Analytical solutions for assessing pressure buildup during CO_2 injection in geological reservoirs

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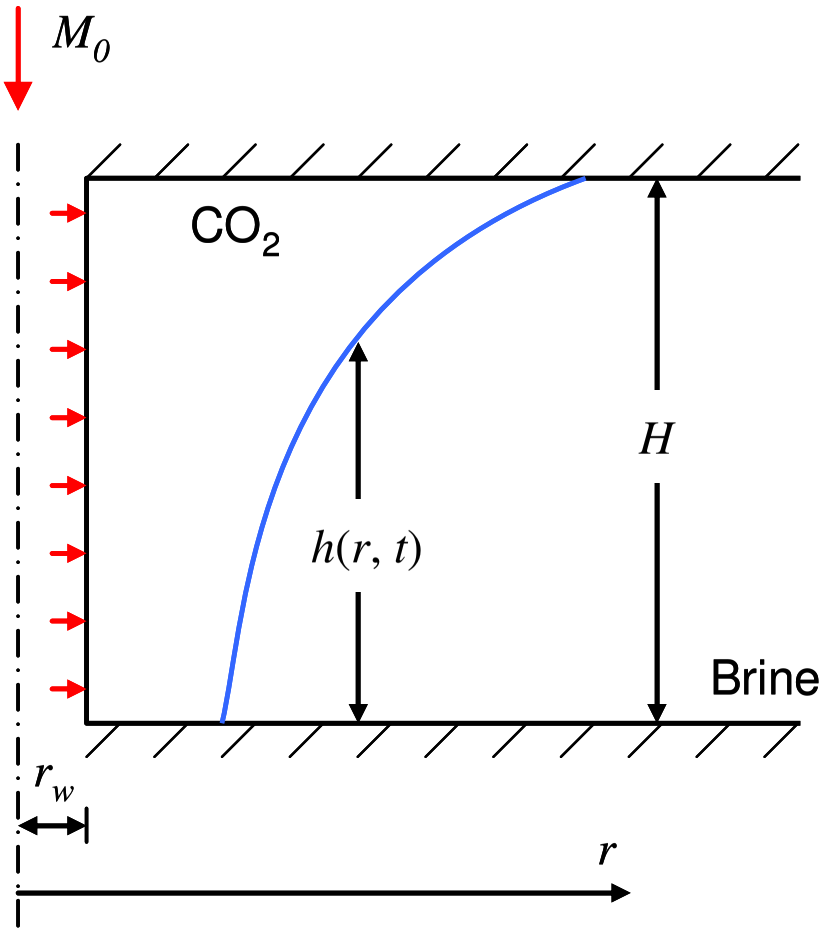
