *discord* arises;
      ii Where the process of assigning an object to a class at all is open to interpretation, then the problem is *non-specificity*.

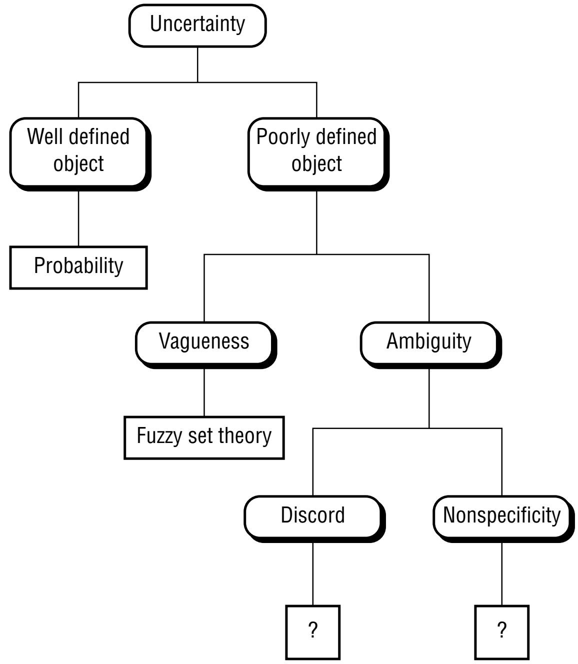

Fig 1. A conceptual model of uncertainty in spatial data (adapted from Klir and Yuan 1995: 268).

In the context of spatial databases, only vagueness as expressed by fuzzy set theory and error as represented by probability theory have been researched, and these are the primary focus of the discussion below. The list is necessarily not exhaustive: however, the volume of research and the amount of interest in this area continues to increase.

If a chapter had been written in this form for the first edition of this book, it would have focused on only one variety of uncertainty, namely error (Chrisman 1991). A few years later there are two equally important strands to be discussed. Although the strands discussed here seem to explain the majority of the long-recognised causes of uncertainty in spatial information, it is already possible to identify other types of uncertainty that should be addressed in future research.

2 THE PROBLEM OF DEFINITION

The principal issue of geographical uncertainty is the understanding of the collector and user of the data as to the nature of that uncertainty. There are three facets to this, namely uncertainty in measurement of attributes, of space, and of time. In order to define the nature of the uncertainty of an object within the dimensions of space and time, a decision must be made as to whether or not it is clearly and meaningfully separable from other objects in whichever dimension is of interest – ideally it will be separable in both. This is a complex intellectual process, one which draws on the history and the critical appraisal of subject-specific scientists. This conceptual model has been complicated and muddied by conventions which influence the perception of geographical information. Foremost among these is the historical necessity of simplification of information for map production; what Fisher (1996) denotes the paradigm of 'production cartography'. Equally important are the concepts of classification, commonly based on hierarchies, in which objects must fall into one class or another, and of computer database models in which objects are treated as unique individuals and form the basis to analysis.

If a spatial database is to be used, or to be created from scratch, then investigators or users have to ask themselves two apparently simple questions:

1 Is the class of objects to be mapped (e.g. soils, rocks, ownership, etc.) clearly separable from other possible classes?
2 Are the geographical individuals within the class of objects clearly and conceptually separable from other geographical individuals within the same class?

If it is possible to separate unequivocally the phenomenon to be mapped into mappable and spatially distinct objects using the spatial distribution of some individual attribute or collection of attributes, at a given time, then there is no problem of definition. A phenomenon which is well defined should have diagnostic properties for separating individuals into classes based on attributes and into spatially contiguous and homogenous areas.

If it is not possible to define the spatial extent of an object to be mapped or analysed, there is a problem of definition, and it can be said to be 'vague' (Williamson 1994). In this circumstance, while specific properties may be measured and these measurements may be precise, no combination of properties allows the unequivocal allocation of individual objects to a class, or even the definition of the precise spatial extent of the objects. Most spatial phenomena in the natural environment share this problem of definition to some extent. Error analysis on its own does not help with the description of these classes, although any properties which are measured may be subject to errors just as they are in other cases.

2.1 Examples of well-defined geographical objects

In developed countries *census geographies* tend to be well defined; even in less developed countries the geographical concepts are generally well defined, if less clearly implemented. They usually consist of a set of regions each with precise boundaries within which specific attributes are enumerated (Openshaw 1995). The areas at the lowest level of enumeration (city blocks, enumeration districts, etc.) are grouped with specific instances of other areas at the same level to make up higher level areas, which in turn are grouped with other specific areas to form a complete and rigid hierarchy (e.g. see Martin, Chapter 6). The attributes to be counted within the areas are typically based on property units, individuals, and households: although the definitions of 'household' may differ between different surveys (Office of National Statistics 1997) and there is rarely any perfect correspondence between households and property units (e.g. houses in multiple occupation), each definition is nevertheless quite transparent and unambiguous. The data collection process in the western world relies on a certain level of cooperation and literacy amongst those being counted, and while there are frequently legal sanctions for non-cooperation these cannot easily be enforced if people are reluctant to cooperate. The primary errors associated with the US Census of Population arise out of underenumeration of groups such as illegal immigrants and the homeless (Bureau of the Census 1982).

A second example of a well-defined geographical phenomenon in western societies is *land ownership*. The concept of private ownership of land is fundamental to these societies; therefore the spatial and attribute interpretation of that concept is normally quite straightforward in its spatial expression. The boundary between land parcels is commonly marked on the ground, and marks an abrupt and total change in ownership. In point of fact, at least in the UK, the surveyed boundary is only deemed indicative of the actual position of the boundary, and so any property boundary has a defined uncertainty in position, otherwise it would require resurveying every time the boundary marker is rebuilt (Dale and McLaughlin 1988). Even in instances of collective ownership in which two groups may own two adjoining parcels and one person may belong to both groups, the question of ownership and responsibility remains clear in law.

Well-defined geographical objects are essentially created by human beings to order the world they occupy. They exist in well-organised and established political and legal realms. Some other objects in our built and natural environments may seem to be well defined, but they tend to be based on a single measurement, and close examination frequently shows the definition to be obscure. For example, the land surface seems well defined, and it should be possible to determine its height above sea level rigorously and to specified precision. But even the position of the ground under our feet is being brought into question. This is caused by the increasing availability of elevation models derived from photogrammetry to sub-centimetre precision, when the actual definition of the land surface being mapped must come into question, and whether the field was ploughed or the grass was cut, become serious issues in defining the so-called land surface. Most, if not all, other geographical phenomena are similarly poorly defined to some extent.

