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REPLY

Reply to ''Comment on 'Investigation of the hydrodynamics of flash floods in ephemeral channels: Scaling analysis and simulation using a shock-capturing flow model incorporating the effects of transmission losses' by S.M. Mudd, 2006 (Journal of Hydrology) 324, 65–79'' by Cao and Yue

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Overview

In their comment, Cao and Yue (2006) present a criticism of the momentum source term due to infiltration presented in Mudd (2006). Cao and Yue (2006) argue that because water infiltrating into the bed of a channel during a flash flood has no downstream component of velocity, the momentum sink term due to infiltration described in Mudd (2006) should be equal to zero. In addition, they argue that because rainfall has no downstream velocity component, it too has no effect on momentum, and the source terms that appear in El-Hames and Richards (1998) are also incorrect. Because there are inconsistencies in the terms used in the momentum equations for infiltration and rainfall (e.g., Bras, 1990; Cunge et al., 1980; El-Hames and Richards, 1998; Mudd, 2006), here I address the comments of Cao and Yue (2006) and clarify the role of rainfall at the surface and infiltration into the bed in the momentum equations by depth integrating the Reynolds-averaged Navier–Stokes equations.

Depth integration reveals that the manner in which rainfall and infiltration affect the momentum balance are fundamentally different. Further, the depth integration shows that there are errors in my original manuscript, as well as the rainfall term in El-Hames and Richards (1998), and also the assertion by Cao and Yue (2006) that water entering a channel with no downstream momentum will not affect the momentum balance. Cao and Yue (2006) are correct in stating that the momentum sink due to infiltration presented in Eq . (1b) of Mudd (2006) is incorrect; the scaling analysis of Mudd (2006) should be adjusted accordingly.

Using the newly derived source terms, I repeat the simulations of Mudd (2006) and demonstrate that the results using the corrected momentum equation are negligibly different from simulations using the equations in Mudd (2006). Thus the conclusions presented by Mudd (2006) based on simulations are still fundamentally valid. All future

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simulations, however, should be carried out using the corrected momentum equations because, as demonstrated by Cao and Yue (2006), errors could be significant in channels with lower gradients and more porous sediment than those simulated by Mudd (2006).

Finally, I reiterate and demonstrate the conclusion of Mudd (2006) that infiltration plays a fundamental role in determining the speed of flood propagation; if flash floods are to be accurately predicted infiltration must be taken into consideration.

Depth integration of the Navier–Stokes equations in two Dimensions

In order to derive the source terms in the momentum equation, I begin with the Reynolds averaged Navier–Stokes equations for a flow in the x–z plane, where x is oriented downslope (parallel to the bed) and z is oriented normal to the bed. The continuity equation is

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0, \tag{1}$$

where u is the downstream velocity and w is the velocity normal to the bed. The momentum equations are

$$\frac{\partial u}{\partial t}+\frac{\partial}{\partial x}u^2+\frac{\partial}{\partial z}uw=-\frac{1}{\rho}\frac{\partial p}{\partial x}+g_x+\frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{zx}}{\partial z}\right) \tag{2}$$

and

$$\frac{\partial w}{\partial t}+\frac{\partial}{\partial x}uw+\frac{\partial}{\partial z}w^2=-\frac{1}{\rho}\frac{\partial p}{\partial z}+g_z+\frac{1}{\rho}\left(\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{zz}}{\partial z}\right), \tag{3}$$

where p is the pressure, ρ is the density of the water, τ_{xx}, τ_{zx}, τ_{xz}, and τ_{zz} are turbulent stresses (viscous stresses are assumed to be negligible). Because the coordinate system is tilted such that the x coordinate is oriented parallel to the bed and the z coordinate is oriented perpendicular to the bed, the gravitational components, g_x and g_y are

$$g_x=g\sin\theta,\quad g_z=-g\cos\theta \tag{4}$$

where g is the scalar value of acceleration due to gravity (defined as positive) and theta is the angle between a horizontal surface and the bed (e.g., the bed slope S is equal to tan θ). The bed angle, θ, is defined as always positive and less than $\pi/2$.

Scaling of Eqs. (1)–(3) shows that the terms involving τ_{zx}, τ_{xz}, and τ_{zz} may be neglected (e.g., Furbish, 1997). In addition, scaling analysis also indicates that the terms to the left of the equality in Eq. (3) are negligible. These assumptions reduce Eq. (3) to the hydrostatic approximation

$$\frac{\partial p}{\partial z}=-\rho g\cos\theta. \tag{5}$$

Eqs. (1), (2) and (5) are depth integrated from the channel bed, located at elevation $z=\eta$ to the water surface at elevation $z=\zeta$. Depth integration proceeds using both the Leibniz integral rule and the mean value theorem (e.g., Vreugdenhil, 1994), so, for example, depth integration of the first term in Eq. (1) proceeds as

$$\int_\eta^\zeta \frac{\partial u}{\partial x}\,dz=\frac{\partial}{\partial x}\int_\eta^\zeta u\,dz-u_\zeta\frac{\partial \zeta}{\partial x}+u_\eta\frac{\partial \eta}{\partial x}, \tag{6}$$

and

$$\int_\eta^\zeta u\,dz=\frac{1}{\zeta-\eta}\bar{u}=\frac{\bar{u}}{h}, \tag{7}$$

where u_ζ and u_η are the downstream velocities evaluated at the water surface and the bed, respectively, the overbar denotes a depth averaged quantity, and h is the water depth.