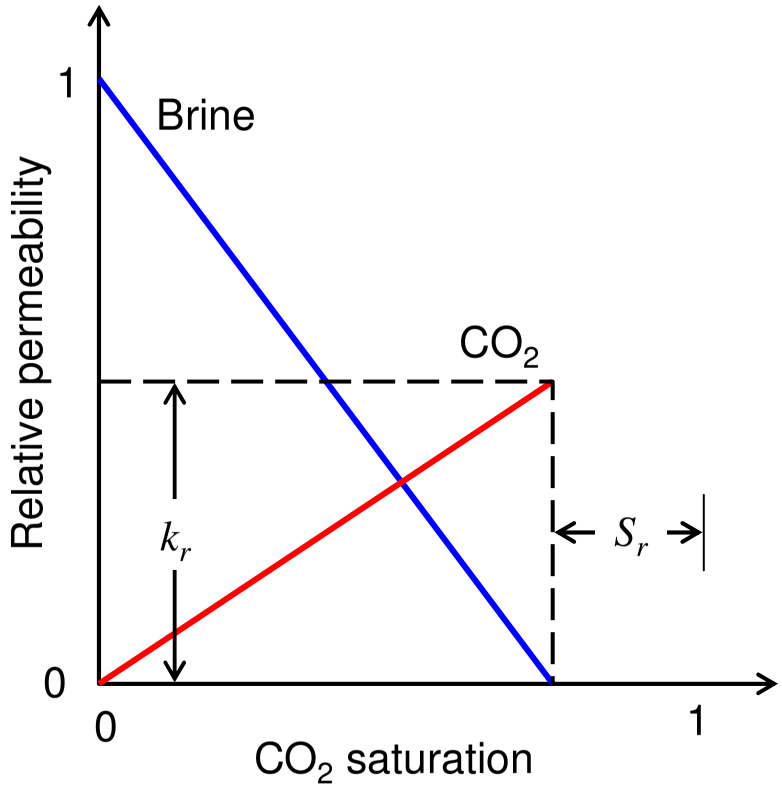
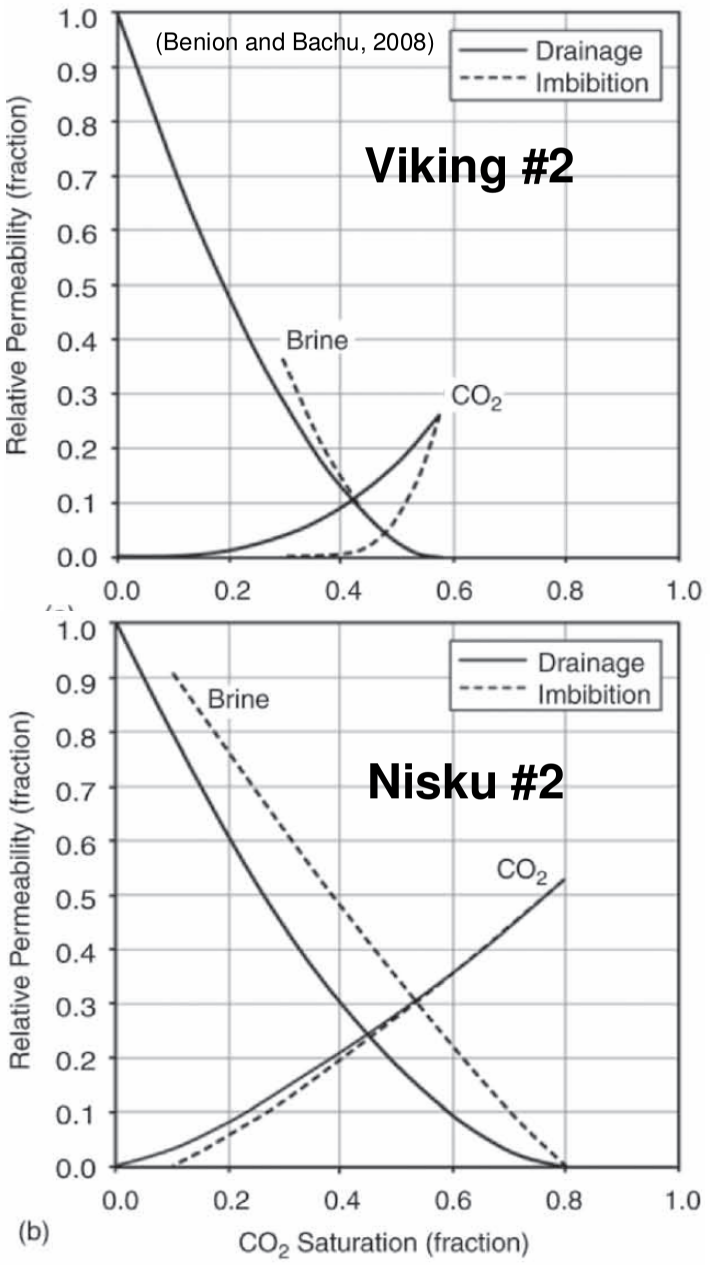
Approximate solution for pressure-buildup during CO2 geo-sequestration