2.2 Examples of poorly-defined geographical objects

In aboriginal societies the concept of ownership is much less clear than in western society. There are many different native cultures, but many have a conception of the land owning the people, and responsibility for nurturing the land is a matter of common trust within a group (Native North Americans and Australians, for example: Young 1992). Areas of responsibility are less well defined, with certain core areas for which a group or an individual may be responsible (e.g. the sacred sites of the Australian Aborigines: Davis and Prescott 1992), and other regions for which no one is actually responsible but many groups may use (so-called ‘frontier zones’). Among both North American and Australian native groups, the spatial extents of these core and peripheral areas have been shown to be well known to the groups concerned, although they may not be marked, precisely located, or fixed over time (Brody 1981; Davis and Prescott 1992). There are therefore acknowledged divisions of space, but the spatial location of the divider may be uncertain. The extent of the zones of uncertainty can be resource dependent, so that when resources are plentiful there may be relatively precise boundaries, and when scarce there may be very diffuse frontiers (Davis and Prescott 1992; Young 1992). Alternatively, ties of kinship between groups may create less specific frontiers, and lack of kinship hard boundaries (Brody 1981). These aboriginal territories have much in common with the documented ‘behavioural neighbourhoods’ of western individuals. Such neighbourhoods are also poorly defined both spatially and temporally: they may be discontinuous and will inevitably overlap with others, and while possibly unique to an individual or family, may nonetheless make up part of a geographical region that is occupied by a group.

Complexity is also inherent in the mapping of vegetation (Foody 1992). The allocation of a patch of woodland to the class of oak woodland, for example – as opposed to any other candidate woodland type – is not necessarily easy. It may be that in that region a threshold percentage of trees need to be oak for the woodland to be considered ‘oak’, but what happens if there is one per cent less than that threshold? Does it really mean anything to say that the woodland needs to be classed to a different category? Indeed, the higher level classification to woodland at all has the same problems. Mapping the vegetation is also problematic since in areas of natural vegetation there are rarely sharp transitions from one vegetation type to another, rather an intergrade zone or ecotone occurs where the dominant vegetation type is in transition (Moraczewski 1993). The ecotone may occupy large tracts of ground. The attribute and spatial assignments may follow rules, and may use indicator species to assist decisions, but strict deterministic rules may trivialise the classification process without generating any deeper meaning.

In discussion of most natural resource information we typically talk about central concepts and transitions or intergrades. Figure 2 shows a scatter plot of some remotely-sensed (LANDSAT) data from Band 3 and Band 5 (which record the amounts of reflected electromagnetic radiation in the wavelength ranges 0.63–0.69 and 1.55–1.75 µm, respectively). This is part of the information used in the assignment of pixels in an image to land covers. The conceptualisation of the land covers is as Boolean objects (discussed below), and yet it is clear from Figure 2 that there are no natural breaks in the distribution of points in the 2-dimensional space shown. This is typical of satellite imagery. Although LANDSAT actually records information in seven spectral bands which can give identification to some natural groups of pixels, the number of identifiable groups very rarely corresponds with the number of land cover types being mapped (Campbell 1987). The classification process involves the identification of prototypical values for land cover types, and the extension of that mapping from the attribute dimensions shown to the spatial context. Conceptually, the same basic process is executed in almost all traditional mapping operations, and the problem of the identification of objects is fundamental. It is apparent from Figure 2 that the intergrades (all possible locations in attribute space which are between the prototype or central concepts) are more commonly and continuously occupied than the prototypical classes.

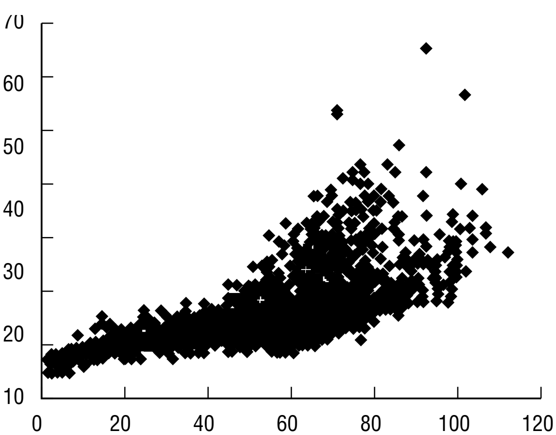

Fig 2. A scatterplot of Bands 3 and 5 of a LANDSAT TM image.

The problem of identification may be extended into locations. Figure 3 shows a soil map of part of the Roujan catchment in France with numbers indicating soil map units (soil types) and the shading indicating the extent of boundary intergrades between types. The width of intergrades is based on the knowledge of soil surveyors who prepared the map (Lagacherie et al 1996).

Within natural resource disciplines the conceptualisation of mappable phenomena and the spaces they occupy is rarely clear cut, and is still more rarely achieved without invoking simplifying assumptions (see also Veregin, Chapter 12). In forestry, for example, tree stands are defined as being clearly separable and mappable; yet trees vary within stands by species density, height, etc., and often the spatial boundary between stands is not well defined (Edwards 1994). Although theorists may recognise the existence of intergrades, the conceptual model of mapping used in this and other natural resource disciplines accepts the simplification and places little importance on them, although the significance has not been assessed. In other areas, such as soil science and vegetation mapping, some of the most interesting areas are at the intergrade, and these are rightly a focus of study in their own right (Burrough 1989; Burrough et al 1992; Lagacherie et al 1996). The interest in intergrades as boundaries is not a preserve of natural resouce scientists, however, and in discussion of urban and political geography considerable attention is paid to these concepts (Prescott 1987; Batty and Longley 1994).