Using this procedure, depth integration of Eqs. (1) and (2) results in

$$\frac{\partial h\bar{u}}{\partial x}-u_\zeta\frac{\partial \zeta}{\partial x}+u_\eta\frac{\partial \eta}{\partial x}+w_\zeta-w_\eta=0. \tag{8}$$

At this point the appropriate kinematic boundary conditions (e.g., Crank, 1984; Vreugdenhil, 1994) must be applied at the surface and the bed. At the surface, the kinematic boundary condition is

$$\frac{\partial \zeta}{\partial t}=w_\zeta-u_\zeta\frac{\partial \zeta}{\partial x}+r\cos(\theta), \tag{9}$$

where r is the rate of rainfall in dimensions L/T (that is, the volume of rainfall per time per unit area of channel) and the factor of $\cos(\theta)$ is due to the inclined coordinate system in which θ is the angle between the channel bed and horizontal. If the bed is considered to be stationary during the flood, then $\partial\eta/\partial t=0$, and since $h=\zeta-\eta$ then $\partial\zeta/\partial t=\partial h/\partial t$. During flash floods in ephemeral channels, there is vertical infiltration into the bed, such that the vertical velocity at the bed is nonzero. At this point I consider the assumption that infiltration is dominantly in the vertical direction, as suggested by Cao and Yue (2006). At the bed, the z-direction velocity w_η is equal to the infiltration rate f multiplied by a factor of $\cos(\theta)$ due to the inclined coordinate system:

$$w_\eta=-f\cos(\theta). \tag{10}$$

Note that the infiltration rate f is defined as positive down (e.g., positive for infiltration and negative for exfiltration). Typically, the no-slip condition is invoked at the base of the flow, such that u_η is equal to zero. If, however, the channel is inclined and infiltration is in the vertical direction there will be a component of streamwise velocity at the channel bed due to infiltration:

$$u_\eta=f\sin(\theta). \tag{11}$$

Whereas the velocity at the bed is nonzero in the inclined coordinate system, the change in bed elevation with respect to downstream distance ($\partial\eta/\partial x$) is, by definition, zero, such that the third term in Eq. (8) is equal to zero. If Eqs. (9)–(11) are inserted into Eq. (8), and the bed is considered stationary during the flood, then the continuity equation becomes

$$\frac{\partial h}{\partial t}+\frac{\partial h\bar{u}}{\partial x}=(r-f)\cos(\theta). \tag{12}$$

If Eq. (12) is multiplied by the channel width (assuming a rectangular channel), the resulting equation is

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=(R-q)\cos(\theta), \tag{13}$$

where A is the cross sectional area, Q is the discharge, R is the rainfall rate in dimensions L^2/T (that is, the volume of rainfall per time per unit length of channel), and q is the infiltration rate in dimensions L^2/T (the volume of infiltrated water per time per unit length of channel).

The momentum equation is closed using the kinematic boundary conditions, the assumption of a stationary bed, and the hydrostatic approximation. In addition, it is assumed that the shear stress at the surface is zero and the shear stress at the channel bed is approximated by a friction slope term

$$\tau_{xz}|_{\eta} = \rho g h S_{\mathrm{f}}, \tag{14}$$

where S_{f} is the friction slope. Finally, it is assumed that the integration of the second term in Eq. (2) is approximated by

$$\frac{\partial}{\partial x}\int_{\eta}^{\zeta} u^2\,\mathrm{d}z = h\bar{u}^2. \tag{15}$$

Eq. (15) is equivalent to assuming the Boussenesq coefficient (see Cao and Yue, 2006) is equal to unity. These approximations result in the following momentum equation:

$$\frac{\partial h\bar{u}}{\partial t} + \frac{\partial h\bar{u}^2}{\partial x} - u_{\eta}w_{\eta} - r\cos(\theta)u_{\zeta} = -ghS_{\mathrm{f}} - g\cos(\theta)h\frac{\partial \zeta}{\partial x} + gh\sin(\theta). \tag{16}$$