- We consider a fluid pressure, p [$ML^{-1}T^{-2}$] that includes an assumption of negligible capillary pressure, and which applies over the entire thickness of a confined porous formation of vertical extent H [L].
- The CO_2 and brine are assumed to be separated by a sharp interface, located at an elevation h [L] above the base of the formation.
- The CO_2 zone is fully saturated with CO_2 whilst the brine zone is fully saturated with brine.



What does the relative permeability look like?





Key parameters

Dimensional analysis reveals that there are three important dimensionless parameter groups:

$$\alpha = \frac{M_0 \mu_o (c_r + c_w)}{2\pi(1 - S_r) H \rho_o k_r k} \qquad \beta = \frac{M_0 k_r k b}{2\pi H r_w \mu_o} \qquad \gamma = \frac{\mu_o}{k_r \mu_w}$$

compressibility parameter

inertial parameter

viscosity ratio

b = Forchheimer parameter [L^{-1}]

c_r = compressibility of formation [$M^{-1}LT^2$]

c_w = compressibility of brine [$M^{-1}LT^2$]

H = formation thickness [L]

k = permeability [L^2]

S_r = residual brine saturation [-]

M_0 = mass injection rate [MT^{-1}]

μ_o = viscosity of CO_2 [$ML^{-1}T^{-1}$]

μ_w = viscosity of brine [$ML^{-1}T^{-1}$]

ρ_o = density of CO_2 [ML^{-3}]

ρ_w = density of brine [ML^{-3}]

k_r = relative permeability [-]

Large time solution

$$p_D \approx -\frac{1}{2}\ln\left(\frac{x}{2\gamma}\right) - 1 + \frac{1}{\gamma} - \frac{1}{2\gamma}\left[\ln\left(\frac{\alpha}{2\gamma^2}\right) + 0.5772\right] + \frac{\beta}{r_D}$$

Microsoft Excel - WorkSheetR4a.xls

	A	B	C	D	E	F	G
1	Aquifer unit	Madison Group			Lower Cretaceous		
3	Site	A	B	C	A	B	C
5	Depth to top, D (ft)	9000	3500	7000	2000	2000	6000
6	Aquifer thickness, H (ft)	1700	450	650	1600	3200	2600
7	Transmissivity, T (ft^2/day)	320	200	960	2200	4500	3700
8	Hydraulic head (ft asl)	2900	1850	3900	1750	1800	2800
9	Ground elevation (ft asl)	2000	1900	4600	2000	1900	4600
16	Temperature, T_c (^oC)	75	35	100	55	30	80
18	Salt concentration, C (kg/l)	0.3	0.3	0.3	0.01	0.01	0.01
32	Poisson's ratio, ν (-)	0.25	0.25	0.25	0.25	0.25	0.25
33	Biot coefficient, η (-)	1	1	1	1	1	1
34	Density of rock, ρ_s (kg/m^3)	2260	2260	2260	2260	2260	2260
35	Vertical stress, σ_v (MPa)	61	24	47	14	14	41
36	Horizontal stress, σ_h (MPa)	44	16	31	8	8	22
38	Injection rate, M_0 (t/day)	4000	8000	7000	4000	8000	7000
39	Time, t (years)	50	50	50	50	50	50
40	Porosity, f (-)	0.1	0.1	0.1	0.1	0.1	0.1
41	Formation compressibility, c_r (GPa^{-1})	0.45	0.45	0.45	0.45	0.45	0.45
42	Well radius, r_w (m)	0.1	0.1	0.1	0.1	0.1	0.1