3 ERROR

If an object is conceptualised as being definable in both attribute and spatial dimensions, then it has a *Boolean* occurrence; any location is either part of the object, or it is not. Yet within GIS, for a number of reasons, the assignment of an object or location to the class may be expressed as a probability. There are

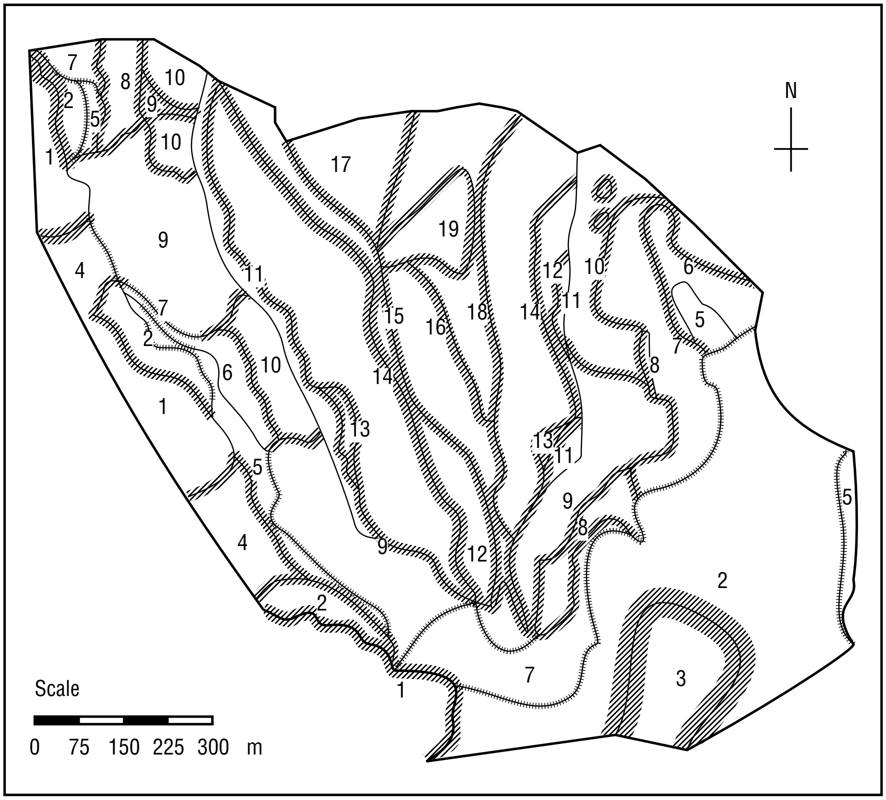

Fig 3. Soil map of the Roujan catchment in France showing the extent of soil intergrades (after Lagacherie et al 1996: 281).

any number of reasons why this might be the case. Three are briefly discussed here:

1 probability owing to error in the measurement;
2 probability because of the frequency of occurrence;
3 probability based on expert opinion.

Errors occur within any database, and for any number of reasons; some reasons are given in Table 1. They are given more complete treatment by Fisher (1991b) and Veregin (Chapter 12). The simplest to handle are those associated with measurement, because well-advanced error analysis procedures have been developed (Heuvelink, Chapter 14; Heuvelink et al 1989; Heuvelink and Burrough 1993; Taylor 1982). If the true value of a property of an object were precisely known, then it would be possible to identify the distribution of 'real world' measurement error by making repeated measurements of the property (which would each differ from the true value by a variable measurement error). It would then be possible to estimate the distribution of the error in its measurement, and thus to develop a full error model of the measurement error. This is, in fact, the basis of the 'root mean square' reporting of error in digital elevation models (see also Beard and Buttenfeld, Chapter 15). Yet there are many instances in which such reductionist measures of error are over-simplistic and aspatial, failing to identify the spatial distribution of the error in GIS-based modelling (Monckton 1994; Walsby 1995).

A further means of describing aspatial error is to create a confusion matrix which shows the cover-type actually present at a location crosstabulated against the cover-type identified in the image classification process. Typically the matrix is generated for a complete image. It reports errors in the allocation of pixels to cover types (Campbell 1987; Congalton and Mead 1983). However, the confusion matrix is of limited use if the precise interpretation of either the classification process or the ground information is not clear cut.

A different view of probability is based on the frequency of the occurrence of a phenomenon. The classic applications of probability in this area include weather and flood forecasting. Floods of a particular height are identified as having a particular return period which translates as a particular probability of a flood of that level occurring.

A third view of probability is as a manifestation of subjective opinion, where an expert states a 'gut feeling' of the likelihood of an event occurring. Much geological and soil mapping is actually the result of Boolean classification of subjective probability, since it is impracticable to observe directly either of these phenomena across the entire countryside: rather inference is made using sampled points such as outcrops and auger borings. Between those locations it is expert opinion as to what is there; so long as a Boolean model of soil and rock occurrence is applied, the map is implicitly a matter of the expert's maximum probability (Clarke and Beckett 1971).

Probability has been studied in mathematics and statistics for hundreds of years. It is well understood, and the essential methods are well documented. There are many more approaches to probability than the three described here. Probability is a subject that is on the syllabus of almost every scientist qualified at degree level, and so it pervades the understanding of uncertainty through many disciplines. It is not, however, the only way to treat uncertainty.

Table 1 Common reasons for a database being in error.

Type of error	*Cause of error*
Measurement	Measurement of a property is erroneous
Assignment	The object is assigned to the wrong class because of measurement error by field, or laboratory scientist, or by surveyor
Class generalisation	Following observation in the field and for reasons of simplicity, the object is grouped with objects possessing somewhat dissimilar properties
Spatial generalisation	Generalisation of the cartographic representation of the object before digitising, including displacement, simplification, etc. (see Weibel and Dutton, Chapter 10)
Entry	Data are miscoded during (electronic or manual) entry to a GIS
Temporal	The object changes character between the time of data collection and of database usage
Processing	In the course of data transformations an error arises because of rounding or algorithm error

4 VAGUENESS

In contrast with error and probability which are steeped in the mathematical and statistical literature, vagueness is the realm of philosophy and logic and has been described as one of the fundamental challanges to those disciplines (Williamson 1994; Sainsbury 1995). It is relatively easy to show that a concept is 'vague', and the classic pedagogic exposition uses the case of the 'bald' man. If a person with no hair at all is considered bald, then is a person with one hair bald? Usually, in any working definition of 'bald', the answer to this would be 'yes'. If a person with one hair is bald, then is a person with two hairs bald: again, 'yes'. If you continue the argument, one hair at a time, then the addition of a single hair never turns a bald man into a man with a full head of hair. On the other hand, you would be very uncomfortable admitting that someone with plenty of hair was bald, since this is illogical (Burrough 1992; Burrough 1996; Zadeh 1965). This is known as the *Sorites Paradox* which, little by little, presents the logical argument that someone with plenty of hair is bald! A number of resolutions to the paradox have been suggested, but the most widely accepted is that the logic employed permits only a Boolean response ('yes' or 'no') to the question. A graded response is not acceptable. And yet there is a degree to which a person can be bald. It is also possible that the initial question is false, because 'bald' would normally be qualified if we were examining it in detail, so we might ask whether someone was 'completely bald', and we might define that as someone with no hair at all. Can we ever be certain that individuals have absolutely no hair on their heads? Furthermore, where on their neck and face is the limit of the head such that we can judge whether there is any hair on it? You are eventually forced to admit that by incremental logical argument, it is impossible to specify whether someone is 'completely', 'absolutely', 'partially', or 'not at all' bald, given a count of hairs on their head, even if the count is absolutely correct. So no matter the precision of the measurement, the allocation to the set of people is inherently vague.