Eqs. (10) and (11) are then inserted into Eq. (15):

$$\frac{\partial h\bar{u}}{\partial t} + \frac{\partial h\bar{u}^2}{\partial x} + f^2\cos(\theta)\sin(\theta) - r\cos(\theta)u_{\zeta} = gh\sin(\theta) - ghS_{\mathrm{f}} - g\cos(\theta)h\frac{\partial \zeta}{\partial x}. \tag{17}$$

The hydrostatic term (the second term to the right of the equality in Eq. (17)) may be rewritten as 0.5 $g\cos(\theta)\,\partial h^2/\partial x$ by noting that $h = \zeta - \eta$ and $\partial\eta/\partial x = 0$ due to the inclined coordinate system. If Eq. (17) is multiplied by the channel width, it results in

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A} + g\cos(\theta)\frac{A^2}{2b}\right) = gA(\sin(\theta) - S_{\mathrm{f}}) - \frac{q^2}{b}\cos(\theta)\sin(\theta) + R\cos(\theta)u_{\zeta} \tag{18}$$

where b is the channel width. Note that Eq. (18) assumes a rectangular channel and no downstream changes in the channel width. In channels with gentle gradients, it is commonly assumed that $\sin(\theta)$ is equal to the channel slope (S) and $\cos(\theta)$ is equal to unity. In this case, Eq. (18) becomes

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A} + g\frac{A^2}{2b}\right) = gA(S - S_{\mathrm{f}}) - \frac{q^2}{b}S + Ru_{\zeta}. \tag{19}$$

Alternatively, Eq. (12) may be multiplied by the mean velocity and inserted into Eq. (16). Assuming a gentle channel gradient and performing some algebra this yields

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + g\frac{\partial h}{\partial x} = g(S - S_{\mathrm{f}}) - \frac{\bar{u}}{h}(r - f) + \frac{ru_{\zeta}}{h} - \frac{f^2 S}{h}. \tag{20}$$

As noted by Cao and Yue (2006), the source term due to infiltration in equation (1b) of Mudd (2006) is incorrect. This equation should be replaced by Eq. (19). Eq. (20) is of the same form as Eq. (5) in El-Hames and Richards (1998), although with additional source terms for both rainfall and infiltration (note that the final term in Eq. (5) of El-Hames and Richards (1998) should be preceded by a leading plus sign). With the exception of cases where the channel is steep and the bed sediment is extremely porous, the source term in Eq. (19) that is related to infiltration (q^2S/b) will be negligible. The term (f^2S/h) in Eq. (20) will also be negligible in most cases. In Eqs. (19) and (20), however, rainfall terms cannot be neglected. These terms are introduced during the depth integration process (e.g., Crank, 1984) due to

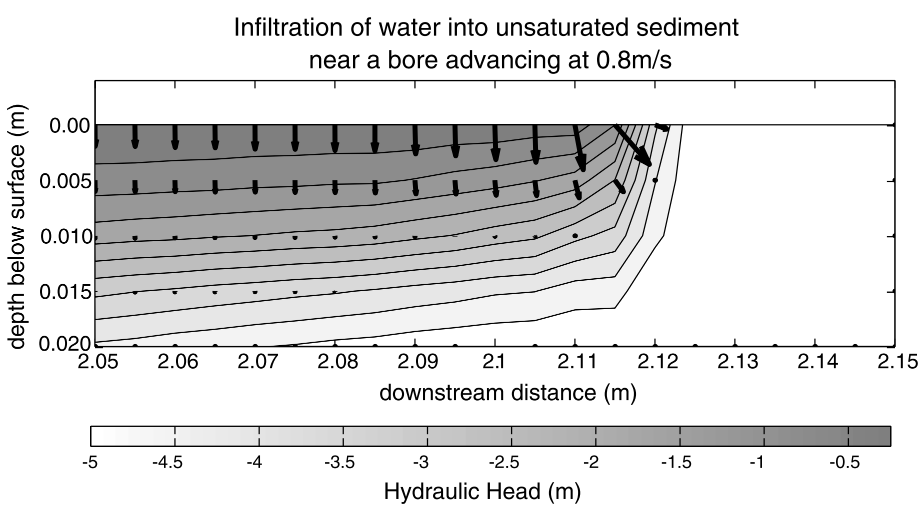

Figure 1 Simulation of the 2-D flow field in an unsaturated sediment near an advancing flood bore. Vadose zone is simulated using a finite element discritization of the Richards' equation following the method of Celia et al. (1990). The shock is modeled using a step change in hydraulic head at the surface of the sediment from −5.0 m to 0.5 m; this shock advances downslope at a speed of 0.8 m/s. The bed sediment is inclined at a slope of 0.01; this grid has been rotated for plotting such that the coordinate system has longitudinal coordinate that is parallel to the bed surface and a vertical coordinate that is perpendicular to the bed surface. Shading shows the spatial variation in hydraulic head, arrows plot the direction of groundwater flux (in the rotated coordinate system), the size of the arrows is proportional to the flux. Sediment characteristics are the same as those used in simulations in Mudd (2006).