$$h_D = h / H$$

$$r_D = r / r_w$$

$$t_D = \frac{M_0 t}{2\pi(1-S_r)\phi H r_w^2 \rho_o}$$

$$x = r_D^2 / t_D$$

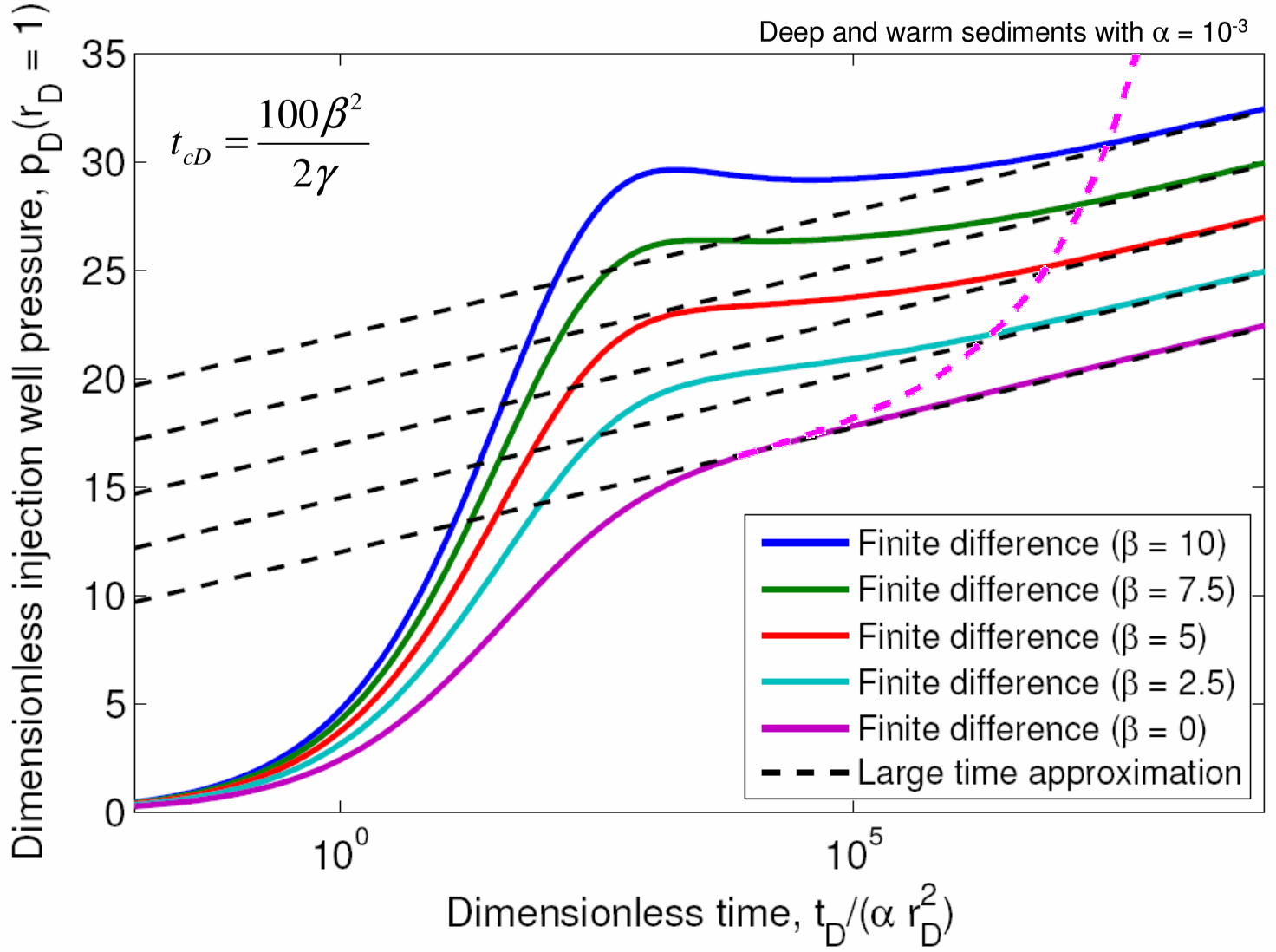
$$p_D = \frac{2\pi H \rho_o k p}{M_0 \mu_o}$$

$$\alpha = \frac{M_0 \mu_o (c_r + c_w)}{2\pi(1-S_r) H \rho_o k_r k}$$

$$\beta = \frac{M_0 k_r k b}{2\pi H r_w \mu_o}$$

$$\gamma = \frac{\mu_o}{k_r \mu_w}$$

Pressure buildup



Relevant references

Mathias, SA, Hardisty, PE, Trudell, MR & Zimmerman, RW 2009. Approximate Solutions for Pressure Buildup During CO_2 Injection in Brine Aquifers. *Transport in Porous Media* **79**(2): 265-284. (View publication online)

Mathias, SA, Hardisty, PE, Trudell, MR & Zimmerman, RW 2009. Screening and selection of sites for CO_2 sequestration based on pressure buildup. *International Journal of Greenhouse Gas Control* **3**(5): 577-585. (View publication online)