The Sorites Paradox is one way which is commonly used to define vague concepts. If a concept is 'Sorites susceptible', it is vague. Many geographical phenomena are 'Sorites susceptible', including concepts and objects from the natural and built environments (e.g. see Band, Chapter 37). When, exactly, is a house a house; a settlement, a settlement; a city, a city; a podsol, a podsol; an oak woodland, an oak woodland? The questions always revolve around the threshold value of some measurable parameter or the opinion of some individual, expert or otherwise.

Fuzzy set theory was introduced by Zadeh (1965) as an alternative to *Cantor* (*Boolean*) *sets*, and built on the earlier work of Kaplan and Schott (1951). Membership of an object to a Cantor set is absolute, that is it either belongs or it does not, and membership is defined by integer values in the range $\{0,1\}$. By contrast, membership of a fuzzy set is defined by a real number in the range $[0,1]$ (the change in type of brackets indicates the real and integer nature of the number range). Definite membership or non-membership of the set is identified by the terminal values, while all intervening values define an intermediate degree of belonging to the set, so that, for example, a membership of 0.25 reflects a smaller degree of belonging to the set than a membership of 0.5. The object described is less like the central concept of the set.

Fuzzy memberships are commonly identified by one of two methods (Robinson 1988):

1 the *Similarity Relation Model* is data driven and involves searching for patterns within a dataset similarly to traditional clustering and classification methods, the most widespread method being the Fuzzy *c* Means algorithm (Bezdek 1981). More recently, fuzzy neural networks have been employed (Foody 1996);
2 the *Semantic Import Model*, in contrast, is derived from a formula or formulae specified by the user or another expert (Altman 1994; Burrough 1989; Wang et al 1990).

Many studies have applied fuzzy set theory to geographical information processing. There are several good introductions to the application of fuzzy sets in geographical data processing, including books by Leung (1988) and Burrough and Frank (1996) – see also Eastman (Chapter 35).

Fuzzy set theory is now only one of an increasing number of soft set theories (Pawlak 1982), in contrast to hard, Cantor sets. However, a number of authorities consider that fuzzy set theory is mistakenly used for problems which more correctly fall within the realm of subjective probability (Laviolette and Seaman 1994). They have, however, primarily addressed fuzzy logic rather than fuzzy sets, and illustrated their arguments with Boolean conditions and decisions. As such, they have failed to address the nature of the underlying set and any inherent vagueness which may be present, as Zadeh (1980) has shown. Moreover, Kosko (1990) has argued that fuzzy sets are a superset of probability.

5 AMBIGUITY

The concepts and consequences of ambiguity (Figure 1) in geographical information are not well researched. Ambiguity occurs when there is doubt as to how a phenomenon should be classified because of differing perceptions of it. Two types of ambiguity have been recognised, namely *discord* and *non-specificity*. In other areas of study some partial solutions have been suggested, but they are not reviewed here because of the lack of specific research with geographical information.

Within geography the most obvious form of discord through ambiguity is in the conflicting territorial claims of nation states over particular pieces of land. History is filled with this type of ambiguity, and the discord which results. Examples in the modern world include intermittent and ongoing border conflicts and disagreements in Kashmir (between India and China) and the neighbouring Himalayan mountains (between China and India). Similarly, the existence or non-existence of a nation of Kurds is another source of discord. All represent mismatches between the political geography of the nation states and the aspirations of people (Horn, Chapter 67; Prescott 1987; Rumley and Minghi 1991).

As has already been noted, many if not most phenomena in the natural environment are also ill-defined. The inherent complexity in defining soil, for example, is revealed by the fact that many countries have slightly different definitions of what a soil actually constitutes (cf. Avery 1980; Soil Survey Staff 1975), and by the complexity and the volume of literature on attempting to define the spatial and attribute boundaries between soil types (Webster and Oliver 1990; Lagacherie et al 1996). Furthermore, no two national classification schemes have either the same names for soils or the same definitions if they happen to share names. This causes many soil profiles to be assigned to different classes in different schemes, as shown in Table 2 (see also Isbell 1996; Soil Classification Working Group 1991; Soil Survey Staff 1975). Within a single country this is not a problem, yet ambiguity arises in the international efforts to produce supra-national or global soil maps. The individual national classifications cause considerable confusion in the process and the classification scheme becomes part of the national identity within the context. There is also rarely a one-to-one correspondence between classification systems (soil type x in this classification corresponds to soil type a in that), but rather a many-to-many classification (soil types a and b correspond broadly to soil type x, but some profiles of soil type a are also soil types y and z). This leads to different placement of soil boundaries in both attribute and spatial dimensions, and generates considerable problems in mapping soils across international and interstate boundaries (FAO/UNESCO 1990; Campbell et al 1989), as has been exemplified in the creation of the Soil Map of the European Communities (Tavernier and Louis 1984).

Several measures of social deprivation have been suggested which are based upon information from the UK Census of Population (Table 3). Enumeration areas are assigned to one class or another, and the classes have been used in the allocation of resources for a range of social and economic programmes. The fact that there are different bases to the measurement of deprivation means that enumeration areas may be afforded special policy status using one indicator, but not using another, and this is a source of potential discord.

Ambiguity through non-specificity can be illustrated from geographical relations. The relation 'A is north of B' is itself non-specific, because the concept 'north of' can have at least three specific meanings: that A lies on exactly the same line of longitude and towards the north pole from B; that A lies somewhere to the north of a line running east to west through B; or, in common use, that A lies in the sector between perhaps north-east and north-west, but is most likely to lie between north-north-east and north-north-west of B. The first two definitions are precise and specific, but equally valid. The third is the natural language concept which is itself vague. Any lack of definition as to which should be used means that uncertainty arises in interpreting 'north of'.