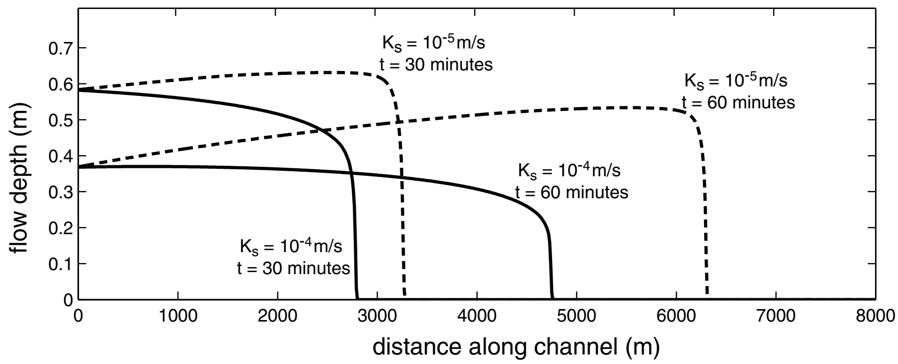

Figure 2 Two floods with identical triangular inflow hydrographs with a peak discharge of 6 m^3/s. Both floods are in channels with a bed slope of 0.01 and a channel width of 4.0 m. Roughness characteristics are the same as those used in simulations in Mudd (2006). The solitary difference between the two floods in this figure is the saturated hydraulic conductivity of the bed sediment.

the kinematic boundary condition at the water surface. If the downslope velocity of the flow is nonzero at this boundary, the momentum source terms introduced by the kinematic boundary conditions will be nonzero; thus fluid with zero downslope velocity (such as vertical rainfall) will affect the momentum balance of the flow.

An important region where the assumption of vertical groundwater flow used to arrive at Eq. (19) may not be applicable is near the flood bore. The bore will cause a steep gradient in the hydraulic head in the downslope direction; this may lead to greater downstream flow velocities at the bore leading to greater momentum losses (e.g., Fig. 1). Investigation of this effect, however, is beyond the scope of this reply.

Simulations

The simulations performed in Mudd (2006) were rerun using the updated source terms. In Mudd (2006), the simulations were performed in channels with a gradient of 0.01, which is a typical bed slope in upland reaches. As described by Cao and Yue (2006), and confirmed by repeat simulations of the model of Mudd (2006) using the corrected source term, at slopes of 0.01 the corrected source term predicts floods that have negligibly different behavior than the original simulations. For example, a flood with a channel width of 4 m and a peak discharge of 6 m^3/s will have traveled only ~5 m farther than floods predicted with the original source term after 80 min of flow (over a total flow distance of ~6.5 km, an error of less than 0.1%). The curves for the old and updated simulations are indistinguishable at the scale of figures plotted in Mudd (2006). Cao and Yue (2006) also identified a sign error in the equations of Mudd (2006) in the friction slope term; this error appeared only in the text and did not affect the original simulations. Although Cao and Yue (2006) have identified an important error in the source term due to infiltration in Mudd (2006), simulation results using the corrected source terms are essentially the same and I can report with confidence that all conclusions drawn from the simulations in Mudd (2006) are still valid. As described by Cao and Yue (2006), however, any future simulations, particularly at lower bed slope and with less permeable bed sediment, should be performed using Eq. (19).

Conclusion

Cao and Yue (2006) identified an erroneous source term that was attributed to infiltration in the momentum equation of Mudd (2006). Cao and Yue (2006) are correct in asserting that in most cases this term will be negligible; however it is not the case that this source is zero because water entering or exiting the channel has zero or near zero downstream velocity. This situation is appropriate for infiltration at the bed, but not for rainfall onto the water surface, as revealed by the depth integration of the Reynolds averaged Navier–Stokes equations presented in this reply. Simulations performed using the corrected equations reveal negligible differences between floods predicted using the incorrect source term in Mudd (2006) and the corrected source term derived here; thus all conclusions based on the simulations in Mudd (2006) are still valid.

Mudd (2006) concluded that the sink term due to infiltration in the momentum equation presented could be significant; this is revealed to be incorrect, as pointed out by Cao and Yue (2006). It is imperative; however, to reiterate the fundamental conclusion of Mudd (2006) that infiltration must be simulated in order to correctly predict the passage of a flash flood, this can be seen in Eq. (20). Eq. (20) shows that the velocity of the flood (or momentum, when multiplied by density) is affected by infiltration. In addition, infiltration also leads to shallower flows, which then results in more flow resistance. Thus floods with the same inflow hydrographs in channels with the same geometry will propagate through the channel significantly slower if the bed material is more permeable (Fig. 2).

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