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Applying Different Normalisation Models to Improve Core Flow Inversion

1. Introduction

The Earth's magnetic field is generated by fluid motion of liquid iron in the outer core. Flows at the top of the outer core are believed to be responsible for the secular variation (SV) observed at the surface of the Earth. Modelling of this flow is open to considerable ambiguity, though methods adopting different physical assumptions do lead to similar flow velocity regimes. The aim of the project is to develop methods that predict the SV more precisely than previous methods (such as non-linear extrapolation).

This project will investigate the use of different ‘best-fit’ or ‘minimisation’ methods with core flow models based on past records of SV, in order to produce a better prediction of the time evolution of the field over the short term (i.e. 5-10 years). The project will initially test if one-norm minimisation models better describe the secular variation than the current two-norm models. Here we present the preliminary results of flows generated using the dataset modelled by Wardinski (2005).

2. Method

It has been suggested that the use of a 1-norm measure of misfit of the model data to secular variation should be used to infer core fluid flow. It has previously been assumed that the noise within observations of the magnetic field has a Gaussian normal distribution. However, empirical evidence appears to show that the noise follows a Laplacian (double exponential) distribution [Walker and Jackson (2000)].

This motivates the use of a one-norm measure of the misfit of weighted residuals between observations and model estimates. Following Whaler *et al.*(2004), the one-norm model solves the equation of the form:

$$(\mathbf{C}^{\mathbf{T}}\mathbf{E}^{\mathbf{T}}\mathbf{R}\mathbf{E}\mathbf{C} + \lambda\mathbf{D})\mathbf{m}^{\mathbf{k+1}} = \mathbf{C}^{\mathbf{T}}\mathbf{E}^{\mathbf{T}}\mathbf{R}\mathbf{E}\mathbf{y}$$

where there is a linear relation between the spherical harmonic coefficients of the flow and secular variation observations. **C** is the equations of condition matrix, **E** is diagonal with the inverse of the standard deviations along the diagonal, λ is the damping parameter, **D** is the regularization matrix, and **y** the data vector. $\mathbf{m}^{k+1}$ is the *(k+1)*th solution iteration. The solution is non-linear because the diagonal misfit matrix **R** has elements $\sqrt{2}/|\varepsilon_i|$, where ε_i is the normalised residual to the *i*th datum. The system becomes ill-conditioned if the residuals are very small; any normalised residual smaller than 0.01 are reset to that value. The MATLAB backslash routine was used in place of a Cholesky decomposition and back substitution. This was verified as equivalent to the previous method used in Whaler *et al.*, to within machine precision.

Note that in the two-norm model, **R** becomes the identity matrix and only one iteration is required to solve the system. This is equivalent to the least-squares solution.

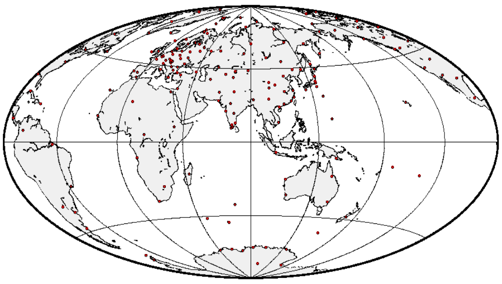

Figure 1: Magnetic Observatory Distribution – 172 (red) points

3. Results

All flows have been calculated with the GUFM main field model with data from the 1990 epoch. Magnetic field observatory data has been adapted from Wardinski (2005). There are 172 data points (**Figure 1**) unevenly spaced across the globe. The flows are calculated up to degree and order 14. A steady flow has been assumed.

Figures 2 and **3**, were generated using a damping parameter, $\lambda = 0.001$, for each flow inversion. The RMS velocity for the two-norm solution is 36.15 km/yr. The solution norm is: $5.28*10^6$. For the one-norm solution, the solution converged after approximately 20 iterations. The RMS velocity is 15.68 km/yr. The solution norm is: $2.56*10^5$.

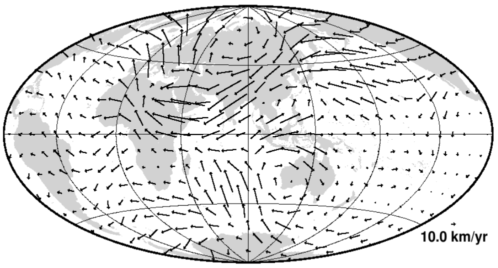

Figure 2:

Two Norm minimisation

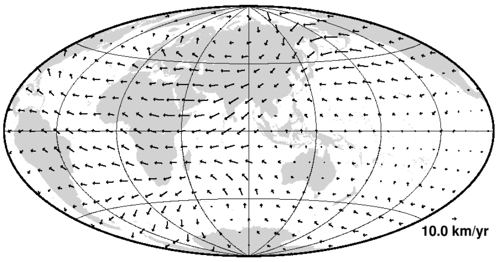

Figure 3:

One Norm minimisation

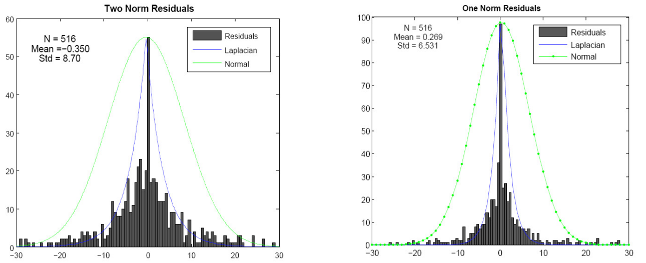

Figure 4: Two Norm and One Norm minimisation Residuals with Laplacian and Normal Distribution probability density curves

4. Discussion / Future Work

The one-norm solution shows a distinctly slower rate of flow than the two-norm solution. The smooth westward drift is the most prominent feature of the one-norm solution. The two-norm solution contains a number of large structures which are not visible in the one-norm solution.

The residuals of both solutions (**Figure 4**) show a better fit to the Laplacian Distibution than the Gaussian. This can justify the use of a one-norm minimisation solution.

The code for computing the one and two-norm flow has been updated from FORTRAN to MATLAB. The code will be further developed to investigate the differences between the one-norm and two-norm estimates of flow. The next objective is to incorporate data from range of time-steps to model flow evolution. Prediction of the flow (and hence SV) both backward and forward in time will be the primary objective for the future.

5. References:

- Wardinski, I., 2005, PhD Thesis, *Core Surface Flow Models from Decadal and Sub decadal Secular Variation of the Main Geomagnetic Field,* GFZ Postdam.
- Walker, M. R. and Jackson, A., 2000. Robust modelling of the Earth’s magnetic field, *Geophys. J. Int.*, 143, 799-808.
- Whaler, K, Holme, R., Jackson, A., 2004, One-norm modelling of flow at the core-mantle boundary, Poster at SEDI Conference 2004