Arguably, soil classification is a process whereby modern schemes have removed the problem of non-specificity which was inherent in earlier schemes and replaced it by supposedly objective, globally applicable diagnostic criteria. The remaining problems arise out of creating Boolean boundaries in a vague classification environment and the problem of discord.

None of this should be taken to imply that ambiguity is inappropriate or intrinsically 'wrong'. The England and Wales soil classification scheme at the scale of England and Wales is possibly the most relevant classification scheme for the soils in that country. Similarly, the United States Department of Agriculture scheme (Soil Taxonomy) was the best scheme for the US when it was finalised in 1975 (although it does claim a global application). The problem of ambiguity arises when we move to a higher level, and data from the British Soil Survey

Table 2 Alternative soil classification schemes for global and national use.

US Classification (Soil Survey Staff 1975)	*Australian Classification (Isbell 1996)*	*Soil Map of the World (FAO/UNESCO 1990)*		*British Soil Classification (Avery 1980)*
Entisol	Anthroposol	Fluvisol	Kastanozem	Terrestrial raw soil
Inceptisol	Organosol	Gleysol	Chernozem	Hydric raw soil
Spodosol	Podsol	Regosol	Phaeozem	Lithomorphic soil
Mollisol	Hydrosol	Lithosol	Greyzem	Pelosol
Oxisol	Kurosol	Arenosol	Cambisol	Brown soil
Ultisol	Sodosol	Rendzina	Luvisol	Podzolic soil
Alfisol	Chromosol	Ranker	Podzoluvisol	Ground-water gley soil
Aridisol	Calcarosol	Andosol	Podzol	Surface-water gley soil
Histosol	Ferrosol	Vertisol	Planosol	Man-made soil
Vertisol	Dermosol	Solonchak	Acrisol	Peat soil
	Kandosol	Solonetz	Nitosol	
	Rudosol	Yermosol	Ferrasol	
	Tenosol	Xerosol	Histosol	

Table 3 Measures of social deprivation used in the UK, with the associated census variables used in their calculation (Openshaw 1995).

Variable	*Jarman*	*Townsend*	*Department of the Environment*
Unemployment	X	X	X
No car		X	
Unskilled	X		
Overcrowding (more than 1 person per room)	X	X	X
Lacking amenities			X
Not owner occupied		X	
Single-parent household	X		X
Children under 5 years old	X		
Lone pensioners	X		
Ethnic minorities	X		

have to be fused with data from neighbouring countries or countries further afield. In preparing the Soil Map of the European Community, for example, the FAO/UNESCO classification was employed with some amendments.

In a like manner, there is nothing wrong with there being three different methods of defining deprived regions in Britain. Deprivation is a social construct and any quantitative index can only be an approximation which is deemed relevant and acceptable within its own terms of reference. If the constituent attributes of a particular index happen not to be measured in another country, that index simply ceases to have international application. (In fact, with regard to the use of the Jarman Index *within* the UK, housing indicators of deprivation replace ethnic indicators in Wales.) Ambiguity nevertheless does come into play in the allocation of social and economic programme resources, and can lead to contention between local, national, and (in the case of EU programmes) supra-national, politicians over the issue of the basis to financial support.

6 CONTROLLED UNCERTAINTY

Many agencies distribute and allow access to spatial information which is degraded deliberately through creating uncertainty. Two examples of this are discussed (see also Heuvelink, Chapter 14, and Hunter, Chapter 45, for a discussion of the management of uncertainty).

If the exact locations of rare or precious objects such as nesting sites of endangered birds or archaeological sites are recorded in a dataset, any more widely distributed versions may introduce a systematic or random error introduced into the locational component. This may be done by only reporting information for large areal aggregations (e.g. 4 km^2 in the county flora of Leicestershire, England: Primavesi and Evans 1988; and 100 km^2 grid in the state flora of Victoria, Australia, distributed on CD-ROM: Viridians 1996). In some cases both systematic and random elements are introduced in order to protect the phenomenon reported, and although the error may be inconvenient, the consequences of not introducing it may be worse.

Uncertainty is also deliberately introduced into census data in order to preserve confidentiality. If only a few people living within any one enumeration area have a particular characteristic – for example high income – and incomes are reported, it may be very easy to identify exactly which person that is. This is not socially acceptable, and so most census organisations withhold or falsify small counts. For example, in the USA, data for areas with small counts are withheld (Bureau of the Census 1982), whereas in the UK small counts have had a random value between +1 and -1 added (Dewdney 1983).

7 DISTINGUISHING BETWEEN VAGUENESS AND ERROR

Appropriate conceptualisation of uncertainty is a prerequisite to its modelling within GIS. In this section two areas of previous study are examined, and the reasons for the use of either vague or error models of uncertainty are discussed.

7.1 Viewshed

The viewshed is a simple operation within many current GIS, which, in its usual implementation, reports those areas in a landscape which are in view and those which are not (coded 1 and 0 respectively), whether in a triangulated grid or dataset (De Floriani and Magillo, Chapter 38; De Floriani et al 1986; Fisher 1993). Fisher (1991a) has shown how, for a variety of reasons, the visible area is very susceptible to error in the measurement of elevations in the Digital Elevation Model (DEM). (While Fisher used a rectangular grid in his 1991 study, the same would be true for a triangulated model.) The database error is propagated into the binary viewshed because of error in the elevation database (Fisher 1991a) and uncertainty in determination of visibility because of variation between different algorithms (Fisher 1993). Fisher (1992, 1993, 1994) has proposed that it is possible to define the error term from the Root Mean Squared Error (RMSE) for the DEM such that the error has a zero mean and standard deviation equal to the RMSE. This is not in fact true and provides insufficient description of the error for a fully justifiable error model since the mean error may be biased (non-zero) and must have spatial structure. Spatial structure of the error may be identified through spatial autocorrelation measures or full specification of the variogram of the error field (Journel 1996). If the error field is generated using this method then it can be added to the DEM, yielding a revised DEM which includes the known error. If the viewshed is determined over that DEM with error, then a version of the Boolean viewshed is generated. If the process is repeated, then a second version of the Boolean viewshed, a third, a fourth, and so on are generated. If each Boolean viewshed image is coded as 0 and 1 indicating areas which are out-of-sight and in-sight, then using map algebra to find the sum of Boolean viewsheds, a value between 0 and the number of realisations will be found for all locations depending on the number of realisations in which that location is visible. Dividing by the number of realisations will then give an estimate of the probability of that location actually being within the viewshed. The probability of any pixel being visible from the viewing point, or the probability of the land rising above the line of sight somewhere between the viewer and the viewed is given by:

$$p(x_{ij}) = \frac{\sum_{k=1}^{n} x_{ijk}}{n} \tag{1}$$

where

$p(x_{ij})$ is the probability of a cell at row i and column j in the raster image being visible; and
x_{ijk} is the value at the cell of the binary-coded viewshed in realisation k such that k takes values 1 to n.

This is illustrated in Plate 9.

In contrast, using a Semantic Import Model it is possible to define a number of different fuzzy viewsheds (Fisher 1994; note that the term is used incorrectly by Fisher 1992) from a family of equations relating the distance from the viewer to the viewed to the fuzzy membership function (Plate 10). Any number of different circumstances can be described, and two are included here: Equation 2 represents normal atmospheric conditions, and Equation 3 describes the visibility through fog.

$$\mu(x_{ij}) = \begin{cases} 1 & \textit{for } d_{vp \to ij} \le b_1 \\ \dfrac{1}{\left(1+\left(\dfrac{d_{vp \to ij} - b_1}{b_2}\right)^2\right)} & \textit{for } d_{vp \to ij} > b_1 \end{cases} \tag{2}$$

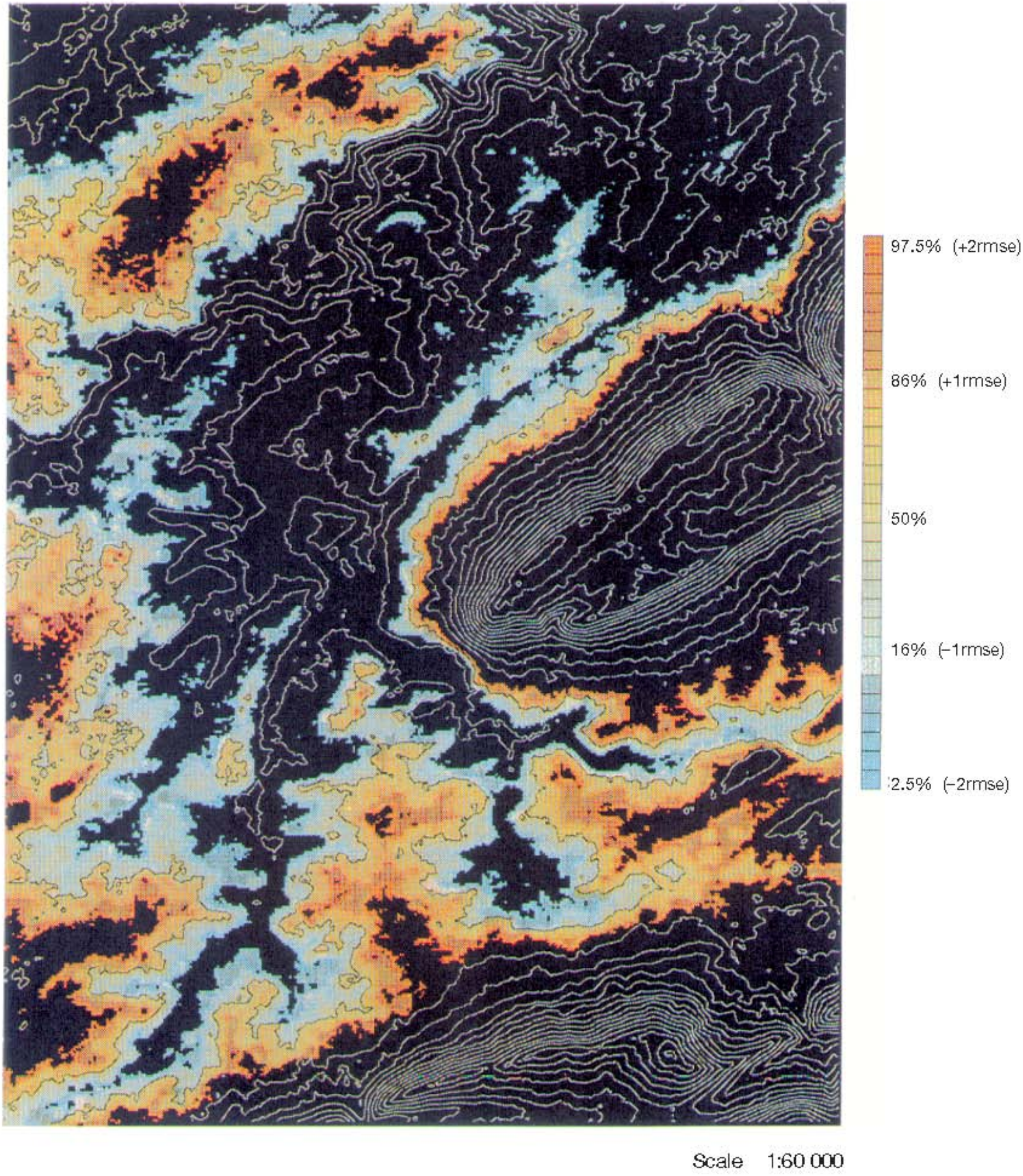

Plate 9 Graphic depiction of the error in a single-elevation value.
(*Source*: Hunter and Goodchild 1995a)

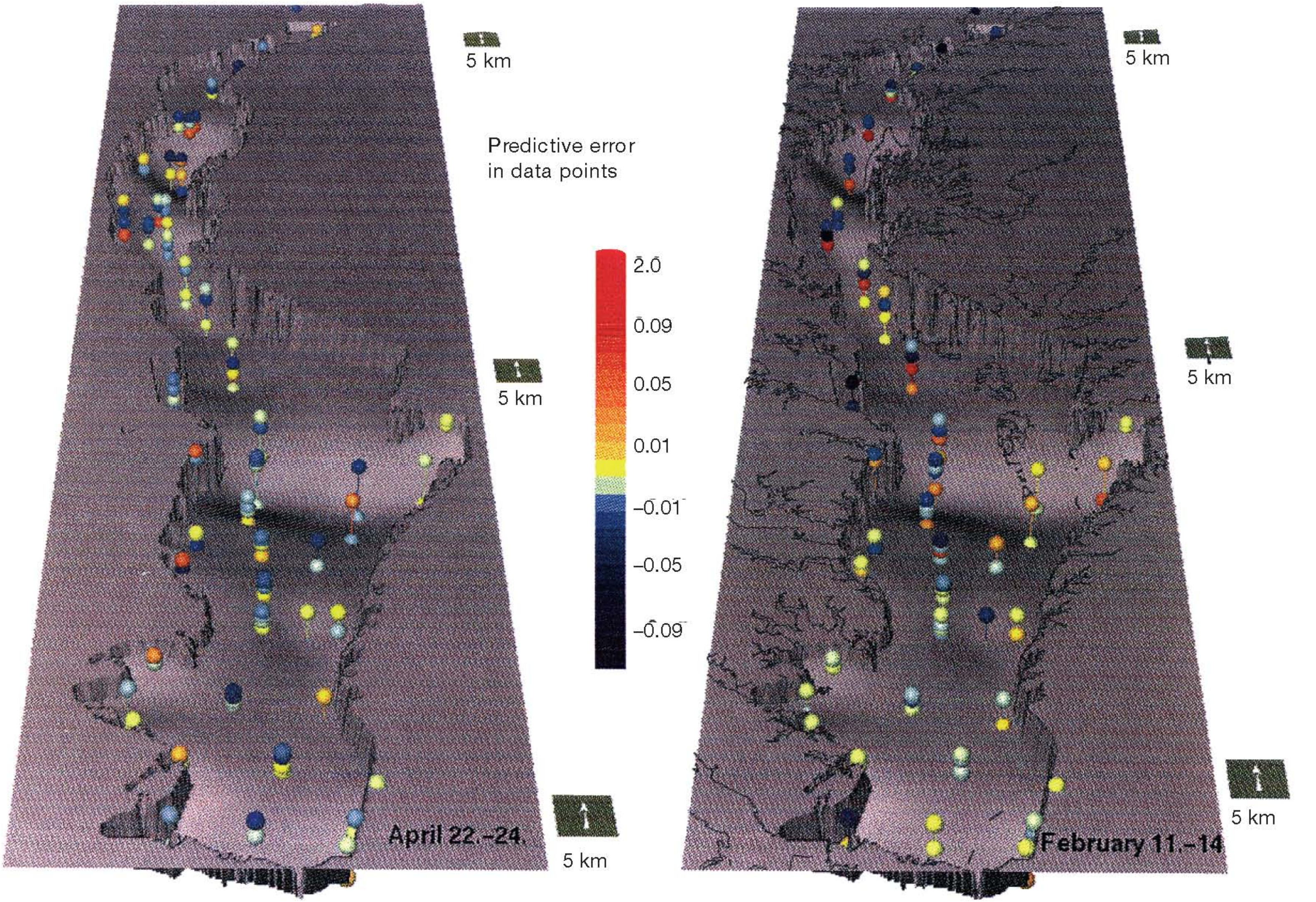

Plate 10 The cross-validation error shown separately from the data in a side-by-side display for two different periods. The cross-validation error is displayed as glyphs (coloured balls on pins) with the colour of each ball representing the error at sampled depths.

(*Source*: Mitasova et al 1995)

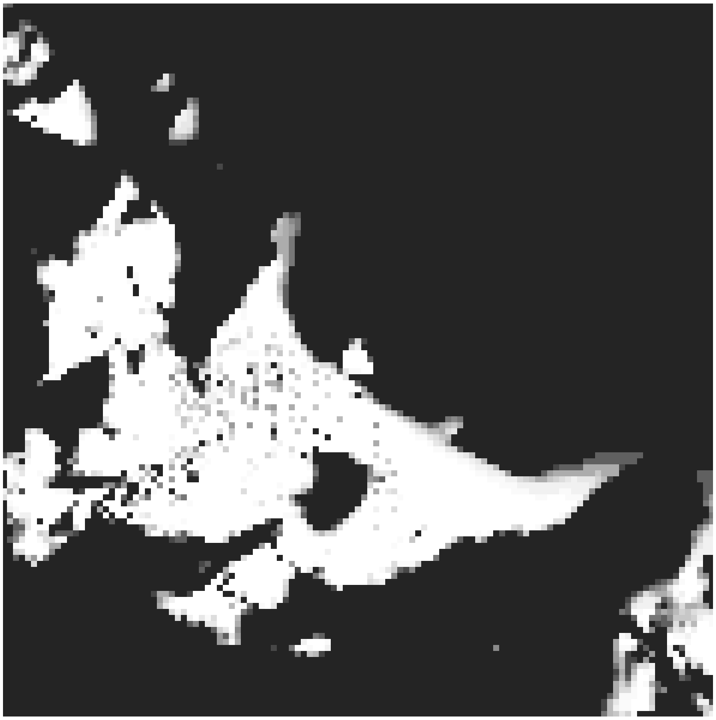

Fig 4. Probable viewshed based on Equation 1.

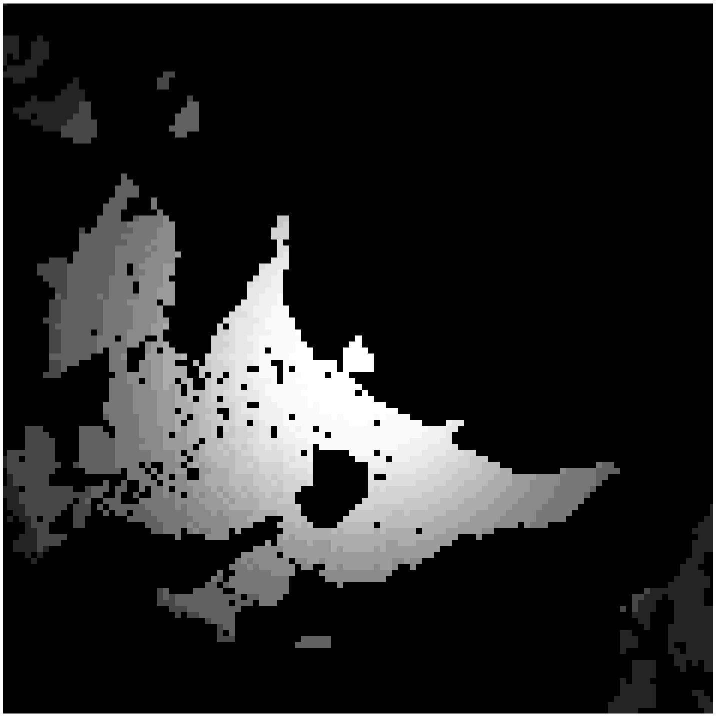

Fig 5. Fuzzy viewshed based on Equation 2.

$$\mu(x_{ij}) = \begin{cases} 1 & for\ d_{vp} \rightarrow {}_{ij} \leq b_1 \\ 0 & for\ d_{vp} \rightarrow {}_{ij} > b_1 + 2 \bullet b_2 \\ \sin\left(\left(\dfrac{d_{vp} \rightarrow {}_{ij} - b_1}{2 \bullet b_2}\right)\dfrac{\pi}{2}\right) & for\ d_{vp} \rightarrow {}_{ij} > b_1 \end{cases} \tag{3}$$

where

$\mu(x_{ij})$ is the fuzzy membership at the cell at row i, column j;
$d_{rp} \rightarrow_{ij}$ is the distance from the viewpoint to row i, column j;
b_1 is the radius of the zone around the viewpoint where the clarity is perfect, and the target object can be seen at the defined level of detail;
b_2 is the distance from b_1 to fuzzy membership of 0.5, sometimes called the cross-over point.

The distinction between fuzzy and probable viewsheds is that the first describes the probability of a location being visible, while the second portrays the degree to which objects can be distinguished. Thus there is an objective definition of the first, and only subjective versions of the second which may describe group or even personal circumstances.

7.2 Remote sensing

Classification of remotely-sensed data has been a major source of land cover and land-use information for GIS. The basic methods, based on a number of discriminant functions from numerical taxonomy, are well known and widely documented (Campbell 1987). The assumptions implicit in this approach are threefold:

1 the cover type itself is a well-defined phenomenon with clear breaks reflected by there being more similarity within cover types than between them;
2 the digital numbers recorded in the original satellite image allow the discrimination of land cover/use types, mapping on a one-to-one basis between reflectance and cover type;
3 the area of the pixel on the ground can be identified as having a single cover type (that area can be assigned to one and only one land-cover or land-use).

From these assumptions it is possible to allow the conceptualisation of both the spatial extent of the pixel and the land cover attributes to be determined as Boolean concepts. Therefore uncertainties can be described by probability, and functional methods such as the maximum likelihood classifier are applicable. Unfortunately, all the assumptions are made for the convenience of the operator, and none matches the actual situation either pragmatically or theoretically.

It is a fact of life that the spatial extent of geographical objects is not coincident with the image pixel, hence the class types on the ground are often hard to define precisely (many are Sorites susceptible), and the digital numbers do not show greater similarity within cover type than between. Therefore, arguably, fuzzy set theory (as an expression of concepts of vagueness) is a more appropriate model for working with satellite imagery and has been the subject of a number of explorations (Foody 1992, 1996; Fisher and Pathirana 1991; Goodchild et al 1994). Both Foody (1992, 1996) and Fisher and Pathirana (1991) have shown that the fuzzy memberships extracted from digital images can be related to the proportion of the cover types within pixels. This can be seen as a step towards a full interpretation of the fuzzy memberships derived from the imagery, since in the work reported the land covers analysed are still well-defined Boolean concepts; the vagueness is introduced by the sensor characteristics (Fisher and Pathirana 1991; Foody 1996). On the other hand, Foody (1992) uses the fuzzy sets to examine a zone of intergrade between vegetation communities, where both the communities and the intergrade are vague concepts.

The confusion between land cover and land use is also problematic (see also Barnsley, Chapter 32). Land use has a socioeconomic dimension to it, which cannot be sensed from satellites. Land cover, on the other hand, pertains to directly observable physical properties of the Earth's surface, and so can be classified directly. Indeed, one reason for the poor results of classification accuracy is the confusion in the conceptualisation of this transformation, and the opacity of the relationship between the surface reflectance of land covers and land use. The most successful attempts at land-use mapping from satellite imagery have adopted rule-based (Wang et al 1991) or graph theoretic approaches to the problem (Barnsley, Chapter 32; Barr and Barnsley 1995), and a combination of fuzzy set theory with these other methods may well further improve the results.

Within remote sensing, it can therefore be seen that the conceptualisation of the problem is the controlling influence. If the assumptions as to the spatial and attribute discrimination of land cover within a pixel noted above are accepted, then there is a Boolean mapping between land cover and digital number which can be extracted by classification, and uncertainty can be expressed probabilistically. If they cannot be accepted, then the Sorites susceptibility of the subjects of mapping indicates their vagueness, and so fuzzy set theory is a more appropriate approach to analysis. A clear conceptualisation of the nature of the phenomenon to be mapped and the approach to be taken is essential to the successful analysis of satellite imagery.

8 CONCLUSION: UNCERTAINTY IN PRACTICE

Through citing a number of different examples, this chapter has argued that within geographical information there are a number of different causes of and approaches to uncertainty. Anyone using uncertain information (i.e. the overwhelming majority of GIS users) needs to think carefully about the possible sources of uncertainty, and how they may be addressed. Uncertainty is a recurrent theme throughout many of the chapters of this book (e.g. Hunter, Chapter 45; Martin, Chapter 6; Raper, Chapter 5); the particular contribution of this chapter is to relate our conceptualisation of the nature of uncertainty to GIS-based data models. Analysis without accommodating data uncertainty (both error and vagueness) can quite severely limit its usefulness. Yet an appropriate conceptualisation of uncertainty and the application of related analytical methods creates a rich analytical environment where decision making based on spatial information is facilitated not only by objective orderings of alternatives but also by giving confidence in those alternatives. New analytical products are beginning to appear as a result of processing, and not ignoring, uncertainty (Burrough 1989; Burrough et al 1992; Davidson et al 1994; Wang et al 1990).

It is crucial to the correct use of geographical information systems that all aspects of uncertainty should be accommodated. This can only be achieved through awareness of the issues and a thorough and correct conceptualisation of uncertainty. The subject of uncertainty in spatial information has developed rapidly, and is still changing, particularly with the increasing use and exploration of alternative, soft set theories (Pawlak 1982).

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